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MECHATRONIC REDESIGN OF SLIDER CRANK MECHANISM

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ABSTRACT
Mechatronic design efforts have been and continue to be heavily investigated in the development of robotic manipulator arms. However, little effort has been devoted to mechatronic redesign of traditional two-dimensional mechanisms which mechanical engineers get exposure to when they study subjects such as kinematics and mechanism design.

In this paper a feasibility study for controlling the motion of the popular slider crank mechanism with appropriate sensing and actuation is elaborated. The results indicate that a variety of motion profiles can be derived from the same mechanism without involving any mechanical redesign. Many of the control approaches that have been heavily investigated in the field of robotics are readily applicable to such mechanisms. The synergistic combination of mechanical design, soft computing, sensing, instrumentation, and control is likely to bring about unprecedented versatility and performance levels in the hardware realization of machines based on these mechanisms.

INTRODUCTION
There has been a growing number of applications and product development at the interfaces of traditional disciplinary boundaries of mechanical, electronics and computer engineering fields within the broad area of engineering education in recent years. This has necessitated the development of a new and interdisciplinary field of pedagogy - "Mechatronics".

"Mechatronics" synergistically combines engineering mechanics, electronics, sensors, actuators, soft computing and control [1,2].

Traditionally, mechanisms, linkages and cams are designed with the assumption that "input motion" will be provided by a constant velocity motor [3,4]. The design involves developing a mechanical arrangement that will provide the desired "output motion" utilizing a motor running at a set number of revolutions per minute (rpm). Any change of desired "output motion" therefore required the development of a new mechanical device with this approach. Mechatronic redesign not only allows these mechanisms to enhance performance but to exhibit some degree of flexibility to adapt to varying of task requirements.

The slider crank mechanism is used in a variety of machines when there is a need to convert rotary motion into reciprocating motion and vice versa. As with most other mechanisms the input motion to a slider crank mechanism is typically from a motor or other rotating device running at constant rpm. The resulting motion is therefore a reciprocating motion of the slider with every cycle of the crank. Also the forward and return motion of the slider has the same duration. An educational project involving the study of dynamic model and motion characteristics of such a device has been reported in reference [5]. For all requirements involving slow forward motion (such as, may be necessary for cutting action, photocopying etc.) and a faster return motion (no action, just to reset quickly to save time before the next action stroke), traditional approach has been a complete mechanical redesign resulting in what are popularly called "Quick Return Mechanisms". Offset slider-crank, Drag link, Whitworth, Crank Shaper etc.[3] are some examples of such mechanisms.

Traditionally no–effort has been made to integrate appropriate sensors, to determine the crank angle, and appropriate controllable actuator that can apply variable torque to the crank, with such mechanisms. Such "Mechatronic redesign" with appropriate sensing and control action enables the traditional slider crank mechanism to perform a variety of motion patterns.

SLIDER MOTION PROFILES

The following motion characteristics have been simulated with a slider crank mechanism in the present study:

(i)Desired Cartesian space motion of the slider with dwell times at the end of forward and return strokes.

(ii) Desired duration forward motion of the slider blended with a quick return motion with or without the "dwell period" in between.

(iii) Equal duration forward and reverse strokes but with reduced transient time compared to pure mechanical design without any sensing and control of the crank motion as reported in reference [5]. The resulting motion is the same as that would be obtained using a traditional constant rpm crank. The control action however allows the transient time reach the steady state constant rpm motion to be reduced.

The trajectory profiles are tabulated in Table 1.

For (i) and (ii) continuously differentiable cycloidal motion patterns have been used to generate smooth position, velocity and acceleration characteristics of the slider in the Cartesian space.

The desired crank motion is developed using the following inverse kinematics relationships from the desired slider motion in Cartesian space:

$$\varphi = \cos^{-1}\left[\frac{(x_1(t)^2 + r^2 + l^2)}{(2x_1(t)r)}\right] \qquad (1)$$

where $x_1(t) = l + r - x(t)$, as illustrated in Figure 1.

Also from Figure 1 the Cartesian position of slider can be expressed as:

$$x(t) = r(1-\cos(\varphi)) + (r^2/2l)\sin^2(\varphi) \qquad (2)$$

Also differentiating with respect to time:

$$\frac{dx}{dt} = \left[\frac{dx}{d\varphi}\right]/\left[\frac{d\varphi}{dt}\right] = J\,\frac{d\varphi}{dt}$$

where,

Thus, $J = \frac{dx}{d\varphi} = r\sin\varphi + (r^2/2l)\sin(2\varphi)$

$$\frac{d\varphi}{dt} = J^{-1}\frac{dx}{dt} = \left(\frac{dx}{dt}\right)/[r\sin(\varphi) + (r^2/2l)\sin(2\varphi)] \qquad (3)$$

$$\frac{d^2\varphi}{dt^2} = J^{-1}\left[\frac{d^2x}{dt^2} - \frac{dJ}{dt}\frac{d\varphi}{dt}\right] = \frac{\left[\frac{d^2x}{dt^2} - \left(\frac{d\varphi}{dt}\right)^2\{r\cos(\varphi) + (r/l)\cos(2\varphi)\}\right]}{[r\sin(\varphi) + (r^2/2l)\sin(2\varphi)]} \qquad (4)$$

In Equations (1-4), l represents the connecting rod length and r the crank radius. Note that Equation (4) is substantially different from the equation that can be obtained from the acceleration term in the "Constant angular velocity crank motion" in Table 1. The difference can be attributed to the non-zero crank acceleration ($d^2\phi/dt^2$) due to control action of the actuator and desired trajectory requirements.

MOTION DYNAMICS AND CONTROL SCHEME

The dynamics of slider crank mechanism can be expressed as [2,5]

$$\tau = K_1\ddot{\varphi} + K_2(\dot{\varphi})^2 + \nu\dot{\varphi} + \kappa\,\mathrm{sgn}(\dot{\varphi}) \qquad (5)$$

where, $\dot{\varphi} = \frac{d\varphi}{dt} = \omega_1$ and $\ddot{\varphi} = \frac{d^2\varphi}{dt^2} = \alpha_1$

$$K_1 = I + mr^2[\sin\varphi + (r/2l)\sin 2\varphi](\sin\varphi + \cos\varphi\tan\theta)$$

$$K_2 = mr^2[\cos\varphi + (r/l)\cos 2\varphi](\sin\varphi + \cos\varphi\tan\theta),$$

and, $\tan\theta = \frac{r\sin\varphi}{(l^2 - r^2\sin^2\varphi)}$.

ν,κ are the coefficients of viscous and coulomb friction.

To carry out the computer simulation studies the following parameter values were chosen: Crank radius, r = 0.04 m, connecting rod length l = 0.16 m, reciprocating mass m = 0.6kg and a crank moment of inertia of 1kg-m^2. The coefficients of viscous and coulomb friction were chosen as 1 N.m.s/rad and 0.5 N.m respectively.

The dynamics of the slider crank mechanism is highly nonlinear. There is a vast body of research and implementation schemes in the field of nonlinear control algorithms for robotic systems that can be adapted for controlling systems such as the slider crank mechanism. Mechatronic redesign of slider crank mechanism and other similar mechanisms can benefit a great deal from the control approaches developed and implemented in the field of robotics in the last decade. The study reported in this article uses PID (proportional plus integral plus derivative control) control of the motor driving the crank using appropriate sensor feedback. The results are very encouraging. In future nonlinear control approaches such as intelligent control utilizing fuzzy logic and neural networks [6-8] as well as Model Reference Adaptive Control (MRAC) using parameter based or performance based adaptation schemes [9-11] will be used for improved performance under demanding dynamic conditions. Moreover, determining accurate dynamic model for slider crank mechanism seems to be a less formidable task than robotic arms hence, computed torque control strategy[12] with appropriate feedback gains for disturbance rejection also seems plausible.

CONTROLLER IMPLEMENTATION

The control scheme utilized in the present study is illustrated in Figure 2. The control torque at the crank is developed using proportional plus integral plus derivative (PID) control scheme as expressed below:

$$\tau = \ddot{\varphi}_d + K_p E + K_v \dot{E} + K_I \int E dt \qquad (6)$$

where K_p, K_v and K_I are the proportional, derivative and integral gain ; $\ddot{\varphi}_d$ is the desired acceleration of the crank. E and $\dot{E}$ are angular position and velocity errors of the crank. This scheme is similar to resolved acceleration control scheme that is heavily utilized in the field of robotics[12].

The simulation has been performed using the MATLAB software. The control update rate (h) is chosen as 0.01 sec. The dynamic behavior of the system is simulated using both Runge-Kutta (3rd order) and Euler methods of numerical integration. Both yield very similar results. The control input(torque) is kept the same during the update interval, but the system dynamics is evaluated at 1/10th of the control bandwidth for accurate representation of realistic behavior.

The discrete implementation of PID control scheme is performed using following MATLAB function to calculate the input torque at every control update :

```
function τ = cntltor(E, τ1)

global E1;

h = 0.01; Kp=15; Kv=2.5; KI = 0.9;

 a = Kp +(h/2)* KI +(1/h)* Kv ;
 b =(h/2)* KI -(1/h)* Kv ;
 c = KI ;
 τ = a*E + b*E1 + c* τ1;
 E1 = E;
```

where,

h is the control update rate;

τ, τ_1 are the control torque for the current and the previous control update respectively;

E, E_1 are the position errors in the current and previous control update.

The control gains have been tuned via trial and error to give accurate results. Approaches indicated in reference[6] and [7] can be readily adopted for improved performance in hardware implementation to be performed in the future.

RESULTS

The results of the simulation runs corresponding to the trajectory profiles of the slider given in Table 1 are discussed below:

(i) Slider motion with dwell time at two ends: There are several applications that involve to and fro motion with dwell times at the end of forward and return portion of the motion. A popular example of such a motion pattern can be found in the valve timing system of an automobile engine. Typically the task is handled by a cam-follower system and the cam profile is designed to produce the desired motion of the follower. In this study for one complete cycle of the crank, the slider is commanded to execute a motion similar to that of the follower. The desired motion begins with a 1 second dwell time, a forward motion lasting 4 seconds using a cycloidal motion pattern, another dwell time of 1 second at the end of forward motion followed by a return motion of 4 seconds duration similar to the forward motion. The same motion is to be repeated for subsequent cycles of the crank. With appropriate tuning of the PID gains excellent results are achieved as illustrated in Figures 3,4 and 5. In Figure 3 the dashed line corresponds to the desired Cartesian motion of the slider position and the solid line indicates the actual motion achieved by controlling the torque input to the crank. Figure 4 corresponds to the desired Cartesian velocity of the crank and actual Cartesian velocity of the crank. Figure 5 illustrates correspondence between the desired crank angle (obtained using Equation 3) with the actual crank angle. It may be noted from Figure 5 that as desired the crank is stationary for 1 second when the crank angle is 0 as well as when the crank angle is π radians (180 degree).

(ii) Quick return motion of slider: As mentioned before there are a number of mechanisms which can achieve a slow forward stroke and a quick return stroke using a constant angular velocity crank by careful mechanical design. However, mechatronic redesign with appropriate control of the torque input to the crank enables slider crank mechanism to execute such motion. Figures 6,7 and 8 demonstrate that with appropriate tuning of PID gains the slider executes a forward motion in 4 seconds , for the first 180 degree rotation of the crank and quick return motion in 2 seconds for the remaining 180 degrees of the crank rotation for one complete cycle. The actual motion (solid) closely follows the desired motion (dashed). Figures 6, 7 and 8 illustrate the corresponding motion profiles as in Figures 4, 5, and 6.

(iii) Constant angular velocity motion of crank: For a similar slider crank mechanism (no control) as demonstrated in reference [5] it takes a few cycles of transient time before the crank settles to a constant velocity motion. Figures 9a and 9b demonstrate that with appropriate control using properly selected PID gains the crank settles to a constant velocity motion much more

quickly. Figure 9a shows the Cartesian back and forth motion of the slider during the first eight cycles of the crank. Figure 9b shows a plot of the crank angular velocity against the cycles of crank. It can be readily observed the crank achieves angular velocity values very close to the desired angular velocity of 105 rad/s within the first cycle and maintains it thereafter.

CONCLUSION AND FUTURE WORK

Mechatronic redesign enables the mechanism to exhibit variety of motion patterns with variable speed and duration. Future applications in this field can also benefit by embedding pre-programmed computer chips in the mechanisms. It is likely that the feedback gains will need to be adjusted for different motion attributes for improved performance. Fuzzy gain-scheduling and other adaptive algorithms can be effectively utilized for intelligent control of such versatile mechatronic devices**.** Future efforts by the author will investigate these approaches both in simulation and hardware for a variety of linkages and cams.

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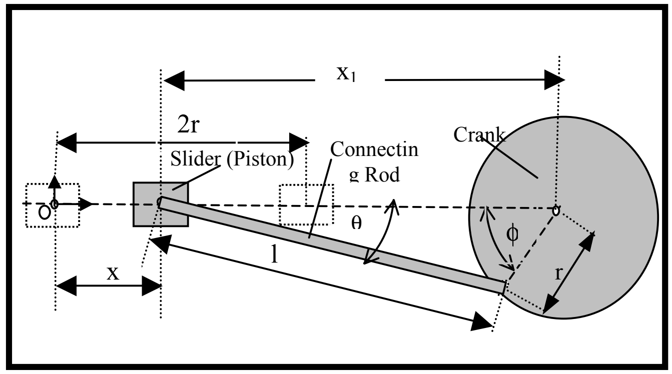

Figure 1 : Slider Crank Mechanism

r - radius of the crank, l - connecting rod length

(i) Slider motion with dwell time at two ends

$x(t) = 0$
for $0<=t<=1$ (Dwell for 1 sec. @ x=0)

$x(t) = 2r/2pi[2\pi (t-1)/4 - \sin(2\pi(t-1)/4)]$
for $1<=t<=5$ (Forward Stroke for 4 sec.)

$x(t) = 2r$
for $5<=t<=6$ (Dwell for 1 sec. @ x=2r)

$x(t) = 2r/2pi[2\pi (t-1)/4 - \sin(2\pi(t-1)/4)]$

(ii) Quick return motion of slider

$x(t) = 2r/2pi[2\pi (t)/4 - \sin(2\pi(t)/4)]$
for $1<=t<=4$ (Forward Stroke for 4 sec.)

$x(t) = 2r/2pi[2\pi (t-4)/4 - \sin(2\pi(t-4)/4)]$
for $4<=t<=6$ (Return Stroke for 2 sec.)

(iii) Constant angular velocity motion of crank

$x(t) = r(1-\cos(\phi)) + (r^2/2l)\sin^2(\phi)$
$V = dx/dt = r\, d(\phi)/dt[\sin(\phi) +(r/2l)\sin(2\phi)]$
$A = d^2xt/dt^2 = r(d(\phi)/dt)^2[\cos(\phi)+(r/l)\cos(2\phi)]$

Table 1 : Cartesian Trajectory profile of Crank

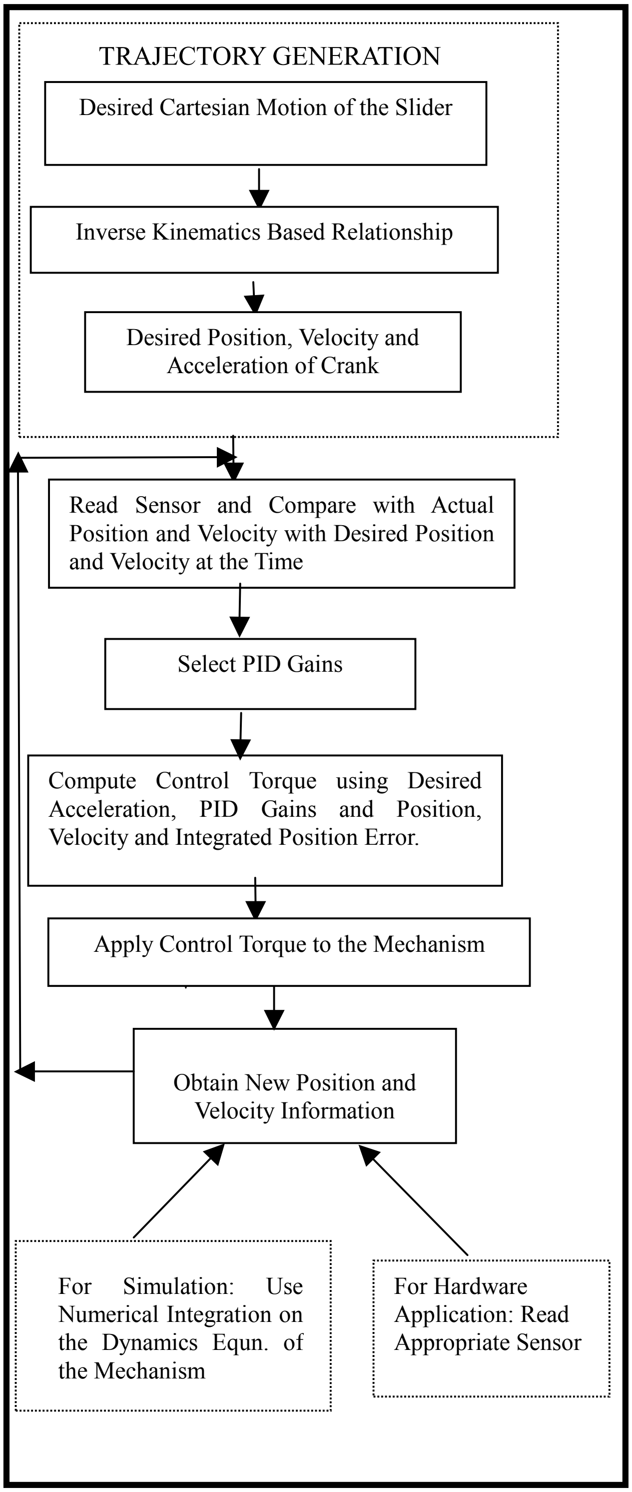

Figure 2 : Simulation and Control Scheme

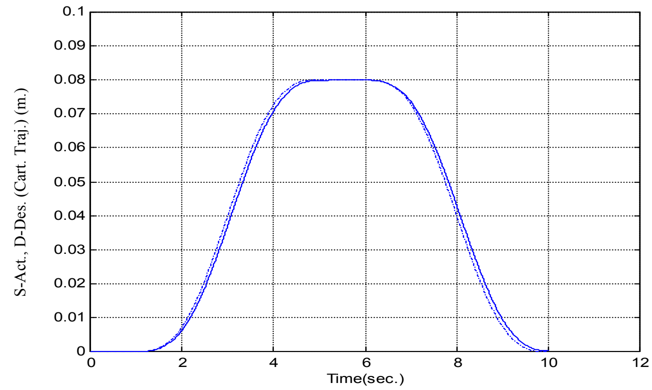

FIGURE 3: Desired and Actual Position of the Slider (Cartesian) for the first Crank Cycle [Motion profile (i) in Table 1]

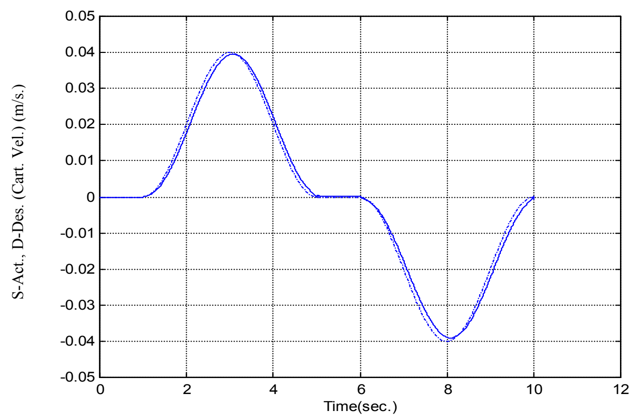

FIGURE 4: Desired and Actual Velocity of the Slider for the first Crank Cycle [Motion profile (i) in Table 1]

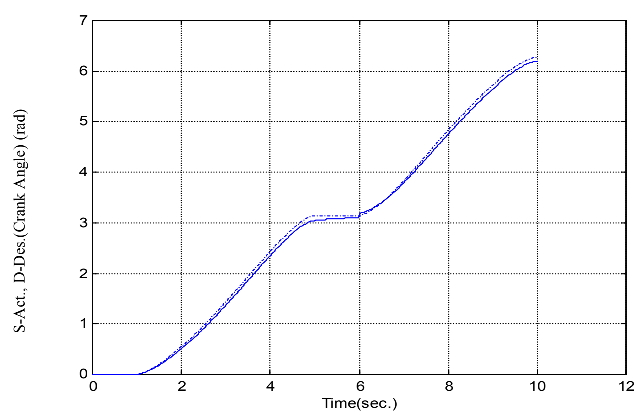

FIGURE 5: Desired and Actual Angular Position of Crank from 0 rad – 2pi rad. (1st Cycle) [Motion profile (i) in Table 1]

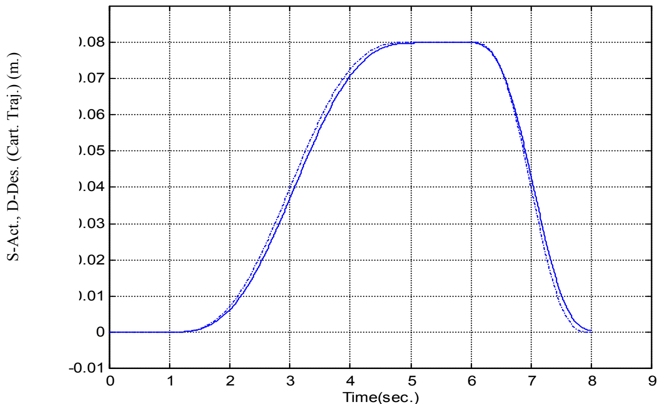

FIGURE 6: Desired and Actual Position of the Slider (Cartesian) for the first Crank Cycle [Motion profile (ii) in Table 1]

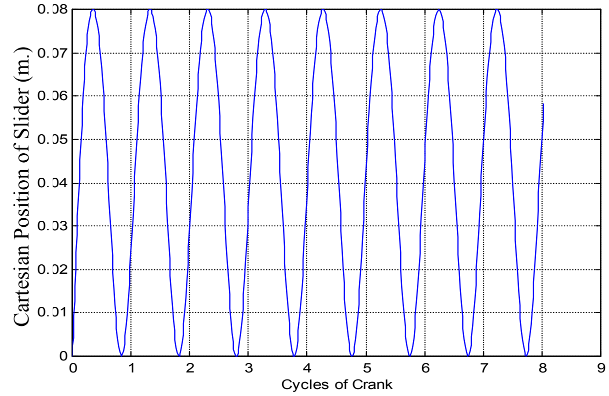

FIGURE 9(a): Slider Motion Vs. Cycles of Crank (Time represented in units of cycles of crank) [Motion Profile (iii)]

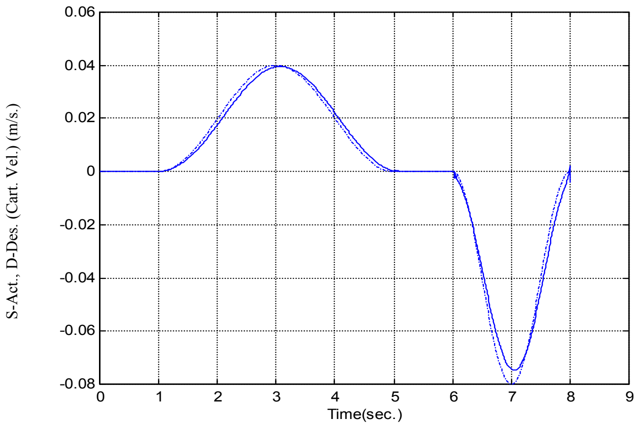

FIGURE 7: Desired and Actual Velocity of the Slider for the first Crank Cycle [Motion profile (ii) in Table 1]

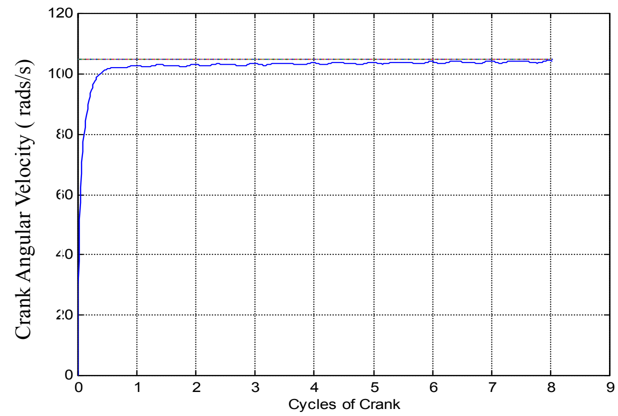

FIGURE 9(b): Crank Angular Velocity Vs. Cycles of Crank (Time represented in units of cycles of crank) [Motion Profile (iii)]

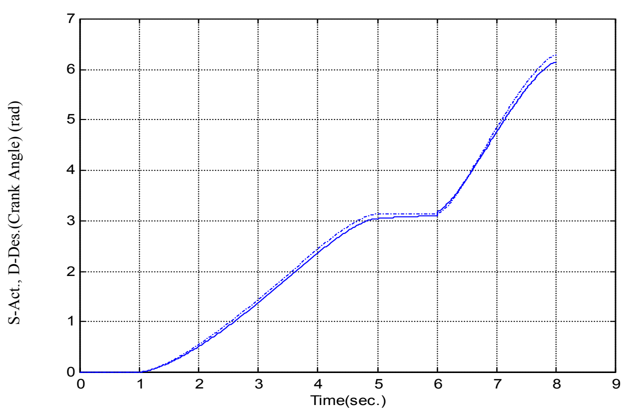

FIGURE 8: Desired and Actual Angular Position of Crank from 0 rad – 2pi rad. (1st Cycle) [Motion profile (ii) in Table 1]