

# From Utterances to Genres in Mathematics Education (and vice versa) - a dynamic triadic (socio-)semiotics exemplified

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## Abstract

This paper is an appreciative critique of research on Mathematical Education (ME) which has searched a theoretical basis beyond Saussurean perceptions of language (Mellin-Olsen 1987, Ernest 1998). Hence it works along similar lines as Sáenz-Ludlow (2001) developing a semiotics for interpreting utterances in educational sites, such as classroom, although from a different platform. A general problem for a Peircean semiotics, giving priority to the sign, is to explain how utterances are related to embodied contexts (or genres in Bakhtinian terms).

The paper is divided in three. In part I triadic approaches in the works of Bühler (1934), Bakhtin (1986), Habermas (1984), and Halliday (1978) are combined. Two basic principles are connected in a model, a) the inevitable semiotic reciprocity between expressing, referring and addressing when uttering, and b) so-called theme-rheme processes between utterances and genres, allowing for understanding how given and new merge (semiosis as learning). Central concepts touched upon are *utterance*, *genre*, *theme-rheme*, *positioning(s)*, and *ideology*.

In part II some illustrating examples of these semiotics concepts are given, among others from the project *Wholeness in Teacher Education?* researching texts in curricula and textbooks in the disciplines ME and Mother Tongue Education (MTE). The paper ends (part III) focusing (seemingly) paradoxical outcomes of a triadic approach, by briefly contrasting some fundamental differences between Peirce, Habermas, and Ernest.

## PART I - THEORIES AND MODELS

### Mathematics - from logic monads to semiotic triads?

Mathematic education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language (Durkin and Shire 1991:3).

But in an advanced culture much which has to be learned is stored in symbols (Dewey 1916:8).

According to Maddy, mathematics, in its most puristic form, tends to be seen as monadic, that is, it deals with singletons or closed numerable entities that can be handled by mono-logic:

"(...) *every physical thing is already mathematical, and every mathematical thing is based on the physical. In place of the customary dualism of mathematical and physical, this pared-down set theoretic realism offers a version of monism* (Maddy 1990:157)".

Following this view mathematics can be treated as a 'box' of monads or single elements where phenomena such as 1, +, =, 0, -, :, x,  $\pi$  etc are logical challenges that should be focused as mathematical *objects*. In an educational perspective the implication is that learning

mathematics is accumulation of a series of logical elements that needs to be portioned out one by one relative to learners' level of understanding and capacity.

A somewhat different position is what has been called logical positivism or logical empiricism, represented by for instance Carnap and the early Quine. According to Hersh mathematics for this 'ism' is nothing but a *language* for science, a contentless formal structure (Hersh 1997:167, my italics). Hersh refutes their positioning by claiming that the positivists regarded mathematics as a tool for science, not as a mental activity in itself. A sign based understanding of this kind will probably be *dyadic*. It basically sees mathematics as mathematical *signs* based on a Saussurean model of semiology, segregating between the signifier and the signified. According to what one could call a neo-Platonistic understanding, mathematics can be seen as a sign system, as a language of its own. However even a dyadic understanding will tend towards objectivism if it prioritizes the mathematical object rather than the system, since the need for 'closing' of an object to avoid uncertainty, using axioms and definitions, is at the heart of mathematics as an academic discipline. If we turn more to the *system* though, things become more complex. Semiotic systems, such as language, can be systematic (closed) or systemic (partly open). If one focuses a system in a Saussurean synchronic sense, it is closed. There are given elements and rules. The main paradigm is dyadic, based on digital logic, that is either/or thinking. It will basically consist of *semantics* (content elements) and *syntax* (set of acceptable rules for structures).

However if one focuses on the *use* of a system, and sees it as a tool, the approach may lead to a dynamic and hence a *triadic* understanding. It takes the character of *communication*, not 'language', as it will have to incorporate a third aspect, *pragmatics*. Accordingly a paradigmatic conflict may arise. What is most appropriate 1) to see language is a tool for communication, whereby semantics and syntax work together and pragmatics is added, or 2) to see syntax, semantic and pragmatics as inseparable? This is a dilemma not only for 'language', but even for 'semiotics'. In this situation a move from Saussurean semiology to Peircean semiotics may seemingly help to understand that a *sign* is dynamic, since the Peircean third element, the *interpretant* will function as the crucial element in explaining the (neverending) process of semiosis. It may help understanding how mathematical concepts are developed, how growth in insight in a given phenomenon can be an *object* for teaching and learning.

However this perception is nevertheless still *sign fixated* and not *systemic*. It lacks an understanding of the nature of the *utterance* as different from sign. An utterance cannot be seen as an object without 'killing' it as an utterance (Bateson 1972, Bakhtin 1986), but should be handled both as a triadic phenomenon, that is as dynamics of syntax, semantics and pragmatics, and as a dialogical phenomenon, constantly balancing and blurring given and new, saying new things with given means (Bakhtin 1986). In the following I will present, first a *triadic*, and then a *dialogic* view of utterances (and genres) and combine the principles in a model. Besides I discuss positioning as an approach, and touch upon the notion of ideology.

### Three independent aspects or systemically related triads?

Society exists through a process of transmission quite as much as biological life. This transmission occurs by means of communication of habits of *doing, thinking* and *feeling* from the older to the younger (Dewey 1916:3; my italics).

Through the history of rhetorics, philosophy, communication theory, and education there are developed quite a few sets of tripartite aspects, pathos-logos-ethos, beauty-truth-goodness, I-it-you, form-content-use, teacher-discipline-learner, heart-head-hand, aesthetical-theoretical-practical (Kant 1987/1790). An unpretentious example of such a general thinking in three can be found in the above Dewey quote: feeling, thinking, and doing. Recently other theorists have put together similar three aspects: personal being-physical being-social being (Harré, 1980, 1984, 1990), experience-understanding-action (Hiim and Hippe 1998), psychic (Freud)-cognitive (Piaget)-social (Marx) (Illeris, 2000), child-object-mother (Poulsen 1994), the aesthetic-the cognitive-the moral (Bauman 1995), student-subject-teacher (Künzli 1998, Westbury 1998), aesthetic-scientific-moral (Miller 1999).

A more explicit triadic understanding is found in the work of Habermas (1984, 1988 and 1998/1988). Habermas sees utterances as principally related to three 'worlds', to the objective worlds, about which there can be made true statements, to the social worlds which consist of all interpersonal relations and to the subjective worlds which are all experiences the utterers have privileged access to (Habermas 1984:100). According to Habermas there is a tight connection between the aspect of action in utterances and the establishing of societies. The processes and the relations will inevitably be triadic:

Thus, to the different structural components of the lifeworld (culture, society, personality) there correspond reproduction processes (cultural reproduction, social integration,

socialization) based on different aspects of communicative action (understanding, coordination, sociation), which are rooted in the structural components of speech acts (propositional, illocutionary, expressive). These structural correspondences permit communicative action to perform its different functions and serve as a suitable medium for the symbolic reproduction of the lifeworld. (Habermas 1984:xxv/McCarthy (Trans.'s introduction.) (Related approaches are found in Bühler 1934, Halliday 1978 and 1994, Hernadi 1995, Ongstad 1996, 1997,1999, 2002, and Bordum 2001.)

Based on ideas from text theory and semiotic genre theory, Habermas' more general ideas can be narrowed and made more concrete, leaning to the Bakhtinian dialogical notions of *utterance* and *genre*. According to Bakhtin (1986) an utterance is a delimitable unit of communicational meaning which, when (partly) repeated, enters into a dialogical relationship with genres (types of utterances or kinds of communication). What characterizes this unit is its *opening*, its *positioning* and its strive towards a sufficient *finalization*. This is achieved by the utterers' combining of references (semantics), a strategic speech plan (pragmatics), and their use of more or less stable generic forms (style or syntax) (Bakhtin 1986: 76-99). All of the three aspects, *referentiality*, *addressivity*, and *expressivity* may be taken from available genres to utter in a world that changes (meaning) constantly because of communication, because of uttering and interpreting (or uptake, Coe et al 2002). To do this the utterer has to be positioned in relation to the three mutual aspects of *referring*, *expressing*, and *addressing*. Thus the utterance will inevitably have three reciprocal dimensions or aspects which will link the utterance to *the utterer*, *the world* and to *others*, in other words a Habermasian encounter of '*self*', '*world*', and '*society*', or in our context, an educational, institutional meeting of the mathematics *teacher*, the *school subject* mathematics, and *learner* of mathematics.

There are clear affinities and similarities with a famous, classic communicational triad, Bühler's so called *organon model* (Bühler 1934).<sup>1</sup> The use of the Greek word 'organon' hints that language is seen as *a means*, not just as a system or as dyadic opposition between a signifier and a signified, as we find in Saussure's influential work (Saussure 1916). Bühler's paradigmatic new pattern puts him in some sense in line with later scholars such as Wittgenstein, Austin, Searle and Halliday, all stressing language as *use*, in other words a *functional* view.

### **Objects and states of affairs**

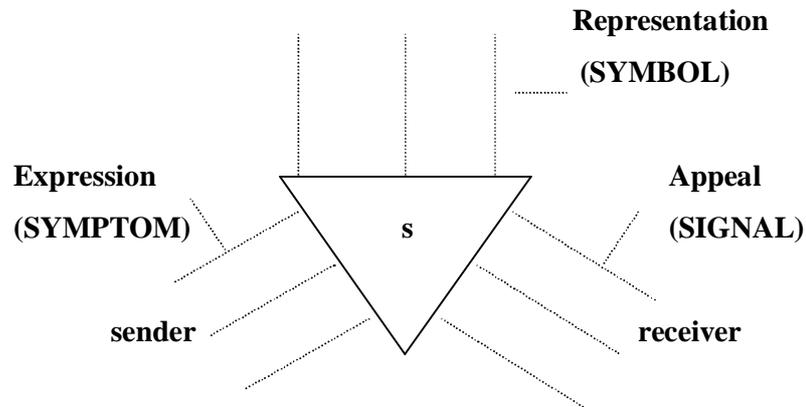


Fig. 1. A simplified version of Bühler's organon model of language. For a fuller version, see Nöth (1990:186) and Bühler (1934:28). (S = sign.)

According to Bühler there are three characteristic functions of language and of signs in general: *representation*, *expression* and *appeal* respectively related to *objects*, *sender* and *receiver*. Focusing gives each function a dominance. Hence the sign is a *symbol* when the representative function dominates, a *symptom* when expression dominates, and a *signal* when appeal dominates. All aspects are at work at the same time in any communication (Bühler 1934:28, Nöth 1990:185). Bühler is the direct source and inspiration for Habermas. In a rather late article, *Toward a Critique of the Theory of Meaning*, Habermas underlines that a validity-theoretic interpretation of Bühler's functional scheme (..) *leads to the assumption that with a speech act 'MP', S takes up relations simultaneously to something in the objective world, to something in the subjective world, and to something shared in a social world* (Habermas 1998/1988:76, Habermas' own italics). Accepting a (triadic) simultaneity will provoke or disturb any traditional validity regime, which builds on a dyadic/digital/dual thinking. I will return to this question at the end of the paper.

'Translated' into more traditional terms Bühler's triad equals or parallels a traditional triad of *form*, *content*, and *use*, notions now used in some textbooks for genre knowledge (in Mother Tongue Education). Hence a sign, an utterance, a text, a genre, a discourse, a context, in short, any communication will have a *structural* (form), *referential* (content), and an *addressive* (use) aspect. These are always intertwined, which in practice means that meaning and real communication happens *within* this dynamic triangle. Or in other words, the triadic mutuality functions as *the definition of communication*. They are 3, and not 2, 5, 7 or (n), because 3 is the lowest possible number of aspects to define communication (as different from 'language'):

*Someone says something to someone.* The three are not categories, but rather aspects or 'vectors', they create a semiotic 'universe' *within* which utterers must communicate.

Knowledge of form, content, and use are resources taken from the embodied semiotic system that each communicator will bring to communication.

Thus a rough overview over triads we have met so far may, in a somewhat disorderly fashion contain the following sets:

pathos	logos	ethos
beauty	truth	goodness
I	it	you
teacher	discipline	learner
heart	head	hand
aesthetical	theoretical	practical
doing	thinking	feeling
experience	understanding	action
psychic	cognitive	social
Freud	Piaget	Marx
child	object	mother
the aesthetic	the cognitive	the moral
student	subject	teacher
the subjective worlds	the objective worlds	the social worlds
personal experiences	true statements	interpersonal relations
personality	culture	society
socialization	cultural reproduction	social integration
sociation	understanding	coordination
expressive	propositional	illocutionary
generic forms	references	speech plan
syntax	semantics	pragmatics
expressivity	referentiality	addressivity
expressing	referring	addressing
the utterer	the world	the others
'self'	'world'	'society'
mathematics teacher	the school subject mathematics	learner of mathematics
sender	objects	receiver
symbol	symptom	signal
expression	representation	appeal
form	content	use
structural	referential	addressive
someone	something	someone

What is claimed about all these aspects and patterns is, for the time being, just that there is a *systemic* relationship between them, horizontally as well as vertically, or in other words *within* and *between* these sets of communicational (and quite a few other) triads. For a more orderly presentation, see attachments. For more thorough theoretical discussion of triads, see Habermas (1984), Hernadi (1995), Borum (2001) (in Danish), and Ongstad (in preparation). These triads form a *systemic* (open) universe *inbetween* which communication proceeds.

Dynamic triads and theme-rheme processes in utterances - a model

Society not only continues to exist *by* transmission, *by* communication, but it may fairly be said to exist *in* transmission, *in* communication (Dewey 1916:4).

Mellin-Olsen (1987) discusses differences between Saussure (1916) and Ogden and Richards (1923). He is dissatisfied with both since they lack a theory of context, which he, however found in the work of Halliday (1978). Even if he pays much attention to Halliday, Mellin-Olsen seems to overlook that Halliday not only develops a contextual grammar, but offers an approach that *combines* both textual triads *and* dynamics of text and context. Since the Hallidayan double combination represents a rather complex socio-semiotic understanding, I will in the following present a rather similar theory/model based on some other sources.

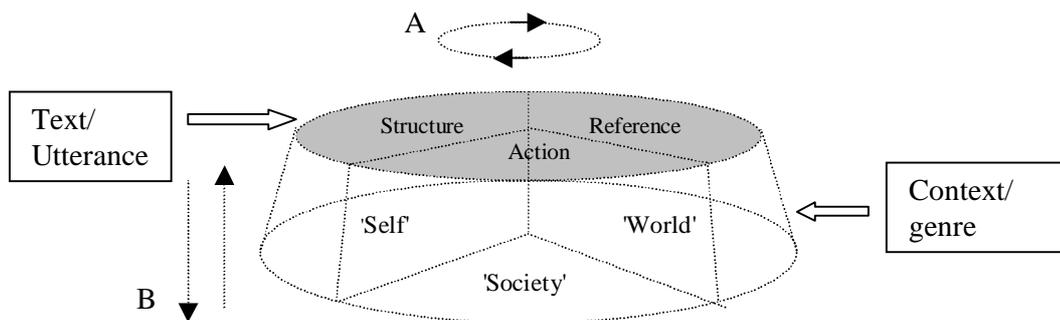


Fig.2. The relationship between the three major aspects on the concrete level of utterance/text (the grey 'surface') and the respective three major aspects of the immanent, level of embodied context/genre. For comparison with a Hallidayan model, see Martin (1997).<sup>2</sup>

There are two main *processes* connected to the model, a 'horizontal' (A) blending of form, content and function, that from an analytical point of view can be studied as dynamics (positionings) of structure, reference and action while uttering. The other process is the 'vertical' (B) where stored elements (meaning potential in Halliday's terms) from the deeper levels (the mind *and* the body) work as active resources in theme-rheme processes (Halliday 1994). These mechanisms are intricate dialogical relationships between utterance (text) and genre (context). To simplify, in an utterance there is a mixture of given parts (theme) and new parts (rheme). The balancing and shifts of these elements happens as the text (the utterance) unfolds. Different semiotic systems and different communicational cultures (and genres) have different syntactic-semantic-pragmatic expectations for these dynamics.

Utterances are on the one hand partly being created by the use of the already embodied resources ('upwards' processes in figure 2). On the other hand they (re-)turn into stored meaning potential for future production and interpretation of utterances ('downwards' processes). In other words will a genre system function as an advanced, constantly accumulating meaning potential for communication by offering certain (communicationally agreed) pre-balances of form, content and use. While for instance blanks (form), definitions (content) and commands (action) can serve as three prototypical *border* examples (even if they all express, refer and address), most of the thousands of other culturally 'negotiated' genres will be found in the discursive landscape *in between*.

### Theme-rheme

If we imagine the utterance as an equation and decoding as a solving process, we have to find the unknown entity on very different places in the equation depending on what is known inbeforehand (...) The equation and the verbal mediated message can thus be compared both regarding complexity and progression. In the mathematical equation there might be one or more unknown elements, in the message one or more fragments of new information. In the equation the unknown will be made known by a systematic combining of all entities, in the decoding of the message the new information has to be framed by all that we know or believe we know (the free information) (Rommetveit 1972:137, my translation).

There are many different definitions of theme. Halliday (1978, 1994) sees theme as a function in the clause as a message: It is what the message is concerned with: the point of departure for what the speaker is going to say. A more general view, not only related to text and verbal language is that theme elements can be related to genre, since the genre presupposes much of what can be expected. (Much is given.) The rheme-parts are the new parts, and have always to be combined with the given in order to function as real communication. If everything is theme (a total given genre) or everything is rheme (a total new utterance) there can be no communication. Thus the more the theme is (strictly) given, the closer the utterance is to a (stereotyped) genre, and the more the positioning is already pre-scribed for the utterer. The more new elements there are, the more risky or challenging positioning as communication may be for the utterer, as well as for the receiver/uptaker. However the given and new will shift syntactically, semantically, and pragmatically through the whole process of uttering and will normally be difficult to trace analytically in every detail. Halliday holds that one has to choose the level of 'delicacy' according to the analytical function (Halliday 1994.)

In the following example, seemingly a beginning of the genre 'fairy tale', I have marked some possible themes and rhemes as they appear in the text. One should have in mind that grammatical *presentation constructions* mostly are 'new' and normally occur in indefinite form (rheme). Once mentioned however the rheme element is now known and can in the next text step get the definite article and function as a known theme. This kind of shift is also the basic principle theme-rheme dynamics will follow more generally.

[*Once upon a time* = theme] [*a linguist wanted to visit a mighty castle called Mathematics education* = rheme]. [*The castle* = theme] [*had huge trolls guarding* = rheme] [*it* = theme]. [*But the linguist* = theme] [*knocked boldly on a door called semiotics* = rheme] etc. It is noteworthy that in certain well-known genres theme parts can be used without presentation, while in others, for instance in a novel written with artistic aspirations, the theme parts cannot be too dominating. It should be underlined that for linguists theme-rheme is not necessarily identical with new-given, and that there are several different definitions of theme-rheme. My main idea is just to create awareness about how crucial theme-rheme processes are for strategies for planning progression in teaching and learning. Particular genres such as textbooks, definitions, explanations, and proofs for instance, will have an implicit regime for balancing theme-rheme (or given and new).

Theme-rheme balances are also important for *positionings* on a more general level. In fields such as chemistry and linguistics the theme part is expected as a point of departure. And generally in research a substantive or crucial part is expected to be new, to deserve the notion of research. A historical example can illustrate that expectation of given and new is a deep *ideological* question: Up till the 1750s the conception of 'art' in Europe was dominated by the view that art meant skills in repeating the classical, admired pattern of ancient art works ('given'), while the innovative aspects from then on little by little took over as a criterium for real 'art' ('new').

Theme and rheme are neither *objective* nor the same for utterers and uptakers. Nevertheless the more communicators share the elements and the clearer they are presented, the more likely it is that one may find a general agreeable balance of what is theme and rheme (and hence what is the meaning or a possible attention). In the newspaper headline *Bush to Russia* readers need to know the semantic content of each of the three elements. *Bush* can then be seen as

given and *to Russia* as the new information. (As already indicated to there are many ways of blurring given and new in different genres/different semiotic systems.)

### Positioning(s)

The above model is not a categorial, but a *relational* model or theory. Hence we need a *relational* concept to combine the three main aspects as well as the dialogism of micro (the utterance/ text) and macro (the genre/context). *Positioning* is coined for this purpose.<sup>3</sup>

However 'positioning' without any topical focus is just a purely abstract relational concept, and thus in itself 'empty'. An object or a phenomenon therefore has to be *focused* for it to be specific, which will, according to the model, create a figure/ground (and a discursive platform relative to the focus) *within* the framework. This means that the paradigmatic perception of the framework is *systemic* (Halliday 1994). By systemic it is meant that no part of the conceptual framework has priority (before someone is focusing on an object). This principle will obstruct the idea that any aspect or relation can have *general* priority over other aspects. Only when one is communicating can one search for possible dominants. This also means that the 'system' rather should be seen as an open-ended, relational and non-hierarchical *network*. Further it implies that all constitutive concepts and elements are *mutually defined* and that no category or concept within the conceptual network is valid in itself, but only in relation to the whole communicational semiotic framework (Halliday 1994, Habermas 1984, Ongstad 1997). Hence the three main aspects are *not* categories, since concrete communication always will happen *in between*.

Positioning is both a process and a product (positionings) in which sign-users locate themselves, the world and others semiotically in utterances. The processes and the products are framed by the impact of embodied meaning resources, by genres. In principle this means that uttering, in order to function as communication implies to move between structuring form (expressing the uttering self) referring the world and addressing others (or oneself as other). Hence in the processes a self, a world, and a society are established as integrated, reciprocally.

Seen as products (most) utterances are the visible, physical and concrete culture. They are all 'dead' in the sense that they just exist as substance, as pure materiality. (That includes in principle memory traces in sign-users' nerve system.) To (re-)create *meaning*, the processes

have to be (re-)established. This can only happen through the initiation of an immanent resource system. Hence as long as there are active genre systems, there is potential for keeping up meaning related to these concrete forms. When the genres (the former contexts) are dead (or lost) the forms degenerate to materiality, and communication fades out or is lost.

Seen as processes, expressing-referring-addressing are in principle immanent even if the expressing of form can be sensed. In other words, positioning is an interpretative activity. As a process, positioning brings in elements for uttering from the embodied resource system in balanced theme-rheme processes, that is, a certain mixture of old/given and new, based on syntactic-semantic-pragmatic 'rules' or conditions for the sign-system and the genre(s) in question. In principle there is not a unique positioning possibility for the observer, the researcher, the 'spectator'. To consider, to look at, to categorize, to interpret implies the deliberate use of particular genres that contextualize the utterances created from a certain position.

Although a reporter, an observer, an inspector or a researcher does not follow a kind of ideal expected *receiver* role, or is originally perceived as an addressee by the sender/utterer, his or her position is nevertheless a receiver's in the sense that the utterance is primarily related to, even by observers, as *meaning* (or symbol in Bühler's term). Given a triadic view on communication (text-context) it is therefore no sense, within such a model, in giving the researcher a protected position *outside* this communication. In other words, observation and research are just *genres* focusing or giving priority to utterances as 'world' or as objects taken from the 'world'.

If we *focus* and our interest is language, the focused object can be figured as *text*. What is *not* looked at, but immediately connected to it, is the *background* for the focused *figure*. This becomes the *context* or the *co-text*. Context varies from immediate to distanced and quite a few concepts and metaphors have traditionally been used to characterize and to differentiate aspects or kinds of context, such as co-text, cultural context, register, genre (Ongstad 1997). A way of avoiding or omitting a negative discussion of such terms is simply to talk about micro and macro, where the micro level represents the focused object, the figure, and where the macro level represents the co-working background (Ongstad 1999c).

A main reason why these two levels or aspects have to be seen in explicit connection, is that meaning is not only developed exclusively *within* a text or utterance, but is a product of an

interplay of what is explicitly said and implicitly unsaid. Hence a genre helps its users by reducing what needs to be in a text. The tacit expectations in the genre, that help utterers to say more than they actually do, can be seen as *ideologies*, as hidden, but functional values and resources for communication (Ricoeur 1981, Bakhtin 1986). Hence for instance, the *sense* in doing mathematics is not explained every time there is a new topic or task. Thus within a professional community or in a textbook this sense can easily be under-communicated.

## **PART II - EXEMPLIFICATIONS**

### Positioning the three major aspects (simplistically)

Example A.

T: What is the *difference* between 8 and 3?

S: Well... they are *different*. 8 is bigger than 3.

T: If you take away 3 from 8, how much is that?

S: That is 5.

T: If you subtract 3 from 8, how much is that?

S: That is also 5.

T: What is the difference between 8 and 3?

S: I told you before. 8 is bigger than 3

(From Sáenz-Ludlow, 2002:12).

What we see demonstrated here is a *semantic* difference in the conception of the word 'difference' between the teacher who is enculturated to mathematics and to the student who is not (yet). There are several *references* to the form 'difference', the two operate with different *content* dependent of their different embodied *contexts* (register, genre, discourse, code, habitus).

Example B

$$8 - 3 =$$

As teachers and students of mathematics we are enculturated to handle such a structure as a *task*. More or less automatically we will *perform* the calculation because we interpret adressivity as an invitation or an *appeal*, a *signal* to do subtraction based on the interpretation of the operators. If the enculturation is strong, this tends to happen irrespective of which other context the utterance occurs in: "We cannot not subtract". The ideology of this mathematical

genre/discourse thinks for us. Thus a *pragmatic* positioning (reading) of the utterance may reveal such ideologies.

#### Example C

A, E, F...

B, C, D...

Before you read further, spend some time considering what you think will be the next letter in each of the two rows above. The answer is further down in the text. In addition to content and use utterances have a *third* dimension. Their most immediate and direct aspect their appearance, their *form*, depending on the medium or the channel through which they are brought to us. Mathematicians may for instance have an immediate urge to try to solve logical problems of the above kind. They are encultured to see such utterances as challenges or tasks which can be 'solved' based on more advanced knowledge of arithmetic rows. However first grade children, learning the alphabet in mother tongue education, may in the case above see the 'answer' before the mathematician: H in row 1 and G in row 2 (straight vs 'roundish' letters). Sáenz-Ludlow (2001) has a similar example where a mother asks a child "What is half of 8?" and the child answers 3, focusing on the right half part of 8 configured as a shape. (The correct answer is 'of course' 0, since divisions are supposed to be performed horizontally, by the fraction line as a division mark. This is proved by the mathematical fact that the rest this time is not E (the left part of 8) but 0.)

These fabricated examples may seem simple, but the educational importance of the third dimension, form, is often underestimated. Mathematics is not just logic and performance, it has a profound aesthetic aspect, it is hated and loved, it is horrible and beautiful, clear and obscure, negatively frustrating and positively challenging (Kragh 1999). The aesthetic dimension is not general and static, but always shifting through the processes of uttering and interpreting (uptake). The communicator *evaluates* constantly, through which the psyche is inevitably attached to the uttering process. In Bakhtin's term utterers will utter with a certain *expressivity*. In Bühler's term communicators interpret the utterance as *symptom* if we try to interpret from expressivity to utterer. Expressivity is everywhere in an utterance, but will be more explicit in some parts than others.

## New and given in stereotyped mathematical genres

Not only is social life identical with communication,  
but all communication (and hence all genuine social  
life) is educative (Dewey 1916:5).

On the 18th of April 1994 class 7X (age 14) are doing algebra (Ongstad 1997). They are solving textbooks tasks (calculating by insertions). There are quite a few seemingly similar tasks. Suddenly there is an air of confusion and irritation. One of the tasks, on the surface similar to the others, is 'wrong': Calculate  $3x - 7y$  when  $x = 3$  and  $y = 2$ . The frustration is so strong that the teacher, Hedvig, has to stop the competition race and do the calculation in plenum on the blackboard (Ongstad 1997: 363).

This is a so-called 'insertion' task. Seemingly it looks like all the other small 'sums' (arithmetic tasks) they have had the last days. "Calculate" is a clear genre signal. It is not 'find' or 'consider'. A *calculation* is expected. The operation required is the topic (the theme) of the day, *insertion*. The theme part is 'calculate'. The rheme (new) elements are any new logic units running linearly from the left to the right. The sums have stereotypical patterns with little variation, which rapidly establishes safe genre patterns, since all tasks in the task regime look similar. However as Sáenz-Ludlow (2001) points out, the novice learner has an asymmetric pre-knowledge compared to the advanced teacher. In other words the better one knows the genre and the genre system, the better the readiness (zone of expectation) a receiver will have of what *might* appear. This genuine genre knowledge helps, mathematically, to guess how this piece of calculation may unfold in the theme-rheme process.

For the novice learner the task above at a first glance seems trustworthy. It looks like many of the task the class, 7X, has had earlier this semester. Most of the students will therefore not suspect any complex rheme part to occur since this 'genre' has established itself with easily recognizable patterns. The students will consider the task utterance as a combination of safe theme elements they already know, where the only new elements are the specific figures to be filled in. Their problem is therefore experienced rather dramatically. They have never met *negative* numbers as answers in this genre. Most of the students therefore found this task 'wrong' or 'impossible'. They gave up.

The teacher had to explain this carefully. They have of course heard about 'negative numbers' earlier, but they could not imagine the possibility of a negative number in this context. The

original taskmaster of this utterance has placed this 'rhematic' element more or less consciously as a surprise bomb hidden within a seemingly current piece of calculation. To discover this involves incorporating this 'new' element in the genre of 'insertion'. The genre and their genre system grows. Next time, if remembered, this element will occur as known (being thematic, not rhematic) belonging potentially to this particular mathematical genre.

There is in other words an intimate relationship between the dynamics of themes and rhemes on the one hand and learning as combination of given/already acquired knowledge as well as skills and the progression of new learning elements catered by the curriculum, the textbooks and the teacher on the other. In this case it was even a brake with the didactic <sup>4</sup> / traditional educational expectation that any 'major' new element will normally be presented by the teacher *first* (which in the next step often leads to a mechanistic perception of 'calculation'). As soon as there are 'difficulties', students hesitate in trying to solve them.

#### Theme-rheme (given-new) positioning(s) in advanced mathematical texts

For a newborn child the utterance  $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$  does not consist of theme and rheme. Theme has to be given and rheme partly recognizable for it to be meaningful. This particular utterance will function as (partly) meaningful in relation to how communication, language, and mathematical knowledge are built up, stage by stage for potential interpreters. In the following I will give a quite extensive example of how a particular Norwegian textbook in Mathematics Education for student teachers (Nygaard et al 1999) has elected to build a whole chapter towards the above utterance.

#### *Functions and equations*

*In this chapter you will learn about the concept of function, polynomial functions and rational functions. In addition you will get an insight into the concepts 'derivation' and 'integration'.*

*You will learn to solve equations and equation systems. We will also show how we can solve an equation by approximation using spreadsheets. In addition you will gain insight in extracts from the history of equation.*

*If you know the syllabus of functions from upper secondary school very well, you may think that this chapter is strange. It is quite different from what you might expect, we are quite sure about that. However, why learn once again what one already knows?*

*If you do not know much about functions, it is rather doubtful whether it is a good idea to try the same method as the last time you failed to get the point. We therefore want to lay hold of other aspects of the theory of function, and focus more on the concepts themselves and study some different types of functions so that you can see what a function is, and learn to appreciate what a good idea functions represent (Nygaard et al 1999:375, my translation).*

Thus one of the sub-chapters concerns the history of equations, starting with first grade equations. In a balanced mixture of mathematical reasoning, explanations, examples, tasks, and storytelling, we are brought successively from the history of the first grade equation, via first grade function, second grade equations, the history of the second grade equation, the second grade function, the history of the third and fourth grade equation up to the history of the fifth grade equation. We end with the impossibility of solving fifth grade equations with an algebraic formula for solution.

The 'race' between Italian theorists to be the first to solve third grade equations (and to keep the secret) is also described. (Nygaard et al 1999:410-411). When moving from third to fourth grade equations the following lines occur [the (...) indicates that text has been left out]:

*You learned to solve second grade equations in upper secondary school, now it probably will be fun [blir det nok gøy] to learn to solve the third grade equation. The fourth grade equation we are afraid you have to wait to learn [må du dessverre vente med å lære] until you chose a more advanced course in mathematics (Nygaard et al 1999:411). (...)*

#### The history of the fifth grade equation

*We have now seen that during the 16th century formulas were found for solving the general third and fourth grade equation. After this achievement it was natural for mathematicians to start the work finding a formula for the general fifth grade equation:*

$$ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$$

*This was done, and in this history the Norwegian mathematician Niels Henrik Abel (1802 - 1829) played one of the leading parts.*

*Abel first believed that he had found a formula for solving the fifth grade equation, but he soon discovered that it was incorrect. (...) [In the margin there is portrait of Abel accompanied by the text: "Niels Henrik Abel (1802-1829), perhaps the smartest Norwegian ever."] What Abel was able to prove in 1824, was that there does not exist a formula for solving the general fifth grade equation. How could Abel know that there did not exist an extremely complicated formula that he had not tried? (...) What Abel did was find which an ability such a formula had to have, and then prove that irrespective of how you build up the formula for the solution, it will not in any case have the requested quality. Smart, don't you agree?*

Further it becomes clear that for  $n > 4$ , there is no general algebraic formula for the solution of the  $n$ th-grade equation. Hence with this result from the 1900th century we can just forget to try to get famous for finding an algebraic formula of solution for instance the general eight grade equation (Nygaard et al 1999:413).

If we consider this text from the perspective of theme-rheme, we should keep in mind that we are confronted with here a blurring of genres. There is a *story*, and stories have their own expectations regarding the balance of theme and rheme, which is also culturally dependent. Moreover there is a thematical *reasoning* that tries to *explain*. A main point here is that most students may have experienced a rather mechanistic enculturation to algebra. Their genre expectation to *functions* is that tasks appear as something *solvable*, there is always an 'answer'. Hence this is about being right or wrong. Suddenly something occurs that is *not* solveable. A major ideological 'sense' in doing (school) mathematics may abruptly disappear.

However, things turn out not to be that simple. In reduced understanding it is often said that a fifth grade equation does not have a solution or that equations of a higher grade than four are impossible. The rheme strategy of these authors is therefore directed, not only towards an understanding of the 'elegance' of Abel's proof, but function as a stepping stone for the next topic,  $n^{\text{th}}$ -grade functions (Nygaard et al 1999: 413). Hence the theme-rheme regime in the entire chapter is to build up a careful succession of mathematical knowledge by a mixture of verbal and arithmetic utterances.

A didactic point is that the diachronically told history may help some students understand that they are not 'stupid' if they do not grasp this immediately. It has taken two thousand years of human intellectual history to get to the understanding, a knowledge that normally is presented as (synchronically) *given*. This also can make student teachers (and students) aware that the extensive genre knowledge their teachers, and researchers of mathematics at the cutting edge of theory in this field have acquired, has been built up over time, step by step, as theme-rheme alterations in the everyday of learning.

### Emotionality as traces of ideology

(...) the things we take for granted without inquiry or reflection are just the things which determine our conscious thinking and decide our conclusions. And these habitudes which lie below the level of reflection are just those

which have been formed in the constant give and take of relationship with others (Dewey 1916:22).

I have underlined in the textbook extracts places where some kind of 'unexpected' expressivity is breaking through. I have also marked in the text, three formulations: perhaps the smartest Norwegian ever. " (...) Smart, don't you agree? (...) we can just forget to try to get famous. The reason why I interpret the utterances as emotional, having more than 'normal' expressivity, is that, according to my traditional norms for mathematical writing I do not *expect* mathematical texts to be very expressive. In Bühler's terms I have read the text as a *symptom*. Or in Bakhtin's term, I have positioned the text as (a particular) *expressivity*. This particular expressivity can be interpreted as an emotional symptom of the authors' *engagement*. However the marked text-elements can also be overseen/refuted as just referential symbols/words, with secondary importance for the 'main' intention of the text as a whole. Or they can be interpreted as Bühlerian/Bakhtinian forms that *signal* or *address* students (of our time?) in a particular way, and thus would rather be seen as *addressivity*.

There are of course many ways of reading any text. In this particular case I am also interested in *ideologies* of the utterances and the genres, in particular since this is education, and since a general positioning in my own research is that school and education are sites for particular constraints (Braathe and Ongstad 2001). Bakhtin holds that any utterance simply has to contain (tacit) values and will be mainly driven by the genre (Bakhtin 1986). Similarly Ricoeur holds that ideologies are something we think *from*, not on (Ricoeur 1981).

In school there is often a conflict between subject teachers' engagement and students' instrumental attitudes. One particular ideology, that generally is not admitted, is the hidden admiration of smartness (see also my Gauss story, last example). Most teachers of mathematics are 'winners' in the race for good marks and high ranking, and mathematics is the discipline *par excellence* to be used for comparative purposes (not the least internationally). In this particular text a deep structural ideology seems to be that working with mathematics, at the end of the day, is about smartness. The textual strategic positioning of the three expressive elements underpins such a reading.

However, counter-reading and triangulation of different readings and self-critique of the researcher's (my) own positioning should also be conducted: Is this way of writing dominant in their 719 page book? No, but there is a certain tendency towards it (Ongstad and Braathe

2001). Can the particular discourse strategy of the authors be understood differently? Yes, it can be seen as a conscious grip to make their textbook less 'serious' in order to engage more students. Can a researcher's particular interest in the affective domain lead to a lack of understanding of its epistemological value? Yes. In other words - semiotics is multifunctional.

Hence studies from (and of) positionings will not give easy and correct answers. But they can, sensibly used, trace new, hidden aspects. And since the approach is *systemic* it allows for an explicit considering of its own positioning(s) of validities, it can thus be related to over-all ideologies, such as 'subjectivity', 'objectivity' and 'normativity' (Habermas 1984) (See triads in Attachments, *Positioning of professional fields*, etc.) Further the approach is socio-semiotic, which implies that it considers 'findings' as dependent of context/discourse. Finally the triadic basis allows for flexibility and for the multifunctional. I would like to emphasize that analysis of positioning(s) in relation to utterances and genres does not completely refute an *essentialistic* approach. If Popper's third world is chosen as a point of departure (compare Ernest 1998), as well as Popper's claim that the researchable should be anchored in a statement (a proposition) that can be tested (have the ability to be evaluated as false), this is possible to make sense of within the given framework.

Nevertheless such a positioning will often run into trouble in explaining/validating how context is neutralized. This is especially the case once one moves from 'nature' to 'culture' as researched 'objects'. While mathematics and science may still prefer to stick to traditional epistemological validities only, mathematical *education* cannot. Mathematics education is a cultural, that is a semiotic based field. It has to take into consideration, simultaneously, the subjectivity of the learner, the objectivity of the references, and the normativity of the acts not as separate categories, but as inseparable aspects, and hence didactically valid only in their blurred state. It is in the inter-twinned form that learners, teachers and researchers will have to relate to utterances.

#### Positioning simultaneous validities and ideologies

When we utter, we evaluate not only the aesthetic, but also the epistemological and ethical aspects of communication, all at once and blurred. References, content, and knowledge are surveyed by a sort of logical true-false regime. And utterances as acts are considered ethically as good-bad/fair-unfair. Hence *validation* is a question of balancing three unseparable *kinds* of

communication (Ongstad 1997), and, depending of which main aspect will be the point of departure, one will, according to Habermas, have to relate *simultaneously* to the truthfulness/veracity, to truth and to fairness/usefulness. It is of course the context which will help in guiding us to see what is relevant. However, since context is systemic through the impact of genre and genre expectations, we will normally be guided by our genre knowledge.

1. Student: "I hate mathematics!"
2. Student: "The total sum for all whole numbers from 1 to 100 is 5050."
3. Student: "I will sum up all numbers from 1 to 100 by using the calculator."

In context 1 we may understand the utterance as strong expressivity and interpret it as an emotional statement. We have to consider whether the student actually means it, how truthful the utterance is. In context 2 we would tend to ask: "Is it true?" and in 3 we would consider the value of using the calculator. However all three validities/aspects/dimension are present in each of the three utterances. What we do by giving priority to one (seemingly *dominant*) aspect is to *position* the utterance. By positioning we a) take a position, a point of view, a perspective, b) focus, creating a fixated *object*, and c) take for granted an un-uttered context that will help the utterance to give a fuller meaning. Thus in Sáenz-Ludlow's first example about *difference* both the teacher and the student chose a specific position, they both fixated on an object (difference) and they both took for granted a shared context/genre in which the focused word could belong.

There is a famous anecdote about Gauss who as a 10 year old got as punishment from his teacher the task to sum up all whole figures from 1 to 100. Gauss however solved this intuitively by imagining that  $100 + 1$ ,  $99 + 2$ , etc would give 50 times  $101 = 5050$ . This story was my own introduction to arithmetic rows as a mathematical genre. In addition to understanding the pure logic, hearing the story had two ideological outcomes for me: that mathematics could be elegant/smart/beautiful and that in mathematics *functionality* was an important aspect to consider (mathematics was a rational tool, a system for acting).

Nevertheless I also soon understood that there could be a hierarchy of smartness that was embedded in the teaching and learning of mathematics, through which I got an awkward feeling that mathematics in addition or perhaps even primarily was a game for ranking smartness. Besides, time showed, there were also 'fake' functionalities (I never really used arithmetic rows later in my life, outside school mathematics). In other words there were

*hidden ideologies* that were blurred into the everyday of mathematics, and these tacit expectations were again connected to different mathematical/pedagogical genres. <sup>5</sup>

### **PART III - TWO PROBLEMS**

#### Semiotics of mathematics education - departure from Peircean 'interpreter' or Habermasian 'communication'?

I as eye make words of worlds,  
acts of facts and signs of science  
or vice versa? (SO, 1997:434).

In his book *Postmetaphysical Thinking* Habermas has added a new article on *Peirce and Communication*, which originally was delivered as paper at a Peirce conference in 1989 (Habermas 1998/1988). (The English edition, somewhat changed, was first published in 1992.) In this article Habermas raises a basic question:

What considerations could have induced Peirce to turn away from the intersubjective aspects of the sign process? I want to defend the thesis that the interpretant relation of the sign cannot be explained without recourse to the conditions for reaching an intersubjective agreement, however rudimentary these conditions may be (Habermas 1998/1988:92).

In concluding, Habermas holds that the legacy of Platonism is in fact repeated in Peirce's work, even if Peirce obviously takes an anti-Platonist standpoint:

Both allow communication, in which tendency to universalization asserts itself, to be seen from only *one* side: communication is not for the sake of reaching mutual understanding between ego and alter about something in the world; rather, interpretation only exists for the sake of representation and the ever more comprehensive representation of reality. This privileging of the sign's representative relation to the world above the sign's communicative relation to the interpreter causes the full-fledged interpreter to disappear behind the depersonalized interpretant (Habermas 1998/1988:109).

In the first part of the article (p 92) Habermas points to the fact that the young Peirce actually worked with the idea of writing a book about the interplay and the dynamics of 'I', 'Thou', and 'It' as "Elements of thought" (intended book title). However he did not stick to the idea. It seems as if that is the closest Habermas thinks Peirce comes to a communicative/inter-subjective understanding of semiotics, in spite the fact that Peirce meant that "all thoughts

were addressed to a second person" (the so called doctrine of Tuism) (Habermas 1998/1988:111, note 3).

Accordingly, if Ernest in his thorough and quite convincing critique of absolutisms (Ernest 1998:1-38) at the end of the day lands on Peircean semiotics for social constructivism/ mathematical education (Ernest 1997; Ernest, this session), he may end up like Peirce, stopping Platonism at the front door, but letting it in the back-door. The interesting thing is that both Ernest and Habermas find it necessary to position themselves in relation to post-structuralism (respectively positively/critically) (Ernest 1997, Habermas,1988). Post-structuralists notoriously attack structuralists for using binary logic at the same time as they themselves work from it (Ongstad, 1995, 1999b). The model presented in my paper does not accept a simple paradox of either/or as part of a dyadic paradigm. The paradox should rather be seen as a conflict of *both/and between dyads and triads*, in other words, between (dyadic) language and (triadic) communication as phenomena: 'Language' is a true lie of killed context.

Thus I agree with Habermas that the dimension the intersubjective/ society/you has to be brought into a triadic framework, not a dyadic, as psycho-socio or socio-cognitive (or being left out as in a Peircean framework, making acts to thoughts). On the other hand poststructural approaches may be partly relevant when the literary aspect of a text *dominates* (see Habermas' last article, *Philosophy and Science as Literature*, Habermas, 1998/1988) or even in some cases of *essentialistic* positionings in 'hard science'. However in Mathematical Education the overall approach will benefit from a triadic framework. In other words when *communication* tends to be secondary, dyadic understanding may be valid. However it is hard to find any mental, human activity that does not have a communicational aspect. As already quoted: *Not only is social life identical with communication, but all communication (and hence all genuine social life) is educative* (Dewey 1916:5).

### "Texts and the objects of Mathematics"

The above headline is a chapter in Ernest (1998) in which Ernest states that (...) *the social constructivist view is that the discourse of mathematics creates a cultural domain within which the objects of mathematics are constituted by mathematical signs in use* (Ernest 1998:193)". There are good reasons for agreeing that the relationship between sign and discourse is related to their function (use), even in mathematics. Ernest sees two (inter-)

related concerns: How can a positive aspect of Platonism, the ability of explaining the nature of mathematical objects, be kept without being captured by its (negative) ontology? And: What is the relationship between the objects of mathematics and texts?

First I will point to onomatopoeia found in different languages. In Norway the crows 'say' 'kra-kra' and sheep 'bæ-bæ'. These forms are not fully arbitrary, since the sound of an animal is to a certain degree (iconically?) represented in language. As pointed to by Peirce also, icons and indexes are not arbitrary symbols. Given Bühler's Bakhtin's, Habermas', and Halliday's triads of communication however, we may hypothesize that, in a diachronic perspective, all three major instances in principle have the possibility to contribute to uptakes of signifiers in different semiotic systems: each self can, the 'world' can, and 'society' can. These processes of uptakes may change the processes of semiosis. Each main part has to a certain degree influence over the uptake.

Regarding mathematics, where do the geometric figures stem from? I am a collector of minerals, and thus it is hard for me to think that human beings thought out the basic figures (in the first place) when these forms already were developed hundreds of millions of years before there was any human life on earth. Hence it is not problematic to allow for the thought that *originally* some basic figures/forms stem from nature. For these *forms* to become *signs* and then *utterances* though, they need to pass through the semiotic, discursive sign process Ernest is pointing to above.

However they need to, and cannot avoid passing through and developing a double *triadic* process: form being reference being use in micro and macro. These aspects will simultaneously be contextualized and stored in the embodied genre system each time they are 're-used'. In principle the 'system' will change each time one communicates. The personal unfolding utterance (logogenesis) is a fragment of personal growth (ontogenesis) is a fragment of the evolution of the semiotic system(s) (phylogenesis) (Martin 1997). In these processes there are degrees of stability. However a cultural stable phenomenon, such as 'language' does not mean static. Semiosis/positioning(s) will alter systems, even mathematics, and especially mathematics education, by moving from utterance to genre and vice versa. *Mathematic education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language* (Durkin and Shire 1991:3).

**Challenges and advantages using analysis of positioning(s) as method**

### Possible problems

- the different parts of the analyses may be inconsistent
- one may end up with banalities or uninteresting details
- a certain danger for categorial interpretation, even with a relational point of departure
- semiotic analysis beyond the sign level is still not well developed
- analysis of details is more time-and-ressource-consuming
- traditional approaches to validity will be problematic

### Advantages. The approach may...

- give broader, deeper and more inter-related descriptions
- improve the possibilities for integration, generalization and differentiation
- keep principle elements in communication and education together
- strengthen the possibilities to compare or integrate 'self', 'world' and 'society'
- open up for interdisciplinary and cross-scientific understanding
- handle the paradoxical relationship between the parts and the whole
- be used both by teachers and students as analysis and a way of understanding
- give terrain for a dynamic understanding of validity (as process)
- 'reveal' weaknesses of theories of science
- use the same approach for the observed, the object, and the observer
- counter-balance 'logocentrism', that is, the tendency to exaggerate the role of language
- balance aesthetics, epistemology, and ethics within a systemic/relational framework
- accept connections (on certain conditions) to other models of analysis in different fields

### **Considering application of the above triadic, semiotic view on genre, it may be claimed that such a perspective:**

- gives a general (inevitable) framework for theories of genre and general science
- enables the bringing together of theories of self, world, and society
- is an useful tool to explain the rise and decline of cultures and subcultures
- functions as a frame of reference for the problem of the hermeneutic circle
- allows specific and practical research (and pedagogical thinking) on positioning(s)
- is useful for comparative studies of communication and cultures
- gives better access to the studies of power (genre gatekeeping)
- helps understanding the subtle role-giving (and role-avoidance) in the use of genre
- can be used for (self-)critique of ideologies (thus this triadic model is relativistic)

- is basic for the understanding of life-long and institutional socialisation
- is crucial for any practical or theoretical question about validization

## Notes

1. Craig Bandist, Sheffield University, holds in a forthcoming book that Bühler's work was known among key persons in the Bakhtin Circle.

2. Martin (1997) configures the relationship between the two levels and between triadic aspects in the following way:

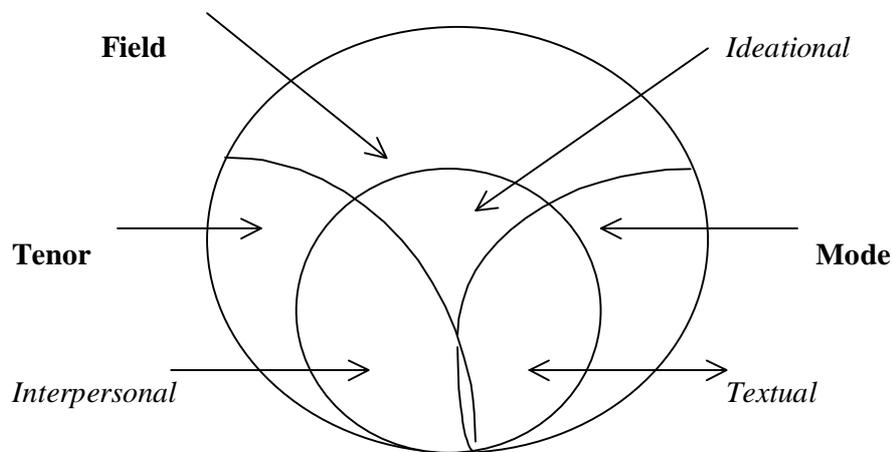


Fig.3. Functional diversification of *language* (the small circle consisting of the aspects textual, interpersonal and ideational) and *social context* (the big circle consisting of mode, tenor and field, respectively) according to Martin (1997:5).

Thus according to Hallidayians (or Systemic Functional Linguistics, SFL) language has three metafunctions: ideational, interpersonal and textual, and their function and relationship should be understood as follows:

*Ideational resources are concerned with representation, interpersonal resources with interaction and textual resources with information flow. In SFL this intrinsic functional organization is projected on to context, redounding with the variables of field, tenor and mode - where field focuses on institutional practices, tenor on social relations and mode on channel* (Martin 1997:4). For further exemplification see Morgan (this semiotic session).

3. The concept of positioning stems from many sources, military and economical strategy, electronic surveillance of the earth by satellites, discursive communicational theory, and study of student strategies in classrooms (Davies and Harré, 1990, Evans and Tsatsaroni 1994). Langenhove and Harré (1993) even speaks about theory of positioning. My use of the concept is more related to Bakhtin's notions of utterance and genre and is supposed to function in a broad semiotic context. For other writing concerned with analysis of positioning and ideologies, see attached list.

4. 'Didaktic' is written with k to illustrate that the concept is semantically related to the edicational field of didaktik. For a fuller explanation, se Ongstad (1999).

5. For a problematization of some ideologies in mathematics education in Norway, see Braathe and Ongstad (2001).

## References

- Bakhtin, M. (1986) *Speech Genres and Other Late Essays*. Austin: University of Texas Press.
- Bateson, G. (1972) *Steps to an Ecology of Mind*. New York: Ballantine Books.
- Bauman, Z. (1995/1993) *Postmodern Etik*. [Postmodern Ethics.] Gothenburg: Daidalos.
- Bednarz, N. (1996) Language activity, conceptualization and problem solving. In H. Mansfield et al. (eds.) *Mathematics for Tomorrow's Children*. Dordrecht: Kluwer Academic Publishers.
- Bordum, A. (2001) *Diskursetik og den positive selvreferense*. Copenhagen: Samfundslitteratur.
- Bourdieu, P. (1989) *Outline of a Theory of Practice*. Cambridge.
- Braathe, H.J. , L. Kværnes and S. Ongstad (2000) *Triads in mathematics education. Mellin-Olsen (1987) and semiotic genre theory - comparisons and exemplifications*. Paper at HiO's/Oxford University's Conference on Mathematics Education. Gausdal. March.
- Braathe, H.J. and S. Ongstad (2001) Egalitarianism meets ideologies of mathematical education - instances from Norwegian curricula and classrooms. *ZDM*, Vol. 33 (5)
- Bühler, K. (1934/1965) *Sprachtheorie*. [Theory of language.] Stuttgart: Fischer.
- Bühler, K. (1990/1934) *Theory of Language. The Representational Function of Language*. (Transl. by D. F. Goodwin.) Amsterdam: Benjamin.
- Coe, R. et al. (Eds.) *The Rhetoric and Ideology of Genre: Strategies for Stability and Change*. Hampton Press.
- Davies, B. & R. Harré (1990) Positioning: The Discursive Production of Selves. *Journal for the Theory of Social Behaviour*. 20/1:43-63.
- Dewey, J. (1916) *Democracy and Education*. New York: The Macmillan Comp.
- Durkin, K. and B. Shire (Eds.) (1991) *Language in Mathematics education - research and practice*. Milton Keynes, England: Open University Press.
- Ernest, P. (1997) Mathematical activity and rhetoric: A semiotic analysis of an episode of mathematical activity. In *Philosophy of Mathematics Education Journal*. 10.
- Ernest, P. (1998) *Social Constructivism as a Philosophy of Mathematics*. NY: State University of New York Press.
- Evans, J. and A. Tsatsaroni (1994) Language and subjectivity in the mathematics classroom. In S. Lerman (ed.) *The culture of Mathematics Classroom*. Dordrecht: Kluwer.
- Hernadi, P. (1995) *Cultural Transactions. Nature, Self, Society*. London: Cornell.
- Habermas, J. (1984) *The Theory of Communicative Action*. (Vol 1.) Boston: Beacon. (Transl. T. McCarthy.)
- Habermas, J. (1988) *Kommunikativt handlande. Texter om språk, rationalitet og samhälle*. Göteborg: Daidalos.
- Habermas, J. (1998) *Postmetaphysical Thinking: Philosophical Essays*. London: Polity Press.
- Halliday, MAK (1978) *Language as socialsemiotic*. London: Arnold.
- Halliday, MAK (1994) *An Introduction to Functional Grammar. Second Edition*. London: Arnold.
- Harré, R. (1980) *Social Being*. Totowa: Rowman and Littlefield.
- Harré, R. (1984) *Personal Being*. Cambridge: Harvard University Press.
- Harré, R. (1991) *Physical Being*. Oxford: Blackwell.
- Harré, R. & L. van Langenhove (1991) Varieties of Positioning. *Journal for the Theory of Social Behaviour* 21/4:393-407.
- Hersh, R. (1997) *What is Mathematics, Really?* London: Jonathan Cape.
- Hiim, H. and E. Hippe (1998) *Læring gjennom opplevelse, forståelse og handling*. [ Learning through experience, understanding and action.] Oslo: Universitetsforlaget.
- Illeris, K. (1999) *Læring - aktuell læringsteori i spenningsfeltet mellom Piaget, Freud og Marx*. Roskilde Universitetsforlag.
- Kant, I. (1987/1790) *Critique of Judgment*. Indianapolis: Hackett.
- Kragh, H. (1999) Det smukke og det sanne. Æstetiske Principper i de Eksakte Videnskaper. [The beauty and the true. Aesthetical principles in the exact sciences.] In J. Holmgaard (ed.) *Æstetik og logik*. [Aesthetics and Logic.] Ålborg: Medusa.
- Künzli, R. (1998) The Common Frame and the Places of Didaktik. In Gundem, B. and S. Hopmann (eds.) *Didaktik and/or Curriculum. An International Dialogue*. NY: Peter Lang.
- Langenhove, L. van & R. Harré (1993) *Positioning and Autobiography: Telling Your Life*. In N. Coupland & J. Nussbaum (Eds.) *Discourse and Life-span Development*. London: Sage.
- Maddy, P. (1990) *Realism in Mathematics*. Oxford: Oxford UP.
- Martin, J. (1997) Analysing genre: functional parameters. In Christie, F. and J. Martin (Eds.) *Genre and Institutions*. London: Cassell.
- Mellin-Olsen, S. (1987) *The Politics of Mathematics Education*. Dordrecht: Kluwer.
- Miller, R. C. (1998) Three versions of objectivity: aesthetic, moral and scientific. In J. Levinson (ed.) *Aesthetics and Ethics. Essays at the Intersection*. Cambridge, UK: Cambridge UP.
- Nygaard, O. et al (1999) *Aha. Matematikk og matematikdidaktikk*. Kristiansand: Høyskoleforlaget.
- Nöth, W. (1990) *Handbook of Semiotics*. Bloomington: Indiana University Press.
- Ogden, C. and I. A. Richards (1923) *The Meaning of Meaning*. London: Routledge and Keagan Paul.
- Ongstad, S. (1995) The concept of 'positioning': bridging a 'false dichotomy' between structuralisms and their 'posts'? Paper at AARE' s conference in Hobart. November. (Ref: <http://www.aare.edu.au/95pap/ongss95.071>)
- Ongstad, S. (1996) Literacies and Mother Tongue Education. The challenge of task ideologies in a semiotic-didactic perspective. In F. Christie and J. Foley (eds.): *Some Contemporary Themes in Literacy Research*. New York and Berlin: Waxmann.
- Ongstad, S. (1997) *Sjanger, posisjonering og oppgaveideologier*. Dr. thesis at NTNU, Trondheim.
- Ongstad, S. (1999a) *Sources of 'didaktization'. On defining disciplines and their '(fag-)didaktik' across borders illustrated with examples from Mother Tongue Education (MTE)*. In Hudson, B. et al (eds.) *Didaktik/Fachdidaktik as Science(s) of the Teaching Profession?* TNEE Publications, Vol. 2, no.1.
- Ongstad, S. (1999b) Self-positioning(s) and students' task reflexivity - a semiotic macro concept exemplified. *Journal of Structural Learning & Intelligent Systems*. Vol. 14(2) pp 125-152.
- Ongstad, S. (2002) Genres - from static, closed, extrinsic, verbal dyads to dynamic, open, intrinsic semiotic triads. In R. Coe et al. (Eds.) *The Rhetoric and Ideology of Genre: Strategies for Stability and Change*. Hampton Press.
- Ongstad, S. and H.J. Braathe (2001) *Analysis of positioning(s) used on studies of textbooks in the didactics of mathematics*. Paper at the symposium "An international study of students' self-positioning" IAIME's conference. Amsterdam. July.
- Poulsen, A. (1994) *Børns utvikling*. Copenhagen: Gyldendal.
- Ricoeur, P. (1981) *Hermeneutics and the Human Sciences*. Cambridge: Cambridge Univ. Press.
- Rommetveit, R. (1972) *Språk, tanke og kommunikasjon*. Oslo: Universitetsforlaget.
- Sáenz-Ludlow, A. (2001) *Classroom Mathematics Discourse as an Evolving Interpreting Game*. Paper at PME25, Utrecht.
- Saussure, F. de (1916/1974) *Course in General Linguistics*. (By. J. Culler.) London: Fontana.
- Westbury, I. (1998) Didaktik and Curriculum Studies. In Gundem, B. and S. Hopmann (eds.) *Didaktik and/or Curriculum. An International Dialogue*. NY: Peter Lang.

## APPENDENCIES

### 1. Overview over triads as consequence of foci on major fields of interest

#### Communicational triads

Utterance, genre, language as...	form	content	use
Utterance, genre, language as...	structure	reference	act
Processes	structuring	referring	acting/addressing
Process as focused characteristics	expressivity	referentiality	addressivity
Fields of language and semiotics	syntax	semantics	pragmatics
Grammatically metaphorized as	adjective	noun	verb
Seen as pronoun	I	it	you
Contexts of the body	self	world	society
Domain of reality (Habermas)	inner nature	outer nature	society
Halliday's language contexts	mode	field	tenor
Contexts of semiotic space	(form) forum	universe (of meaning)	(acting) arena

#### Triads used by some theorists

Bühler (1934) the language sign	expression	representation	appeal
Language sign (in German)	(Ausdruck)	(Darstellung)	(Appell)
Aspect, focused by receiver	symptom	symbol	signal
Ben-Amos (1969)	prosodic	thematic	behavioral
	expressive	cognitive	functional
Halliday (1978) metafunctions/language	textual	ideational	interpersonal
context/register	mode	field	tenor
Miller (1984)	form	substance	action
Fairclough (1992)	identity	ideational	interpersonal
Bakhtin (1986) constituents of utterances	expressivity	referentiality	addressivity
Baumann (1995) processes	aesthetic	cognitive	moral
crucial elements	emotional intensity	knowledge	responsibility

#### Habermas (1984)

Aspect of lifeworld	inner nature	outer nature	society
Form of appearance			
for reality relations	subjectivity	objectivity	normativity
Speech acts	expressive	propositional	illocutionary
General functions	experiences	matters	relations
Mode	expressive	cognitive	interactive
Speech act	representative	constative	regulative
Theme	utterer's intention	propositional content	interpersonal relation
Validity claim	truthfulness/veracity	truth	fairness, usefulness
Components of the lifeworld	personality	culture	society
Reproduction processes	socialization	cultural reproduction	social integration
Communicative action	sociation	understanding	coordination

#### Hernadi (1995)

Major elements	self	nature	society
Other major elements	existence	culture	experience
Rhetorical aims	delighting	teaching	moving
Discourse involve	individual identity	natural facticity	social activity
Psychological capacities	feeling	knowing	willing
Evaluative criteria	beauty	truth	justice

#### Traditional

'Traditional' concepts	emotional	cognitive	social
'Traditional' concepts	existential	informative	addressive
'Traditional' concepts	beauty	truth	goodness
Fields by evaluation of outcomes	aesthetics	epistemology	ethics
Evaluations (ranging from)	ugly - nice	false - true	wrong - right
Societal domains	art	science	politics
Outcome from the domains	artwork	research	decision/regulation
Traditional triads	feelings	thought	will
Alliterations	heart	head	hand
	beauty	brain	brawn
	identity	idea	interrelation

#### Aspects positioned as..

State of the body	being (as a noun)	thinking (as a noun)	doing (as a noun)
Bodily processes	being (as a verb)	thinking (as a verb)	doing (as a verb)
Effect in/of the sign process	impulse	category	process

### Positioning(s) of professional fields, approaches, and isms when dominant

Aspects of language/semiotics	syntax	semantics	pragmatics
General approaches	aesthetics	epistemology	ethics
Professions/disciplines (typical example)	psychology	science	sociology
Different neg. characterizations	subjectivism	objectivism	activism
Different neg. characterizations	expressivism	positivism	functionalism
Different neg. characterizations	formalism	essentialism	pragmatism

### Triads in didaktik and education

Aspects in communication and teaching/learning:

Traditional didaktik triangles	student	content (subject matter)	teacher
	teacher	content (subject matter)	student
	student teacher	content (subject matter)	pupil
	teacher educator	content (subject matter)	student teacher

### Triads in didaktik and education (NB! Just suggested EXAMPLES, not ment as systemic aspects!)

Teacher(s) can be....	the formal	the omni-cogent	the doer
Task(s) can be....	creative writing	cognitivism	change the world
Student(s) can be....	the engaged child	preferred topic	strategic writing
Writing(s) can focus...	modality	content	performance
Evaluation(s) can focus...	faults	maturity	functionality
Evaluator(s) can be...	formalist	semanticist	pragmaticist
Researcher(s) can focus...	psychologist	pedagogue	sociologist
Research text(s) can focus...	attitude/engagement	rationality	change practice
Meta-researcher(s) can focus...	structure of fields	content of fields	function og fields
Meta research text(s) can function(s) as	commitment	'proof'	argument
Didactics	aesthetics	epistemology	ethics
Processes	feeling	knowing	doing
Outcomes	emotions	knowledge	action
The Greek/rhetoric tradition (aims)	pathos	logos	ethos
Historical aims	beauty	truth	goodness
Dewey (1915)	feeling	thinking	doing
Hiim&Hippe (1998)	experience	understanding	action
Illeris (2000)	psychic	cognitive	social
Illeris (2000)	Freud	Piaget	Marx
Curricular goals for the subject Norwegian as first language in L97, The Norwegian Core Curriculum	identity experience	'Bildung' culture	skills communication
Curricular goals for Norwegian for student teachers (Language and text = L&T)	the student's own work on L&T	knowledge of L&T	L&T in a teaching/ learning perspective

## 2. Wholeness in General Teacher Education? Positioning the triad individual-world-society in the profession's (fach-)didaktics and practice

Project responsible: Professor Sigmund Ongstad. E-mail: Sigmund.Ongstad@lu.hio.no  
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Other participants: Assistant professors Hans Jørgen Braathe and Leif Kværnes, FTE/OUC.  
Network partners: Associate professor Inger Anne Kvalbein, FTE/OUC, associate professor Piet Hein van de Ven, University of Nijmegen.  
Project periode: 1.8.2000-31.7.2004 (Supported by NRC to 2003)

The project develops further theories of communication in order to study (im-)balances between orientations towards individual, world and society in fach-didaktics in general teacher education. This part, which concerns the relationship between aesthetics, epistemology and ethics in different fach-didaktics, is more directed to basic (scientific) research. The focus on wholeness is more oriented towards practice by problematizing 'compatibility' and 'wholeness' in and between each didaktics. The ambition is to develop a theory as a basis for analyzing (im-)balances in teacher education as a profession by moving together the didaktik and the communicational triads.

The project analyses textbooks and written curricula in fach-didaktics by focusing the affective domain in the triad aesthetics-epistemology-ethics. This approach implies a focus on specific elements in what can be called a triadic communicational perspective, building on Habermas, Bakhtin and Halliday. The analyses are further related to conceptions among student teachers and teacher educators. Encounters between didaktik texts and these participants in different teaching/learning contexts are studied.

The analysis of texts are connected to the *Sakprosa*-project (at the University of Oslo), the fachdidaktik-part to the course/seminar *Coordinated didaktics of teacher education* (Norgesnettrådet), the EMDID-project (interdisciplinary fach-didaktics) and the studies of the positionings to International Mother Tongue Educational Network (IMEN). Braathe is working on *Integration of Disciplines and interdisciplinarity in education for professions*, focusing mathematics in teacher education. Leif Kværnes' doctoral project is about students teachers' socializing to teachers of mathematics focusing different learning arenas in the education. The overall project is partly coordinated with Inger Anne Kvalbein's project *Profession orientation in general teacher education as a whole* (another project connected to KUPP).

### 3. Some of SO's works/publications (regarding genre, utterance, and positioning)

Braathe, H.J., L. Kværnes and S. Ongstad (2000) *Triads in mathematics education. Mellin-Olsen (1987) and semiotic genre theory - comparisons and exemplifications*. Paper at Oslo University College and University of Oxford's international symposium on mathematics education. Gausdal, Norway. March.

Braathe, H.J. and S. Ongstad (2001) Egalitarianism meets ideologies of mathematical education - instances from Norwegian curricula and classrooms. *ZDM*, Vol. 33 (5).

Ongstad, S. (1996) Literacies and Mother Tongue Education. Genres and literacy in Norwegian as MTE: semiotic-didactic perspectives on students' positionings to task ideologies. In F. Christie and J. Foley (eds): *Some Contemporary Themes in Literacy Research*. New York and Berlin: Waxmann.

Ongstad, S. (1997) *Sjanger, posisjonering og oppgaveideologier. Et teoretisk-empirisk bidrag til et tverrfaglig, semiotisk og didaktisk sjangerbegrep*. Ph D. thesis, NTNU, Trondheim.

Ongstad, S. (1999a) Sources of 'didaktization'. On defining disciplines and their '(fag-)didaktik' across borders illustrated with examples from Mother Tongue Education (MTE). In Hudson, B. et al. (eds.) *Didaktik/ Fachdidaktik as Science(s) of the Teaching Profession?* TNTEE Publications, Vol. 2, no.1.

Ongstad, S. (1999b) Vad är positioneringsanalys? "Själpositionering" i en (post)modern skola som exempel. In C. A. Säfström and L. Østman (eds.): *Textanalys. En introduktion till syftesrelaterade analyser*. Lund: Studentlitteratur.

Ongstad, S. (1999c) Self-positioning(s) and students' task reflexivity - a semiotic macro concept exemplified. *Journal of Structural Learning and Intelligent Systems*. Vol. 14 (2).

Ongstad, S. (2001) *Positioning Aesthetics, Epistemology and Ethics in Didaktik of Subjects*. Paper, ECER 2001. Part of the symposium "Different cultures, different meanings, different didactics?" Lille. September.

Ongstad, S. (2002a) Genres - from static, closed, extrinsic, verbal dyads to dynamic, open, intrinsic semiotic triads. In R. Coe et al. (eds.) *The Rhetoric and Ideology of Genre: Strategies for Stability and Change*. Hampton Press.

Ongstad, S. (2002c) *Didaktik and/as communication. Or aesthetics, epistemology and ethics as mutual*. Paper at NFPF/NERA's conference in Tallinn. March.

Ongstad, S. (2002c) *Wholeness in Teacher Education? Positioning Fach-didaktik*. Paper at NFPF/NERA's conference in Tallinn. March.

Ongstad, S. (in print a) *Didaktik and/as Communication*. Article in an electronic textbook in didaktik/fach-didaktik for European student teachers. Part of the EMDID-project, supported financially by EU. Available autumn 2002 on <http://ive.pa-linz.ac.at/etextbook/>

Ongstad, S. (in print b) *Positioning*. Article in an electronic textbook in didaktik/fach-didaktik for European student teachers. Part of the EMDID-project, supported financially by EU. Available autumn 2002 on <http://ive.pa-linz.ac.at/etextbook/>

Ongstad, S. (in print c) *Mathematics as communication*. Article in an electronic textbook in didaktik/fach-didaktik for European student teachers. Part of the EMDID-project, supported financially by EU. Available autumn 2002 on <http://ive.pa-linz.ac.at/etextbook/>

Ongstad, S. (manuscript) *Outline of a Theory of Triads in Communication and Education. Positioning(s) (in) a theory of general science*. HiO/LU. NFR-prosjektet Helhetlig allmennlærerutdanning? Oslo: Oslo University College, Faculty of Education. 123 pages.

Ongstad, S. and H.J. Braathe (2001) *Analysis of positioning(s) used on studies of textbooks in the didactics of mathematics*. Paper at the symposium "An international study of students' self-positioning" IAIMTE's conference. Amsterdam. July.

## 4. What is (analysis of) positioning(s)?

### 1. Positioning as a concept

- a) Other's use of the concept: Schefflen (1973), McDermott (1976), Davies and Harré (1990), Harré and van Langenhove (1991), Langenhove and Harré (1993), Davies and Hunt (1994), Evans and Tsatsaroni (1994), Heinämaa (1994)
- b) Bakhtin (1986)
- c) Process and/or product: Position - positioning - positioning(s)
- d) Other concepts: Position, agency, motivation, action(s), role...

### 2. Positioning as triadic, semiotic micro-macro processes

- a) 'Triadic':  
Syntactic structuring of form (= structure)  
Semantic referring to content (= reference)  
Pragmatic addressing as use (= action)  
...and these three reciprocal (mutual) complementarity and dynamics on all levels
- b) 'Semiotic':  
Positioning is supposed to be used independently of a particular sign system  
Positioning is going on in many sign systems at once (it is a bodily process)  
Positioning is going on in time and space ('chronotope') (in 'reality')  
Positioning conveys the semiosis
- c) 'Micro-macro':  
Positioning happens in relation to an immanent potential of expressivity (macro)  
Positioning consists of balance and dynamics of theme and rheme (given and new)  
Positioning is a micro-macro process, that is - dynamics of utterance and genre

### 3. Other aspects. Positioning...

is even a process for the receiver, not only for the utterer  
is partly conscious (the focused part) and unconscious (the ideologies of the genre)  
happens with utterers, the observers, the researcher, the readers.. ('chinese boxes')  
is a relational concept (not a category)

### 4. The system of positioning is ...

relational  
systemic  
rhizomatic  
constantly changed by all positioning(s) both systemically and rhizomatically

### 5. Analyses of positioning(s)...

can in principle be done with any method or approach  
has to take point of departure in a concrete relationship of phenomena  
demands an explicit treatment of the question of validity  
may end up in four basic traps: Subjectivism, objectivism, functionalism and relativism

## 5. Positioning Knowledge in the Project "Wholeness in General Teacher Education"

1. All learning happens by and through communication.
2. All learning is semiotic. (Semiotics is the study of self, nature and culture as meaning.)
3. Learning is therefore semiosis, which means change of meaning (as a neverending) process.
4. Semiosis functions as dynamics between (a focused) utterance and its context (figure/ground).
5. Learning is hence an inevitable split in two, between the focused and its context.
6. Knowledge is body-stored learning; as the body is (in) the context and (in) the utterance.
7. Knowledge is thus a potential for meaning and uttering.
8. Formulated knowledge (expressed knowledge) is always just an utterance, not the knowledge.
9. Uttered 'knowledge' is communicated triadically as/by dynamic mutual aspects of form,  
content and use.
10. Knowledge as potential for meaning and uttering is 'stored' simultaneously as utterances  
and genres.
11. Bodily and mental memories take care of ('remember embodied') utterances as such and as  
dynamic context.
12. Judging/evaluating form is aesthetics, content epistemology and use ethics.
13. Hence aesthetics, epistemology and ethics function mutually and cannot be separated.
14. Aesthetics, epistemology and ethics participate as inevitable, but differently balanced aspects  
in all learning.
15. Hence different fachs/disciplines/subjects and fachdidaktiks distributes aesthetics, episte-  
mology and ethics systemically differently according to disciplinary aims and ideologies.
16. The didaktic triad is a specific version of the communicational triad, i.e. it is semiotic.
17. The communicational aspects regulate the basic conditions for the validities of knowledge.
18. The validity of uttered knowledge is bound to genres.
19. Genres are systemic, embodied forms of communication for production and interpretation  
of utterances, characterized by a balance between more open and more closed patterns.
20. Grades and forms of knowledge is directly connected to different genres.
21. Knowledge cannot be separated from its genres.
22. The study of knowledge and learning can be performed by positioning.
23. Positioning incorporates the paradox of the simultaneity of being within and looking at  
utterances and their genres.
24. Hence the approach positioning studies utterances in a double triadic system, since both  
utterances (texts) and genres (contexts) are seen as triadic.
25. The immediate embodied context of form is the 'self'. The immediate embodied context of  
content is 'world' and the immediate embodied context for use is 'society'.
26. Didactic regimes use disciplinary genres to create, evaluate and theorize knowledge.