

Pavement Management Decision Analysis Using Belief Functions in Valuation-Based Systems

N. O. Attoh-Okine
Florida International University
Department of Civil and Environmental Engineering
Miami, Florida 33199
E-mail: okine@eng.fiu.edu

Abstract

Valuation-Based Systems (VBS) for belief-functions theory is applied to Pavement Management Systems (PMS) decision-making. The VBS provides a general framework for representing knowledge and drawing inferences under uncertainty. A VBS network is constructed and potentials are introduced in the form of belief-function (or basic probability assignment) in PMS decision making environment. Valuation network is another method of representing and solving Bayesian decision problems. It is based on the framework of VBS. Valuation networks depict decision variables, random variables, utility functions, and information constraints. The solution method for valuation network is called fusion algorithm, and the Dempster's rule of combination can be successfully applied in this framework. It will be shown that this approach can capture the quantitative, qualitative and incomplete information in PMS decision making.

1. Introduction

A Pavement Management Systems (PMS) is designated to provide information and useful data for analysis so that highway managers and engineers can make more consistent, cost-effective, and defensible decisions related to the preservation of a pavement network [1]. PMS, in its broadest sense, encompasses all the activities involved in planning, design, construction, maintenance, and rehabilitation of a public works program.

In a PMS, the problem setting often involves a large number of uncertain, interrelated quantities, attributes and alternatives based on information of highly vary quality. Pavement management is generally described and developed at two levels. The network level and project level. The primary differences between the

network and project level decision-making tools include the degree or extent for which the decisions are being made and the type and amount of data required. Where network level decisions are concerned with programmatic and policy issues for an entire network, the project level decisions address the engineering and economic aspects of pavement management.

Many important problem in PMS modeling can be addressed using decision analysis, a probabilistic approach for making decisions under uncertainties. Decision trees has been long established method for representing and solving PMS problem under uncertainty and when there are trade-offs among strongly held values. Decision trees graphically depict all possible scenarios of the problem and the outcome. The decision tree representation allows the computation of an optimal strategy by the recursion method of dynamic programming [2]. The strength of the decision tree representation method lies in its simplicity and its flexibility. Decision trees are based on the semantics of scenarios. These semantics are very intuitive and easy to understand.

The weakness of the decision tree representation in PMS analysis is its modeling of uncertainty, informative constraints, it combinatorial explosiveness, and finally the preprocessing of probability that may required prior to the tree representation. A brute-force computation of the desired conditionals from joint distribution for all variables is intractable if there are many random variables [3].

2. Valuation-Based Systems

2.1 General

Valuation-based systems (VBS) have been proposed by Shenoy [4]. Attoh-Okine [5] presented the potential application of VBS in PMS decision-making.

A graphical depiction of a valuation-based systems is called a valuation network. Valuation networks are compact representation emphasizing qualitative features of symmetric decision problems. Valuation networks depict decision variables, random variables, utility functions, and information constraints. They are based on semantics of factorization. The semantics of VBS can represent and solve Bayesian decision and optimization problems. Each probability function is a function of the joint probability distribution function, and each utility function is a function of joint utility function.

A valuation-based system representation of a decision problem consists of a finite set of decision variables X_D , a finite set of random variables X_R , a finite frame W_X for each variable for each variable X in $X_D \cup X_R$, a finite collection of payoff valuations $\{\pi_1, \dots, \pi_m\}$, a finite collection of potentials $\{\rho_1, \dots, \rho_n\}$, and a precedence relation \rightarrow on X_D and X_R . Thus, a valuation-based systems (VBS) for a decision problem can be denoted formally by the 6-tuple:

$\Delta = \{X_D, X_R, \{W_X\}_{X \in X}, \{\pi_1, \dots, \pi_m\}, \{\rho_1, \dots, \rho_n\} \rightarrow\}$. The VBS representation of a canonical decision problem is illustrated in **Figure 1**.

Figure 2 shows the corresponding decision tree representation of the canonical decision problem. A canonical decision problem Δ_c consists of a single decision variable D with a finite frame W_D , a single random variable R with a finite frame W_R , a single payoff valuation π for $\{D, R\}$, a single conditional potential ρ for R given $\{D\}$, and a precedence relation \rightarrow defined by $D \rightarrow R$. The meaning of the canonical problem is as follows. The elements of W_D are acts, and the elements of W_R are states of nature. The conditional potential ρ is a family of probability distribution for R , one for each act $d \in W_D$, that is the probability distribution of random variable $R=r$ given that $D=d$.

In valuation network, circular nodes represent random variables, rectangular nodes represent decision variables, triangular nodes represent potentials, and diamond-shaped nodes represent utility functions. The undirected edges linking variables to potential and utility functions denote the domains of these functions. The directed arcs between variables define the information constraint. If there are no random variables in the problem, it reduces to an optimization problem.

2.2 Variables and Frames

A decision node is represented as a variable. The set of all possible values of variable X , denoted by W_X , is called the frame for X . Given a non-empty subset of h of variables, W_h denotes the cartesian product of W_X for X in h , i.e. $W_h = \times \{W_X \mid X \in h\}$. If h is a subset of variables, potential (or a probability function) α for h is a function $\alpha: W_h \rightarrow [0,1]$. The values of potential α are probabilities, and h is call the domain of α . If h is a subset of variables, a utility function v for h is a function $v: W_h \rightarrow \mathbf{R}$. The values of utility function v are utilities, and is called the domain of v . If a set of variables h is empty, the following convention is adopted; for empty set \emptyset consists of a single configuration, and we use the symbol \diamond to name that configuration $W_\emptyset = \{\diamond\}$.

2.3 Valuations

Valuations are primitives in the VBS framework. Given a subset h of variables in H , there is a set V_h . V_h is called valuations. Intuitively, a valuation for h represents some knowledge about the variable in H . Let V denote the set of all valuations, i.e. $V = \cup \{V_h \mid h \subseteq X\}$. If σ is a valuation for h , then h is the domain of σ . The values of utility valuations are utilities.

2.4 Non-zero Valuations

For each $h \subseteq X$, there is a subset P_h of V_h whose elements are non-zero valuations for h . Let P denote $\cup \{P_h \mid h \subseteq X\}$ the set of all non-zero valuations. Intuitively, a non-zero valuation represents knowledge that is internally consistent.

2.5 Precedence Constraints

Besides acts, events, probabilities and payoffs, an important ingredient of VBS representation is information constraint. Generally, four constraints are needed for precedence relation:

- (a) The transitive closure of \rightarrow , denoted by $>$, is a partial order (irreflexive and transitive) of $X = X_D \cup X_R$. This is called partial order condition;
- (b) For any $D \in X_D$ and any $R \in X_R$, either $R > D$ or $D > R$ this is called perfect recall condition;
- (c) If there is conditional potential for R ($R \in X_R$) gives $h-\{R\}$, and there is a decision variable $D \in h$, then $D > R$;

Suppose $h \subseteq X$, $R \in h$, $R \in X_R$ and ρ is a potential for h . ρ is called a conditional potential for R given

$h\text{-}\{R\}$ if $\rho^{+h\text{-}\{R\}}$ is vacuous potential. In Figure 1 ρ is a conditional potential for R given $\{D\}$.

- (d) If there is potential for h and a decision variable $D \in h$, then $D > R$ for some random variables $R \in h$. The third and fourth conditions are call consistency conditions.

Given the meaning of the precedence relation \rightarrow , for any decision variable D and any random variable R , either R is known when decision D has to be made or not. This translates to either $R > D$ or $D > R$. Finally, the consistency conditions are dictated by the meaning of potentials. If the conditional probability distribution of random variable R depends on the act chosen by the decision maker at node D , then it must be the case that $D > R$.

2.6 Solution of VBS

The main objective in solving a decision problem In PMS is the computation of optimal strategy. This can be approached in two steps. First we compute the maximum expected value of the utilities. Second, we compute an optimal strategy that gives the maximum expected value.

A strategy is a choice of an act for each decision variable D as a function of configuration of random variables $R > D$. Furthermore, each time decision variable is eliminated from valuation using maximization, we store the table of optimal values of the decision variable where the maximums are achieved. This table can be regarded as a function, and it can be called the solution for that decision variable. Suppose $D \subseteq h$, v is a valuation for h , we use $\Psi_D: W_h \rightarrow W_D$ to denote solution for D .

In solving VBS, two operators, combination and marginalization are used: Combination is a mapping $\otimes V \times V \rightarrow V$; such that:

- (i) If ρ and σ are valuations for r and h , respectively, then $\rho \otimes \sigma$ is a valuation for $h \cup r$;
- (ii) If either ρ or σ is not non-zero valuations, then $\rho \otimes \sigma$ is not a non-zero valuation; and
- (iii) If ρ and σ are both non-zero valuations, then $\rho \otimes \sigma$ may not be a non-zero valuation.

$\rho \otimes \sigma$ is called the combination of ρ and σ . The

combination depends on the type of valuations being combined. The combination of two-payoff valuations is a payoff valuation, the combination of two potentials is a potential, and the combination of a payoff valuation and a potential is a payoff valuation. The combination of two payoff consists of point-wise addition, combination of two potentials consist of point-wise multiplication, and finally the combination of a payoff valuation and a potential consists multiplication.

Marginalization: Suppose h and g are subsets of variables, and suppose g is a subset of h . For each $h \subseteq X$, there is a mapping $\downarrow h: \cup \{v_g \mid h \subseteq g\} \rightarrow v_h$, called marginalization to h , such that, if v is a valuation for g and $h \subseteq g$, then $v^{\downarrow h}$ is a valuation on h . Marginalization does not depend on the type of valuation being marginalized. But the definition of marginalization does not depend on the type of variables being eliminated. If the variables being eliminated are random, marginalization is achieved by summing the valuation over the frame of the eliminated variable. If the variable being eliminated is a decision variable, marginalization is achieved by maximization (or minimization depending on the nature of the values of the payoff valuations) over the frame of eliminated variables. With the concepts of marginalization clearly outlined we can now explain the concepts of vacuous potential. Suppose ρ is a potential for g . ρ is a vacuous potential if and only if $v \otimes \rho = v$ for all potentials v for g .

The maximum expected utilities is obtained by:

$$(\otimes \{\pi_1, \dots, \pi_m, \rho_1, \dots, \rho_n\})^{+\Delta} (\diamond) \quad (1)$$

The optimal strategy σ^* that gives us the maximum expected value of Δ is determined as follows:

$$(\pi \otimes \rho)^{+\Delta}(d) = (\otimes \{\pi_1, \dots, \pi_m, \rho_1, \dots, \rho_n\})^{+\Delta} (\diamond) \quad (2)$$

where π , ρ , and D refers to the equivalent canonical decision problem Δ_c .

The solution method for VBS is based on fusion algorithm (3). The basic idea is to successively delete all variables from the VBS. The sequence in which the variables are deleted must respect the precedence constraint. The following theorem describes the fusion algorithm for making inferences in VBS.

Theorem (6)

Suppose $\Delta = \{X_D, X_R, \{W_x\}_{x \in X}, \{\pi_1 \dots \pi_m\}, \{\rho_1 \dots \rho_m\}, \rightarrow\}$, is a well-defined decision problem. Suppose $X_1 X_2 \dots X_k$ is a sequence of variables in $X = X_D \cup X_R$ such that with respect to the partial order

\succ , X_1 is a minimal element of X , X_2 is a minimal element of $X - \{X_1\}$, etc.

Then $\{(\otimes \{\pi_1 \dots \pi_m, \rho_1, \dots, \rho_n\})^{\dagger \emptyset}\} = \text{Fus}_{x_k} \{ \dots \text{Fus}_{x_2} \{ \text{Fus}_{x_1} \{ \pi_1, \dots, \pi_m, \rho_1, \dots, \rho_n \} \} \}$

3. Belief Function in VBS

3.1. Belief functions

Potentials can be defined as belief functions or basic probability assignment in VBS. A basic probability assignment (bpa) function μ for h is a function $\mu: 2^{W_h} \rightarrow [0, 1]$, such that $\sum \{\mu(a) \mid a \in 2^{W_h}\} = 1$ (2^{W_h} denotes the set of all nonempty subsets of W_h).

Intuitively $\mu(a)$ represents degree of belief assigned exactly to a (the preposition that the true configuration of h is in the set a). A bpa function is the belief function equivalent of a probability mass function in probability theory. Whereas a probability mass function is restricted to assigning probability masses only to singleton configurations of variables, a bpa function is allowed to assign probability masses to sets of configurations without assigning any mass to the individual configurations contained in the sets.

The belief function Bel_μ associated with the bpa μ is defined by:

$$\text{Bel}_\mu = \sum \{\mu(b) \mid b \subseteq a\} \quad (3)$$

Another way of expressing the information contained in bpa function μ is in terms of the plausibility function PL :

$$\text{PL}(a) = \sum \{\mu(b) \mid b \cap a \neq \emptyset\} \text{ for each } a \in 2^{W_h}. \quad (4)$$

Intuitively, the plausibility of a is the degree to which a is plausible in the light of the evidence. A zero plausibility for a hypothesis means that we are sure that it is false, but a zero degree for preposition means only that we see no reason to believe the preposition.

3.2 Combination and Marginalization

Two concepts, projection and extension of configurations are needed to explain combination and marginalization. Projection of configuration simply mean dropping extra coordinates; if (w, x, y, z) is a configuration of (W, X, Y, Z) , for example, the projection of (w, x, y, z) to (W, X) is simply (w, x) which

is a configuration of $\{W, X\}$. If g and h are sets of variable, $h \subseteq g$, and X is a configuration of g , then X^{+h} denotes the projection of X to h . If $h = \emptyset$, then $X^{+h} = \diamond$. If h is a subset of W_h , then the extension of h to g , denoted by h^{+g} .

Suppose h and g are subset of $X \in h$. Let v_i , a valuation for g , be represented as a set of $(a \mu(a))$, v_j , a valuation for h , be represented as a set of $(b \mu(b))$ and the resulting valuation $v_i \otimes v_j$, a valuation for $g \cup h$, be represented as a set of $(c \mu(c))$, where c is non empty subsets of $W_{g \cup h}$ and $c \subseteq \{a^{+(g \cup h)} \cap b^{+(g \cup h)}\}$. If both v_i and v_j are belief functions, then their combination is defined by Dempster's rule of combination:

$$v_i \otimes v_j = K \sum \{\mu_i(X^{+h}, a) \mu_j(X^{+g}, b) \mid a^{+(g \cup h)} \cap b^{+(g \cup h)} = c\}; \quad (5)$$

$$\text{where } K = 1 - \sum \{\mu_i(a) \mu_j(b) \mid a^{+(g \cup h)} \cap b^{+(g \cup h)} = \emptyset\}$$

If neither of v_i and v_j are belief functions, then $v_i \otimes v_j$ is obtained by:

$$\sum \{\mu_i(X^{+h}, a) + \mu_j(X^{+g}, b) \mid a^{+(g \cup h)} \cap b^{+(g \cup h)} = c\} \quad (6)$$

Otherwise, $v_i \otimes v_j$ is obtained by:

$$\sum \{\mu_i(X^{+h}, a) \mu_j(X^{+g}, b) \mid a^{+(g \cup h)} \cap b^{+(g \cup h)} = c\} \quad (7)$$

The combination has the following properties:

- Commutativity
- Associativity
- If both v_i and v_j are belief functions, then $v_i \otimes v_j$ is also belief function
- If both are utility valuations, the combination is also utility valuations
- The combination of mixture of belief functions and utility is not associative. In such case the combination are defined as follows (7): Utility valuations are combine d before belief functions. For example, suppose π_1, \dots, π_m are utility valuations and $\text{bel}_1, \dots, \text{bel}_n$ are belief functions. Then $(\otimes \{\pi_1, \dots, \pi_m, \text{bel}_1, \dots, \text{bel}_n\})$ denotes $(\otimes \{\pi_1, \dots, \pi_m\}) \otimes (\otimes \{\text{bel}_1, \dots, \text{bel}_n\})$.

Marginalization: Suppose μ is a bpa function for h , and suppose $X \in h$. The marginal about variables in $h - \{X\}$, denoted by $\mu^{+(h - \{X\})}$, is a bpa function for $h - \{X\}$ defined as follows:

$$\mu^{+(h - \{X\})}(a) = \sum \{\mu(b) \mid b \subseteq W_h \text{ such that } b^{+(h - \{X\})} = a\}.$$

Let the valuation for $h - \{X\}$ be represented by a set of $(c, \mu(c))$. Then if X is decision variable, then for any $x \in c$,

$$\mu(x, c) = \sum \{\text{MAX}[\mu(y, a) \mid y \in a, y^{+(h - \{X\})} = x] \mid$$

$$a^{+(h-\{X\})} = c\} \quad (8)$$

if X is a random variable, then for any $x \in c$,

$$\mu(x, c) = \sum \{\lambda \text{MAX}[\mu(y, a) \mid y \in a, y^{+(h-\{X\})} = x] + (1-\lambda) \text{MIN}[\mu(y, a) \mid y \in a, y^{+(h-\{X\})} = x] \mid a^{+(h-\{X\})} = c\} \quad (9)$$

$\lambda = [0, 1]$ (Weighting factor).

4. Pavement Management System Decision Making

Figure 3 is an example of project level PMS-decision making. The valuation network representation of a project level PMS decision making is specified at three level - graphical, dependence and numeric levels.

Graphical level

Decision Nodes: In the PMS problem, there is one decision node D . D represents the decision to do overlay.

Chance: There are two chance nodes labelled R and K . R represents the dynaflect measurement and K represents pavement performance.

Utility Valuations: Utility valuation represent factors of the joint utility function and depicted by diamond shape nodes. In the PMS problem, there is one additive utility valuation labelled π . π 's domain is $\{R, D, K\}$ and π represent the payoff.

Bpa Valuations: There are two bpa valuations labelled ρ_1 and ρ_2 . ρ_1 domain $\{R, K\}$ and $\rho_2 = \{K\}$.

Information Constraint:

The PMS valuation network at the graphical level includes directed arcs between pair of distinct variables. There are two directed constraints $R \rightarrow D$, meaning that the true value of R is known to the pavement engineer who has to choose an alternative from D 's frame.

Dependence Level:

Frames: Binary acts and decision are assumed. In the PMS problem, $W_R = \{R1, \sim R1\}$, where $R1$ denotes a structurally good pavement, and $\sim R1$ a structurally failed pavement. $W_D = \{D1, \sim D1\}$, where $D1$ denotes overlay and $\sim D1$ no overlay; $W_K = \{K1, \sim K1\}$, where $K1$ denotes a pavement performance level at normal threshold value and $\sim K1$ denotes pavement performance level below the threshold value.

Numeric Level

At this present time no detail numeric computation will

be presented, but apart from the utility valuations two potentials has to be define:

Bel($R \mid K$):

$$\mu(R1 \mid K1), \mu(\sim R1 \mid K1), \text{ and } \mu(\sim R1 \mid \sim K1) = 1.$$

Bel(K):

$$\mu(K1) \text{ and } \mu(\sim K1).$$

5. Solution

The maximum expected utility is $((\pi \otimes \rho \otimes)^{+(R,D)})^{+D}(\diamond)$. An optimal strategy is given by the solution of D with respect to $((\pi \otimes \rho \otimes)^{+(R,D)})$. This is easily achieved using the fusion theorem [6].

6. Concluding Remarks

The paper presents the application of VBS for belief function in PMS decision making. At this stage the paper only provides the general framework and formulation of a problem. One advantage of the present approach is that it can capture both qualitative information (the VBS graphical level) and quantitative formulation based on the potentials. Another important advantage of this approach is that can handle incomplete information through the basic probability assignment (bpa).

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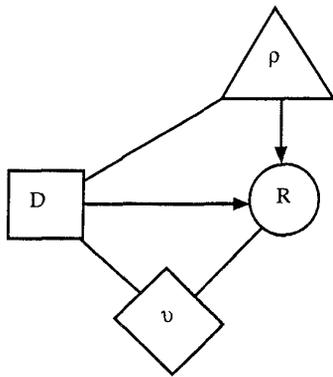


Figure 1. Graphical representation of VBS.

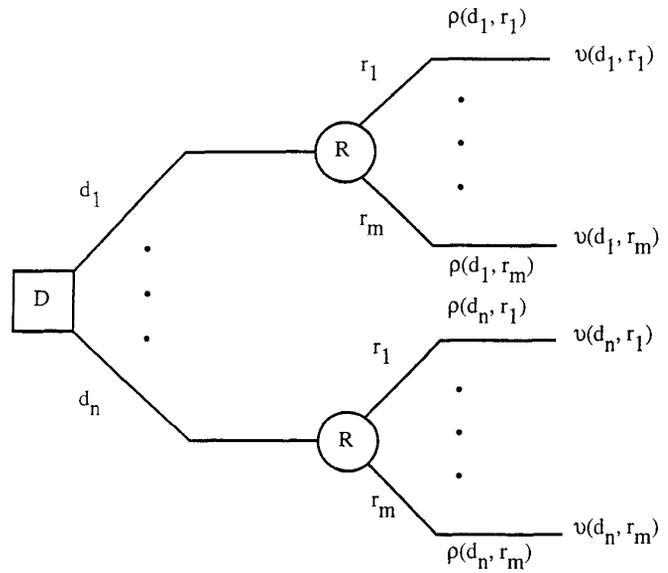
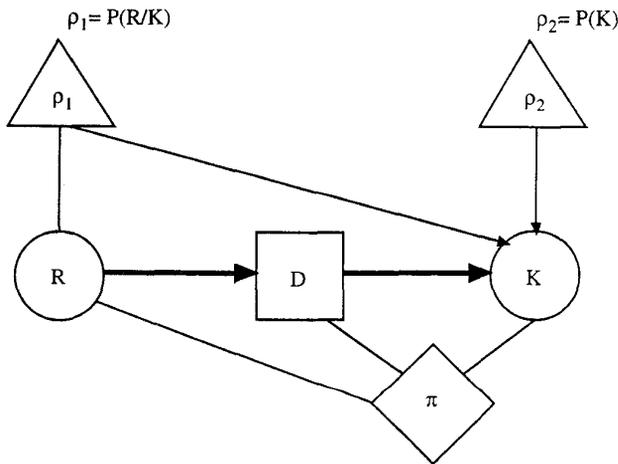


Figure 2. An equivalent decision tree representation of VBS in Fig. 1



- D - Decision to do overlay
- R - Dynaflect Measurements
- K - Pavement performance
- $W_D = \{\text{Overlay (D1), No Overlay (-D1)}\}$
- $W_R = \{\text{Structurally Good (R1), Structurally Failed (-R1)}\}$
- $W_K = \{\text{Above Threshold (K1), Below Threshold (-K1)}\}$

Figure 3. A valuation network for a pavement management system.