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Maximum Likelihood Estimation: A Single and Multi-objective Entropy Optimization Approach

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Abstract: In this paper we first considered a maximum likelihood estimation of trip distribution problem and next use primal-dual geometric programming method the said trip distribution problem converted into an entropy maximization trip distribution problem. Here the generalized cost function is assumed in different form, and then the said formulation is equivalent to single or multi-objective entropy maximization trip distribution problem. We use fuzzy mathematical programming method to show this equivalent problem formulation. The present article we use the concept of multi-objective trip distribution problem.

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Keywords: Trip distribution, Primal-dual geometric programming, Entropy, Multi-objective problem.

1. Introduction:

Entropy models are emerging as valuable tools in the study of various social and engineering problems of spatial interaction. In the study of trip distribution problems or more precisely spatial interaction problems the researcher is often confronted with phenomena, which are a pairing of two locations. These pairing may be, for example, home and business location for a worker, home and school location for a student, home and shopping center locations for a housewife, warehouse and retail shops for a company, origin and destination of a central business district of a transport system etc. In general while we may have some idea about the number of people who live, work, go to school or shop in a various locations it is very difficult to acquired information on the pairing of locations caused by the various social transactions. Because there is many such pairing, which is compatible with the data generally, available it makes sense to choose the most probable set of pairings. This is the 'Principle of Insufficient Reason' of Laplace and the resulting problem is the maximization of entropy with respect to the available information's or data.

The maximum-entropy principle initiated by Jaynes'(1957) is a powerful optimization technique of determining the distribution of random system in the case of partial or incomplete information or data available about the system. This principle has now been broadened and extended and has found wide

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applications in different fields of science and technology [Wilson(1970); Templeman and Li (1989); Kapur (1992, 1993)] .

In conventional mathematical programming, the coefficient or parameters of trip distribution problems are assumed to be deterministic and fixed. But, there are many situations where they may not be exactly known i.e., they may be somewhat uncertain in nature. Thus the decision-making methods under uncertainty are needed. The fuzzy programmings have been proposed from this viewpoint. In Fuzzy programming problems, the coefficients, constraints and the goals are viewed as fuzzy number or Fuzzy sets. It is also assume that their membership functions are known. In decision-making proces, first Bellman and Zadeh (1965) introduced fuzzy set theory. Tanaka et al. (1974) applied the concepts of fuzzy sets to decisions making problems by considering the objectives as fuzzy goals and Zimmermann (1978) showed the classical algorithms could be used to solve multi-objective fuzzy linear programming problems. The non-linear optimization problems have been solved by various non-linear optimization techniques. Among those techniques, geometric programming (GP) is an efficient and effective method to solve a particular type of non-linear problems. Duffin,Peterson and Zener (1967) , Braighter and Philips(1976) developed geometric programming to solve a class of problems called Posynomial problems.

This paper deals with two-type formulation of entropy trip distribution problem from maximum likelihood estimation of trip distribution problem. Here use fuzzy mathematical programming we are to show multi-objective entropy trip distribution problem, which is equivalent, the said formulation of the trip distribution problem.

2. Geometric Programming method:

The constrained Posynomial Geometric Programming (PGP) problem is as follows:

Minimize $$f(t) = \sum_{k=1}^{T_0} u_{k0} = \sum_{k=1}^{T_0} c_k \prod_{j=1}^{m} t_j^{\alpha_{kj}} \qquad (1)$$

subject to

$$g_i(t) = \sum_{r=1}^{T_i} u_{ir} = \sum_{r=1}^{T_i} a_{ir} \prod_{j=1}^{m} t_j^{\alpha_{rj}} \le 1 \text{ , } (i = 1,2,\ldots..,n) \qquad (2)$$

$t_j > 0$, ($j = 1,2,\ldots.,m$).

Here $c_k > 0$, ($k = 1,2,..,T_0$) and α_{kj} be any real number and T_i denotes the number of terms in the i-th constraints.

It is an constrained posynomial geometric programming problem with Degree of Difficulty (DD)=$T_0+\{ T_1 + T_2 + \ldots\ldots + T_n \} - (m+1)$.

Dual Programming (DP) problem is:

Maximize $$v(\delta) = \prod_{k=1}^{T_0}\left(\frac{c_k}{\delta_{k0}}\right)^{\delta_{k0}} \prod_{i=1}^{n}\left[\prod_{r=1}^{T_i}\left(\frac{a_{ir}}{\delta_{ir}}\mu_i\right)^{\delta_{ir}}\right] \qquad (3)$$

subject to

$$\sum_{k=1}^{T_0}\delta_{k0} = 1 \qquad \text{(Normality condition)}$$

$$\sum_{k=1}^{T_0}\alpha_{kj}\delta_k + \sum_{i=1}^{n}\sum_{r=1}^{T_i}\alpha_{rj}\delta_{ir} = 0, \; (j = 1,2,\ldots.,m) \qquad \text{(Orthogonality conditions)}$$

$$\mu_i = \sum_{r=1}^{T_i}\delta_{ir} \text{ ,} (i = 1,2,\ldots,n,)$$

$$\delta_{k0}, \delta_{ir} > 0, \quad (k = 1,2,\ldots\ldots.,T_0,\; i = 1,2,\ldots,n,)$$

For a primal problem with m variables, $T_0+\{ T_1 + T_2 + \ldots\ldots + T_n \}$ terms and n constraints, the dual problem consists of $T_0+\{ T_1 + T_2 + \ldots\ldots + T_n \}$ variables and m+1 constraints. The relationship between these problems, the optimality has been shown (Duffin et. al.(1967))to satisfy

$$u_{k0} = \delta^*_{k0} v^*(\delta^*) \,,\ \ k = 1,2,\ldots\ldots, T_0 \tag{4}$$

$$u_{ir} = \frac{\delta^*_{ir}}{\mu^*_i} \,,\ i = 1,2,\ldots\ldots,n;\ r = 1,2,\ldots\ldots, T_{i.} \tag{5}$$

Taking logarithms in (4) &(5), and putting x_j ($= \log t_j$) for $j = 1,2,\ldots,n$ we shall get a system of linear equations of x_j ($j = 1,2,\ldots,n$). We can easily find primal variables from the system of linear equations. In the next section we shall apply this method to the trip distribution model.

3. Derivation of Entropy based Trip distribution model from maximum likelihood estimations:

Let p_{ij} represent the probability of a trip from zone i to zone j. It is assumed that this probability takes the following form:

$$p_{ij} = r_i s_j f(c_{ij}) \qquad i = 1,2,\ldots\ldots, n,\ j = 1,2,\ldots\ldots\ldots, m \tag{6}$$

where r_i is a parameter representing the ability of zone i to generate trips; s_j is a parameter representing the ability of zone j to attracts trips; $f(c_{ij})$ is a decreasing function of c_{ij}, the unit cost of travel between i and j; n , m are number of origins and destinations respectively. The problem is to estimate r_i, s_j and parameters associated with $f(c_{ij})$ based on a sample of observed trips.

Let x_{ij} be the number of observed trips between zones i and j. Assuming that the p_{ij} 's are independent, the likelihood function given this set of observations is

$$L = \prod_{j=1}^{m} \prod_{i=1}^{n} p_{ij}^{x_{ij}} \tag{7}$$

According to the principle of maximum likelihood, the best estimates for the parameters of p_{ij} can be obtained by maximizing L subject to the constraint

$$\sum_{j=1}^{m} \sum_{i=1}^{n} p_{ij} = 1 \tag{8}$$

Now we replace the probabilities p_{ij} in (7) by its representation as given in (6) and assume that the function $f(c_{ij})$ takes on the form $e^{-\sigma c_{ij}}$ the likelihood function can be rewritten as

$$\prod_{j=1}^{m} \prod_{i=1}^{n} r_i^{\sum_{j=1}^{m} x_{ij}} s_j^{\sum_{j=1}^{n} x_{ij}} e^{-\sigma c_{ij} x_{ij}} \tag{9}$$

By virtue of the maximization process, the quantities $r_i s_j e^{-\sigma c_{ij}}$ will be driven to the highest possible values. Then the equality constraint (8) can be replaced by the inequality constraint:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} r_i s_j e^{-\sigma c_{ij}} \le 1 \text{ Which must be active at the optimum.}$$

Case –1:

Under change of variable $y = e^{\sigma}$, the maximum likelihood formulation becomes:

$$\text{Minimize } L^{-} = \prod_{j=1}^{m} \prod_{i=1}^{n} r_i^{-\sum_{j=1}^{m} x_{ij}} s_j^{-\sum_{j=1}^{n} x_{ij}} y^{c_{ij} x_{ij}} \tag{10}$$

$$\text{subject to } \sum_{j=1}^{m} \sum_{i=1}^{n} r_i s_j y^{-c_{ij}} \le 1 \,,$$

$r_i, s_j, y > 0$.

The corresponding dual of the above geometric programs becomes

$$\text{Maximize } d(w) = \left(\frac{1}{w_0}\right)^{w_0} \prod_{j=1}^{m} \prod_{i=1}^{n} \left(\frac{1}{w_{ij}}\right)^{w_{ij}} T^{T} \tag{11}$$

subject to $w_0 = 1$

$$- w_0 \sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} w_{ij} = 0$$

$$- w_0 \sum_{i=1}^{n} x_{ij} + \sum_{i=1}^{n} w_{ij} = 0$$

$$w_0 \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} x_{ij} - \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} w_{ij} = 0$$

$$w_{ij} > 0$$

The optimal dual variables w_{ij}^* equals to the estimated probability of a trip between zones i and j multiplied by the total number of trips, is therefore the estimated number of trips between the zones. By re-arrangement of terms, taking logarithm of (11) we can see that the objective function actually entropy function and constraint set can be re-written as follows:

$$\text{Maximize } \log d(w) = T \log T - \sum_{j=1}^{m} \sum_{i=1}^{n} w_{ij} \log w_{ij} \tag{12}$$

subject to

$$\sum_{j=1}^{m} w_{ij} = O_i$$

$$\sum_{i=1}^{n} w_{ij} = D_j$$

$$\sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} w_{ij} = C$$

where $O_i = \sum_{j=1}^{m} x_{ij}$, $D_j = \sum_{i=1}^{n} x_{ij}$, $C = \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} x_{ij}$ are represents the number of trips generated from zone i , the number of trips absorbed by zone j and the total cost of traveling for the sample respectively. Ignoring the constant term $T \log T$ of the objective function, then the model becomes

$$\text{Maximize } \log d'(w) = -\sum_{j=1}^{m} \sum_{i=1}^{n} w_{ij} \log w_{ij} \tag{13}$$

subject to the same constraints and restrictions of (12).

Case –II:

When the parameter σ fixed value, then the maximum likelihood formulation becomes:

$$\text{Minimize } L^{-} = \prod_{j=1}^{m} \prod_{i=1}^{n} r_i^{-\sum_{j=1}^{m} x_{ij}} s_j^{-\sum_{j=1}^{n} x_{ij}} e^{\sigma \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} x_{ij}} \tag{14}$$

subject to $\sum_{j=1}^{m} \sum_{i=1}^{n} r_i s_j e^{-\sigma c_{ij}} \le 1$,

$r_i, s_j > 0$.

The corresponding dual of the above geometric programs becomes

$$\text{Maximize } d(w) = \left(\frac{e^{\sigma \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} x_{ij}}}{w_0} \right)^{w_0} \prod_{j=1}^{m} \prod_{i=1}^{n} \left(\frac{e^{-\sigma c_{ij}}}{w_{ij}} \right)^{w_{ij}} T^T \tag{15}$$

subject to $w_0 = 1$

$$- w_0 \sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} w_{ij} = 0$$

$$- w_0 \sum_{i=1}^{n} x_{ij} + \sum_{i=1}^{n} w_{ij} = 0$$

$$w_{ij} > 0$$

The optimal dual variables w_{ij}^* equals to the estimated probability of a trip between zones i and j multiplied by the total number of trips, is therefore the estimated number of trips between the zones. By re-arrangement of terms, taking logarithm of (15) we can see that the this objective function actually entropy function and constraint set can be re-written as follows:

$$\text{Maximize } \log d(w) = \sigma \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} x_{ij} + T \log T - \sum_{j=1}^{m} \sum_{i=1}^{n} w_{ij} \log w_{ij} - \sigma \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} w_{ij} \qquad (16)$$

$$\text{subject to } \sum_{j=1}^{m} w_{ij} = O_i$$

$$\sum_{i=1}^{n} w_{ij} = D_j$$

Ignoring the constant term $\{ \sigma \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} x_{ij} + T \log T \}$ of (16), the objective function re-written the following problems:

$$\text{Maximize } \log d''(w) = - \sum_{j=1}^{m} \sum_{i=1}^{n} w_{ij} \log w_{ij} - \sigma \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} w_{ij} \qquad (17)$$

$$\text{subject to } \sum_{j=1}^{m} w_{ij} = O_i$$

$$\sum_{i=1}^{n} w_{ij} = D_j$$

where $O_i = \sum_{j=1}^{m} x_{ij}$ and $D_j = \sum_{i=1}^{n} x_{ij}$ are represents the number of trips generated from zone i , the number of trips absorbed by zone j respectively.

4. Single and Multi-objective entropy Trip distribution model:

(a) Single objective entropy Trip distribution model:

Wilson, Webber (1970) pioneered the use of entropy models in the study of spatial interaction. Entropy models are commonly used to find the most probable numbers of pairings w_{ij} between locations i and j given the numbers O_i of origins in location i and D_j , of destination in location j, for all locations(i =1,2,..,n ; j =1,2,...,m). In equation form:

$$\sum_{j=1}^{m} w_{ij} = O_i \quad , (\text{i} =1,2,..,\text{n})$$

$$\sum_{i=1}^{n} w_{ij} = D_j \quad , \ (\text{j} =1,2,...,\text{m})$$

In addition to the O_i and D_j we know the cost of a transaction i to j, c_{ij}. We also add this information to the model in the form of the cost equation:

$$\sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij} w_{ij} = C^*$$

where C^* is a fitting parameter to be chosen according to the needs by the model maker. The corresponding entropy, which we want to maximize is:

$$\text{En(w)} = -\sum_{j=1}^{m}\sum_{i=1}^{n} \frac{w_{ij}}{T}\log\left(\frac{w_{ij}}{T}\right) = \frac{1}{T}[T\ln T - \sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}\log w_{ij}] \qquad (18)$$

where $T = \sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij} = \sum_{i=1}^{n} O_i = \sum_{j=1}^{m} D_j$. We can interpret entropy objective function of (18) as follows:

We want to find the matrix M= $[w_{ij}]$ which has the greatest number γ(M) associated with it subject to constraints given above where γ(M) denotes the number of assignments leading to trip matrix M. We can obtain the number of states which give rise to a matrix M as follows. Let T is the total number of workers i.e. $\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}$ =T. Firstly we can select from w_{11} from T, w_{12} from T-w_{11}, w_{13} from T-w_{11}-w_{12}, etc., and so the number of possible assignments or states is the number of ways of selecting w_{11} from T, ${}^{T}C_{w_{11}}$, multiplied by the number of ways of selecting w_{12} from T-w_{11}, ${}^{T-w_{11}}C_{w_{12}}$, etc. Thus

$$\gamma(M) = {}^{T}C_{w_{11}}\;{}^{T-w_{11}}C_{w_{12}} \ldots\ldots\ldots\ldots\; {}^{T-w_{11}-w_{12}-\ldots\ldots-w_{nm-1}}C_{w_{nm}}$$

So, explicitly

$$\gamma(M) = \frac{T!}{w_{11}!(T-w_{11})!}.\frac{(T-w_{11})!}{w_{12}!(T-w_{11}-w_{12})!}\ldots\ldots\frac{(T-w_{11}-w_{13}\ldots\ldots\ldots-w_{nm-1})!}{w_{nm}!(T-w_{11}-w_{12}-w_{13}-\ldots\ldots\ldots\ldots-w_{nm})!}$$

$$= \frac{T!}{\prod_{j=1}^{m}\prod_{i=1}^{n} x_{ij}!} \quad \text{or, } \log\gamma(M) = \log T! - \sum_{j=1}^{n}\sum_{i=1}^{m}\log w_{ij}!$$

Here w_{ij}'s and T are assumed to be sufficiently large. So by using Stirlings approximation formula, we get

$$\log\gamma(M) = \log[e^{-T}T^{T}] - \sum_{j=1}^{m}\sum_{i=1}^{n}\log[e^{-w_{ij}} w_{ij}^{w_{ij}}] = T\log T - \sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}\log w_{ij}$$

Since T is given, maximizing ln γ(M) [=En(w)] is equivalent to maximizing entropy, as defined in the objective function (18). This is one of the reasons why the entropy optimization model is particularly suitable for the trip distribution problem.

So mathematically the model becomes

$$\text{Maximize } \; En_1(w) = -\sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}\log w_{ij} \qquad (19)$$

subject to $\sum_{j=1}^{m} w_{ij} = O_i$, (i=1,2,..,n)

$\sum_{i=1}^{n} w_{ij} = D_j$, (j=1,2,...,m)

$\sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij} w_{ij} = C^*$

$w_{ij} \geq 0$, (i=1,2,..,n ; j=1,2,...,m)

Where $En_1(x) = T * En(w) - T\log T$. The model (19) which is same as the model (13) (Case-1).

(b) Multi-objective entropy Trip distribution model:

In real world, however all trip distribution models are not single objective(i.e not only entropy objective function) Problems. We may have more than one objective function (e.g. minimization of several penalties i.e. minimization of total cost amount, delivery time, deterioration amount of product etc.)) in transportation problem. Let Tc(w) be the total cost of a transaction i to j which we want to minimizes, given that the numbers O_i of origins in location i and D_j , of destination in location j, for all locations(i =1,2,..,n ; j =1,2,...,m). So the Multi-objective entropy Trip distribution model can be stated as the following:

$$\text{Maximize } En_1(w) = = -\sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}\log w_{ij} \qquad (20)$$

$$\text{Minimize } Tc(w) = \sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij} w_{ij}$$

$$\text{subject to } \sum_{j=1}^{m} w_{ij} = O_i \ , \ (i=1,2,..,n)$$

$$\sum_{i=1}^{n} w_{ij} = D_j \ , \ (j=1,2,...,m)$$

$$w_{ij} \geq 0 \ , \ (i=1,2,..,n ; j=1,2,...,m)$$

5. Basic concepts of Fuzzy Set and membership function:

Fuzzy sets first introduced by Zadeh in 1965 as a mathematical way of representing impreciseness or vagueness in everyday life.

Fuzzy set: A Fuzzy set Ã in a universe of discourse X is defined as the following set of pairs $\tilde{A}=\{ (x,\mu_{\tilde{A}}(x)): x \in X \}$, Where $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership function of the fuzzy set Ã and $\mu_{\tilde{A}}(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set Ã . The larger $\mu_{\tilde{A}}(x)$ is the stronger grade of membership form in Ã.

Convex fuzzy set: A fuzzy set Ã of the universe of discourse X is convex if and only if for all x_1 , x_2 in X ,$\mu_{\tilde{A}} (\lambda x_1 + (1 - \lambda) x_2) \geq \min (\mu_{\tilde{A}} (x_1), \mu_{\tilde{A}} (x_2))$ when $0 \leq \lambda \leq 1$.

Normally fuzzy set: A fuzzy set Ã of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that $\mu_{\tilde{A}} (x) = 1$.

Fuzzy number:

A fuzzy number is a special case of a fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature but they all agree on that a fuzzy number represents the conception of a set of ′ real numbers close to a ' where ′a ' is the number being fuzzy field. A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. A fuzzy number Ã is a fuzzy set of the real line ℜ whose membership function $\mu_{\tilde{A}}(x)$ has the following characteristic with $-\infty < a_1 < a_2 < a_3 < a_4 < \infty$

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_L(x) & \text{for } a_1 \leq x < a_2 \\ 1 & ,, \quad a_2 \leq x \leq a_3 \\ \mu_R(x) & ,, \quad a_3 < x \leq a_4 \\ 0 & ,, \quad \text{otherwise} \end{cases}$$

where $\mu_L(x) : [a_1 , a_2] \to [0 , 1]$ is continuous and strictly increasing ; $\mu_R(x) : [a_3 , a_4] \to [0 , 1]$ is continuous and strictly decreasing.

The general shape of a fuzzy number following the above definition is shown in the next page (fig-1).

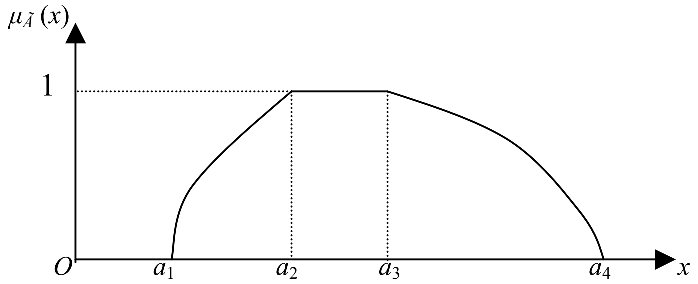

Fig.- 1: Fuzzy Number

Any one example of fuzzy number (Triangular Fuzzy Number (TFN)) and Linear membership function:

Let F(ℜ) be a set of all triangular fuzzy numbers in real line ℜ. A triangular fuzzy number $\tilde{A} \in F(\Re)$ is a fuzzy number with the membership function $\mu_{\tilde{A}} : \Re \to [0,1]$ parameterized by a triplet (a_1, a_2, a_3) TFN. Where a_1 and a_3 denote the lower and upper limits of support of a fuzzy $\tilde{A}$:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{a_2 - x}{a_2 - a_1}, & for \quad a_1 \le x < a_2 \\ 1, & ,, \quad x = a_2 \\ 1 - \dfrac{x - a_2}{a_3 - a_2}, & ,, \quad a_2 < x \le a_3 \\ 0 & ,, \quad otherwise \end{cases}$$

The general shape of a TFN following the above definition is shown below (fig.-2).

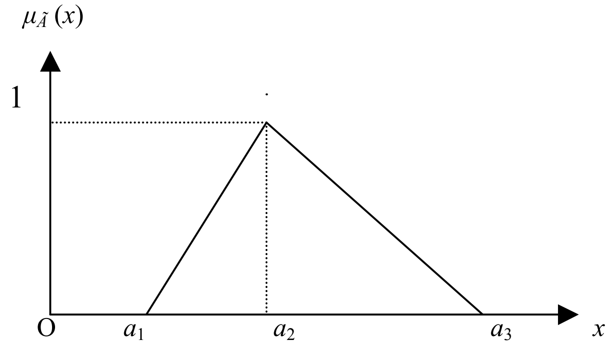

Fig. 2: TFN

6. Fuzzy programming approach to conversion Single Objective Non-linear problem from Multi-Objective Non-Linear problem (MONLP) :

A MONLP or a Vector Minimization Problem (VMP) may be taken in the following form:

$$\text{Minimize } f(x) = [\, f_1(x), f_2(x), \ldots f_k(x) \,]^T \quad (21)$$

$$\text{subject to } x \in X = \{ x \in R^n : g_j(x) \le \text{ or } = \text{ or } \ge b_j \text{ for } j = 1, \ldots, m \; ; \; x \ge 0 \}.$$

$$\text{and } l_i \le x_i \le u_i \; (i=1,2,..,n).$$

Zimmermann (1978) showed that fuzzy programming technique could be used nicely to solve the multi-objective programming problem.

To covert VMP (21) problem as a single objective, following steps are used:

Step 1 : Solve the VMP (21) as a single objective non-linear programming problem using only one objective at a time and ignoring the others. These solutions are known as ideal solution.

Step 2 : From the results of step 1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \cdots \\ x^k \end{array} \begin{array}{cccc} f_1(x) & f_2(x) & \ldots\ldots & f_k(x) \\ \left[\begin{array}{c} f_1^*(x^1) \\ f_1(x^2) \\ \ldots\ldots \\ f_1(x^k) \end{array}\right. & \begin{array}{c} f_2(x^1) \\ f_2^*(x^2) \\ \ldots\ldots \\ f_2(x^k) \end{array} & \begin{array}{c} \ldots\ldots \\ \ldots\ldots \\ \ldots\ldots \\ \ldots\ldots \end{array} & \left.\begin{array}{c} f_k(x^1) \\ f_k(x^2) \\ \ldots\ldots \\ f_k^*(x^k) \end{array}\right] \end{array}$$

Here $x^1, x^2, \ldots, x^k$ are the ideal solutions of the objectives $f_1(x), f_2(x), \ldots, f_k(x)$ respectively. So $U_r = \max\{ f_r(x^1), f_r(x^2), \ldots, f_r(x^k)\}$

and $L_r = \min\{ f_r(x^1), f_r(x^2), \ldots, f_r(x^k)\}$

[L_r and U_r be lower and upper bounds of the r^{th} objective function $f_r(x)$ for r = 1,…,k].

Step 3: Using aspiration levels of each objective of the VMP (21), formulate (21) as follows:

Find x so as to satisfy

$$f_r(x) \underset{\sim}{\leq} L_r \quad (r = 1, 2, \ldots, k) \tag{22}$$

$$x \in X$$

Here objective functions of (21) are considered as fuzzy constraints. This type of fuzzy constraints can be quantified by eliciting a corresponding membership function

$$\mu_r(f_r(x)) = \begin{cases} 0 & \text{if } f_r(x) \geq U_r \\ d_r(x) & \text{if } L_r \leq f_r(x) \leq U_r \quad (r = 1,2,..,k) \\ 1 & \text{if } f_r(x) \leq L_r \end{cases} \tag{23}$$

where $d_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$. Following figure-3 illustrates the graph of the membership function $\mu_r(f_r(x))$

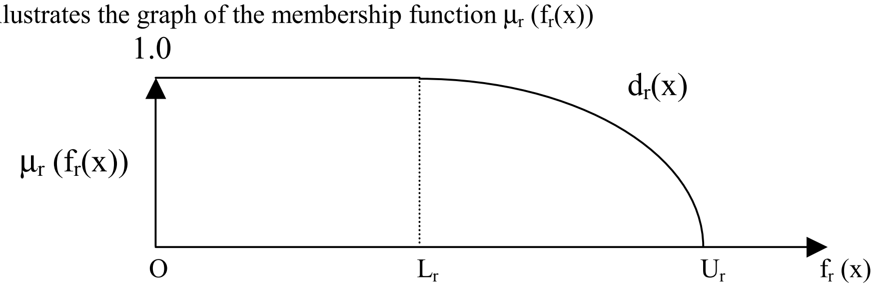

Fig.-3: Membership function for minimization problem

Having elicited the membership functions (as in (23)) $\mu_r(f_r(x))$ for r =1,2,..,k, a general aggregation function

$\mu_{\tilde{D}}(x) = \mu_{\tilde{D}}\left(\mu_1(f_1(x)), f_2(Z_2(x)), \ldots\ldots, \mu_k(f_k(x))\right)$ is introduced. So a fuzzy multi-objective decision making problem can be defined as

Maximize $\mu_{\tilde{D}}(x)$ (24)

subject to

x∈ X

Fuzzy decision (Bellman and Zadeh's (1970)) based on convex operator, the problem (24) is reduced to

Maximize $\mu'_{\tilde{D}}(x) = \sum_{r=1}^{k} \mu_r\left(f_r(x)\right)$ (25)

subject to

x∈ X

$0 \le \mu_r\left(f_r(x)\right) \le 1$ for r = 1, 2, …, k .

7. Fuzzy programming approach to conversion Single Objective Trip distribution problem from Multi-Objective Trip distribution problem:

To convert (20), step-1of fuzzy programming technique is used. After that, according to step-2, Pay-off matrix is formulated as follows:

$$\begin{bmatrix} -En_1(w^1) & Tc(w^1) \\ -En_1(w^2) & Tc(w^2) \end{bmatrix}$$

Now we find, the upper bounds U_{Tc} and U_{En_1} and the lower bounds L_{Tc} and L_{En_1} where

U_{Tc} = max{Tc(w^1), Tc(w^2)}and L_{Tc} = min{Tc(w^1), Tc(w^2)}

and U_{En_1} = max {$En_1(w^1)$,$En_1(w^2)$} and L_{En_1} = min {$En_1(w^1)$,$En_1(w^2)$}.

So $L_{Tc} \le$ Tc(w)≤ U_{Tc} and $L_{En_1} \le$ En_1x) $\le U_{En_1}$ For simplicity the linear membership functions $\mu_{Tc}(Tc)$, $\mu_{-En_1}(En_1)$ for the objective functions Tc(w) and En_1(w) respectively are defined as follows:

$$\mu_{-En1}(En_1) = \begin{cases} 1 & \text{if } En_1(w) \ge U_{En1} \\ \dfrac{L_{En1} - En_1(w)}{L_{En1} - U_{En_1}} & \text{if } U_{En_1} > En_1(w) > L_{En_1} \\ 0 & \text{if } En_1(w) \le L_{En_1} \end{cases}$$

and $$\mu_{Tc}(Tc) = \begin{cases} 1 & \text{if } Tc(w) \le L_{Tc} \\ \dfrac{U_{Tc} - Tc(w)}{U_{Tc} - L_{Tc}} & \text{if } L_{Tc_i} < Tc(w) < U_{Tc} \\ 0 & \text{if } Tc(w) \ge U_{Tc} \end{cases}$$

According to step-3, having elicited the above membership functions crisp non-linear programming problem of (20) is formulated as follows:

Maximize Z(w) = $\mu_{-En_1}(En_1) + \mu_{Tc}(Tc)$ (26)

subject to

$$\mu_{-En_1}(En_1) = \frac{L_{En_1} - En_1(w)}{L_{En_1} - U_{En_1}}$$

$$\mu_{Tc}(Tc) = \frac{U_{Tc} - Tc(w)}{U_{Tc} - L_{Tc}}$$

$$\sum_{j=1}^{m} w_{ij} = O_i \text{ , (i=1,2,..,n)}$$

$$\sum_{i=1}^{n} w_{ij} = D_j \text{ , (j=1,2,...,m)}$$

$$0 \le \mu_{-En_1}(En_1) \le 1 \text{ , } 0 \le \mu_{Tc}(Tc) \le 1$$

$$w_{ij} \ge 0 \text{ , (i=1,2,..,n ; j=1,2,...,m)}$$

The problem (26) can be written as

Maximize Z(w) = $\dfrac{L_{En_1} - En_1(w)}{L_{En_1} - U_{En_1}} + \dfrac{U_{Tc} - Tc(w)}{U_{Tc} - L_{Tc}}$ (27)

$$\sum_{j=1}^{m} w_{ij} = O_i \text{ , (i=1,2,..,n)}$$

$$\sum_{i=1}^{n} w_{ij} = D_j \text{ , (j=1,2,...,m)}$$

$$w_{ij} \ge 0 \text{ , (i=1,2,..,n ; j=1,2,...,m)}$$

$$0 \le \frac{L_{En_1} - En_1(w)}{L_{En_1} - U_{En_1}}, \frac{U_{Tc} - Tc(w)}{U_{Tc} - L_{Tc}} \le 1$$

The problem (27) is equivalent to

Maximize $Z'(w) = -a\sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}\log w_{ij} - b\sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij}w_{ij}$, (28)

subject to the same constraints as in (27).

where, $a = \dfrac{1}{U_{En_1} - L_{En_1}}, b = \dfrac{1}{U_{Tc} - L_{Tc}}$ and $Z(w) = Z(w) + \dfrac{U_{Tc}}{U_{Tc} - L_{Tc}} - \dfrac{L_{En_1}}{U_{En_1} - L_{En_1}}$.

i. e. of the problem

Maximize $Z''(w) = -\sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}\log w_{ij} - \sigma\sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij}w_{ij}$, (29)

subject to the same constraints as in (27). Where $\sigma = b/a$. The problem (29) which is equivalent to the problem (17) (Case-II).

8. Conclusion

In this paper we have analyzed two types of entropy trip distribution problem by using primal-dual geometric programming method. Here we deduce the formulation of trip distribution problem from maximum likelihood estimation of trip distribution and use fuzzy mathematical programming we are to show multi-objective entropy trip distribution problem which is equivalent the said formulation of the trip distribution problem. In this paper we are also use the concept of multi-objective entropy trip distribution problem. Multi-objective entropy optimization method use in various fields of engineering and sciences.

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