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Children's Arithmetical Difficulties: Contributions from Processing Speed, Item Identification, and Short-Term Memory

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Children's arithmetical difficulties are often explained in terms of a short-term memory deficit. However, the underlying cause of this memory deficit is unclear, with some researchers suggesting a slow articulation rate and hence increased decay of information during recall, while others offer an explanation in terms of slow speed of item identification, indicating difficulty in retrieving information stored in long-term memory. General processing speed is also related to measures of short-term memory but has rarely been assessed in studies of children's arithmetic. Measures of short-term memory, processing speed, sequencing ability, and retrieval of information from long-term memory were therefore given to 7-year-old children. When reading ability was controlled for, arithmetic ability was best predicted by processing speed, with short-term memory accounting for no further unique variance. It was concluded that children with arithmetic difficulties have problems specifically in automating basic arithmetic facts which may stem from a general speed-of-processing deficit.

Researchers investigating children's arithmetical difficulties have questioned a number of cognitive mechanisms that may underlie these difficulties. One issue often addressed is whether short-term memory plays a specific role in arithmetic, as short-term memory is thought to have a central role in the acquisition and execution of basic educational skills (Hitch & McAuley, 1991; Siegel & Linder, 1984; Siegel & Ryan, 1989; Swanson, 1993; for a review, see Hulme & Roodenrys, 1995). Much of this research has gained inspiration from the ideas associated with the working memory model. This model was developed by Baddeley and Hitch (1974) and is considered as a measure of

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functional storage capacity, comprising both storage and processing functions (Daneman & Carpenter, 1980; Towse & Hitch, 1995), measured through the use of tasks such as reading span (Daneman & Carpenter, 1980) and counting span (Case, Kurland, & Goldberg, 1982).

The core system thought to be responsible for controlling working memory is the central executive. This system offloads some short-term storage functions to slave systems, freeing a portion of its own capacity for performing more complex information-processing tasks. These slave subsystems are the articulatory loop and the visual–spatial sketchpad. The articulatory loop is involved in the storage of verbal information, which is subject to decay but may be refreshed by subvocal rehearsal (Baddeley, 1986). The visual–spatial sketch pad serves a temporary storage function for visual and spatial material (Logie, 1986; Quinn & McConnell, 1996). When one considers a complex cognitive task such as mental arithmetic, it is evident that it requires the temporary storage of information while new information is being processed and other cognitive tasks are being performed. For example, the central executive monitors and retrieves information about the operation to be used, such as addition, while subsidiary systems store specific numbers involved in the calculation (Logie, Gilhooly, & Wynn, 1994). It is clear therefore why many ideas of children's arithmetical difficulties have appealed to explanations associated with working memory.

A number of studies have reported short-term memory deficits in children with arithmetical difficulties, although there is some debate over the specificity of this deficit. Some researchers report that children with arithmetical difficulties have a short-term memory deficit only for material involving numbers, for example, digit span and counting span (Siegel & Ryan, 1989), while others suggest that such children have a general deficit extending over a range of short-term memory stimuli (Hitch & McAuley, 1991; Swanson, 1993; Turner & Engle, 1989). Work by Dark and Benbow (1990, 1991) with gifted adolescents found that working memory was differentially enhanced, depending on whether the adolescents showed verbal or mathematical precocity. Dark and Benbow (1991) examined three core aspects fundamental to successful memory performance, comparing verbal with mathematical precocity. They investigated the accuracy with which information could be maintained in working memory, the accuracy of manipulating information in working memory, and finally, the speed of activating information from long-term memory into working memory. Stimuli used in the memory-span tasks included digits, letters, words, and spatial locations. Dark and Benbow found that mathematical precocity was best correlated with the ability to manipulate information in working memory. Furthermore, subjects in this group were significantly better on tasks involving digits and spatial location stimuli. In contrast, verbal precocity was found to be best correlated with enhanced retrieval of representations from long-term memory into working memory. Verbally precocious subjects were found to have a significant advantage on span tasks involving word stimuli. This finding lends support to the argument that working memory is specific to different intellectual domains. However, Dark and Benbow do not put this result down to differences in memory performance per se. Rather, they suggest that such differences arise due to differences in item identifiability, such that when items are identified more rapidly, this will lead to enhanced memory performance. This supports the work of Case et al. (1982). For example, digits may have stronger representations (or be more ''compact'' in Dark & Benbow's terms) for individuals who are mathematically talented, meaning that they are identified more quickly, leading to enhanced memory span for those particular stimuli.

Many of the characteristics associated with arithmetical difficulties, such as frequent computational errors and the use of immature, slow counting strategies (e.g., counting using fingers), have also been explained as being the result of poor working-memory resources, leading to the poor representation of arithmetic facts in long-term memory (Geary, 1990; Geary, Bow-Thomas, & Yao, 1992; Geary & Brown, 1991; Geary, Brown, & Samaranayake, 1991; for a review, see Geary, 1993). Theories of the representation of arithmetic facts in long-term memory (e.g., the distributions of associations model, Siegler & Shrager, 1984), point out that performance on simple arithmetic questions depends on retrieval from long-term memory. The strength with which these elements are stored, and hence, the probability of retrieving them correctly, depends on experience, with associations between problems and answers being formed each time a particular arithmetic problem is encountered, regardless of the correctness of the answer. Therefore, it would appear that the ability to utilize working memory resources in order to temporarily store numbers while attempting to reach an answer, typically by means of counting, is of great importance in the early stages of learning arithmetic.

The explanations surrounding these short-term memory deficits are inconclusive. It has often been thought that the availability of working memory resources is dependent on how much information can be rehearsed in a 2- to 3-s span, through subvocal rehearsal utilizing the articulatory loop. The articulatory loop consists of temporarily activated speech items in the phonological buffer, and an articulation process used to silently rehearse items in the buffer or overtly recall them. Speech rate has been taken as a measure of rehearsal rate and has been found to show a strong relationship with short-term memory span and various other measures of short-term memory recall (Baddeley, Thomson, & Buchanan, 1975; Ellis & Hennelley, 1980; Hitch, Halliday, & Littler, 1989). With regard to arithmetic, speech rate (and hence rehearsal time) should be related to counting speed (Baddeley, 1986), such that the faster the counting speed, the longer the short-term memory span for numbers (Kail, 1992). Therefore, the main assumption is that if items are articulated more rapidly, then more items can be refreshed in memory before decay beyond a critical point where there would be no possibility of further rehearsal or recall (Cowan, 1992; Cowan, Keller, Hulme, Roodenrys, McDougall, & Rack, 1994). The use of slow counting procedures could result in the phonological representations of the addends in short-term memory decaying before the count is completed, reducing the likelihood that the addends and the answer will become associated in long-term memory. Evidence to support this comes from Geary, Bow-Thomas, Fan, & Siegler (1993), who attribute the better arithmetic performance of Chinese children, compared to their American counterparts, to their greater digit span. Chinese digits are shorter and hence can be counted more quickly, meaning that the probability of reaching a correct answer to an arithmetic question, while maintaining phonological representations of the addends, is increased. This would result in associations between the addends and the answer being formed in long-term memory.

However, a number of researchers suggest that the ‘‘speech rate’’ explanation of individual differences in short-term memory span may be too simple. Performance on short-term memory tasks depends on a number of factors, and there may be involvement from long-term memory components. Case et al. (1982) proposed that the most important limiting factor on short-term memory is speed of item identification, whereby representations of items are accessed from long-term storage, with developmental increases in memory span being due to extra resources becoming available for storage as a result of a decrease in capacity taken up by item identification. This idea has been supported by findings from recent research showing that factors such as familiarity and the strength of representations in long-term memory have an effect upon short-term memory recall (see e.g., Ericsson & Kintsch, 1995; Gathercole & Adams, 1994; Henry & Millar, 1991; Hitch & McAuley, 1991; Hulme, Maughan, & Brown, 1991; Roodenrys, Hulme, & Brown, 1993). Hitch and McAuley (1991) showed that while children with arithmetical difficulties did have a deficit on tests of short-term memory, they did not have a significantly slower articulation rate. This finding was explained by suggesting that such children may have slower access to number representations in long-term memory, which in turn may lead to slow counting and low digit span. However, another possibility raised by Hitch and McAuley is that children with arithmetical difficulties may have general difficulties in learning sequences, for example, learning the sequence of numbers from one to twenty, affecting the fluency of counting. Such difficulties with sequencing could be seen to be related to procedural deficits shown by mathematics-disabled children, such as committing frequent computational errors in counting (Geary, 1990; see also Geary, 1993). Problems such as this may be linked to lack of knowledge of counting procedures and number sequences. Furthermore, Geary et al. (1992) showed that verbal and finger-counting errors and the frequency of using immature counting strategies was related to counting knowledge. Mathematics-disabled children showed an immature and rigid understanding of some of the basic features of counting.

Established findings associated with short-term memory may also provide support for the involvement of a long-term memory element. For example, the phonological similarity effect and the word-length effect are often seen as evidence for the existence of speech-based coding in short-term memory. With articulatory suppression (the elimination of speech-based coding), both of these effects are abolished, although some serial recall ability remains. It is therefore reasonable to suggest that this remaining recall capacity depends upon activation of information in long-term memory, or may reflect the operation of some other nonphonological information store.

Therefore, although there is evidence relating short-term memory to arithmetical difficulties, the underlying cause of this relationship is not clear. Some research suggests that short-term memory deficits arise from slow articulation rate, while others posit an explanation in terms of the speed of retrieving information from long-term memory, which in turn is dependent upon a number of factors related to the strength of associations in long-term memory for the material to be remembered. Clearly, in order to fully understand the nature of this short-term memory deficit, it is necessary to consider all of these possible explanations.

Yet another mechanism to consider is processing speed. One reason why processing speed differences are potentially important is that tests purporting to measure processing speed are consistently found to be related to performance of other cognitive skills, and to measures of higher-order cognitive processes, for example, short-term memory (Hale, 1990; Kail, 1991; Kail & Hall, 1994; Kail & Park, 1994; Kail & Salthouse, 1994; Rabbitt & Goward, 1994). The nature and the extent of the relationship between processing speed and arithmetical ability has not, as far as we are aware, been examined in previous studies. According to Case (1985), processing efficiency is measured by speed, such that processing which can be completed faster is more efficient. Case suggests that much of the intellectual change observed as the child matures is a direct consequence of the child's increasing ability to process more information in a given period of time. This argument is supported by Kail (1992), who suggests that the maximum rate at which cognitive operations can be executed should be considered as a processing resource, with the resulting level of cognitive performance being dependent upon speed of processing information. Kail maintains that processes requiring control, for example, using algorithms, compete for limited processing resources, while automatic processes do not make such demands on processing resources. In a study by Case et al. (1982), it was found that children showed a strong relationship between the efficiency of executing an operation and the ability to recall products of that operation; for example, higher counting span could be predicted from more efficient counting ability.

Two distinctive views have been put forward to account for changes in processing speed. One view emphasizes experiences that lead to changes in the speed of processing in specific domains, whereby items of information required to deal with a particular problem become more richly interrelated and coordinated into larger chunks, and hence become more readily accessible. Therefore, fewer demands are made on limited processing resources as responses become automated. An alternative view of processing speed emphasizes more global changes (e.g., Pascual-Leone, 1970). Such a view suggests that all information-processing components develop at the same rate. Increased processing speed could be due to more processing occurring in parallel, or may reflect the fact that fewer results of processing must be exchanged between the computing space and long-term memory. Such changes in processing speed would result in differences in performance on most speeded tasks, rather than those specific to certain domains. Evidence for such an explanation comes from Kail (1991), who showed that differences in processing speed occur on a range of cognitive tasks, and also on perceptual-motor tasks, such as the pegboard.

As suggested by Hitch and McAuley (1991), there may be complex interdependencies between cognitive deficits and mathematical ability. The study reported here was carried out in an attempt to pinpoint the cognitive deficits specifically underlying mathematical ability. Taken together, theories relating to processing speed and retrieval of information from long-term memory provide a very rational explanation of children's arithmetical difficulties. Children with poor arithmetical skills show a lack of automaticity in retrieving numbers and number combinations from long-term memory, evidenced through slow item identification and through the use of slow, inefficient counting strategies rather than direct memory retrieval. This slowness may be simply caused by lack of familiarity with the material (Hitch & McAuley, 1991), or may represent a more serious problem in the ability to automate facts in long-term memory (Garnett & Fleischner, 1983; Geary, 1993). The evidence concerning short-term memory difficulties is somewhat inconclusive; it is hypothesized that processing speed and the speed of retrieval of information from long-term memory represent fundamental deficits for children with arithmetical difficulties, and that they mediate the often observed deficits found in short-term memory span.

METHOD

Subjects

Sixty-nine children (mean age 7 years, 5 months, *SD* = 4 months, 39 males and 30 females) were initially screened prior to experimental testing; all children attended two different urban schools in Dundee. One child was later excluded from further analysis due to absence on a number of the experimental testing sessions, leaving 68 children who participated in later experimental testing. No child was excluded due to any intellectual or behavioral difficulties. Children were initially screened for mathematics and reading ability using the Group Mathematics Test (Young, 1970), and the British Ability Scales (BAS) Word Reading Test (Elliott, Murray, & Pearson, 1979). Standard age scores for mathematics and reading ability were calculated using the test norms from the Group Mathematics Test (mean = 100, *SD* = 15), and from the BAS Word Reading Test (mean = 50, *SD* = 10). Results from these tests revealed a range of reading performance from 31 to 73 (mean = 49.84, *SD* = 10.80). Mathematics standard age scores ranged from 70 to 122 (mean = 96.37, *SD* = 6.34). A significant correlation was found between mathematics and reading ability, $r(66) = .67$, $p < .001$.

Children were placed into one of two groups according to their mathematics standard age score, forming a high-ability group whose mathematics standard age scores were above the mean score for the entire sample, and a low-ability group whose standard age scores fell below the group mean of 96. Thirty-six children (21 males and 15 females) were placed into the high mathematics group due to their standard age scores falling above 96, and 32 children (17 males and 15 females) were placed into the low mathematics group, having standard age scores falling below 96. A one-way analysis of variance was conducted comparing groups (high and low mathematicians), which showed there to be no significant difference between the groups in chronological age, $F(1,66) = .61$, $p > .05$. Two-way analyses of variance were conducted examining for differences between the groups and for sex differences on the Group Mathematics Test and the BAS Word Reading Test. Results showed there to be a significant difference between the groups on mathematics ability, $F(1,64) = 149.77$, $p < .0001$, and on reading ability, $F(1,64)$ 30.17, $p < .0001$. There were no significant sex differences in either mathematics ability, $F(1,64) = .008$, $p > .05$, or reading ability, $F(1,64) = .440$, $p > .05$ (see Table 1 for mean mathematics and reading standard age scores for each group). To establish that differences in mathematics ability were still apparent once reading ability had been controlled for, ANCOVA was carried out, controlling for reading differences using performance on the BAS Word Reading Test. This analysis revealed that there were still significant differences between the groups in mathematics ability, $F(1,65) = 86.28$, $p < .0001$. Performance on the BAS Word Reading Test was used for controlling for differences in reading ability in all the statistical analyses reported below.

Materials and Procedure

Experimental testing began approximately one month after the screening tests had been administered. Children were tested individually over six short sessions in a quiet classroom in the school, with tasks being given to each child in the same order. Each group of experimental tasks will be described in turn.

Short-Term Memory Span

Memory span. Memory span for digits and one-syllable words was measured to assess short-term storage capacity, along with counting span as an additional measure of storage and processing efficiency. Digit span was tested using the WISC-R subtest (Wechsler, 1977). One-syllable words were used for the word span test with a mean age of acquisition of 3.1 years (Carroll & White, 1973). The words used were *king, rope, horse, leaf, tree, knife, tent,* and *snake.* Administration of the digit and word span tasks followed the same procedure. After an initial practice session, the digits or words were presented auditorily to the child at a rate of approximately one item per second, starting from a span length of two. Each child was required to recall the words in the order in which they had been presented. If the items were recalled in the correct serial order, the span length was increased by one. If the child recalled the digits or words incorrectly, the span length was repeated with a different set of items. If the child failed on the second attempt of any particular span length, then testing was discontinued. Stimuli for the counting span test consisted of plain white cards with between one and nine green spots and one and nine red spots on each card, each spot being 8 mm in radius. Children were instructed to count the number of green spots on each card presented and to try and remember the number of spots counted on each card. After an initial practice session, children were presented with two cards face down on the table. The experimenter then turned the first card over and after the child had counted the spots, this card was turned face down again, and the next card turned up. This card was then turned face down and the experimenter pointed to the first card and then the second, asking the child to recall the number of spots counted on each card. The child was presented with two sets of cards at each span length. If the child passed on one or both of the trials, the child was then presented with cards of the next span length, i.e., three cards, up to a maximum of eight cards. Testing was discontinued when the child failed both trials at any one span length. Digit span, word span, and counting span were determined to be the length of the last correct serial recall.

Speech rate. This was measured by asking children to repeat two one-syllable words (*king* and *tree*). Children were instructed to continue repeating the word pair until told to stop. This was recorded on audio tape for later analysis. If the child made an error or stopped for any other reason, the child was stopped by the experimenter and instructed to begin again. The time taken by the child to repeat the stimulus pair five times was measured using a stopwatch, with timing being started at the beginning of the second repetition of the words and terminated at the end of the sixth repetition. The same procedure was administered for three syllable words (*banana* and *elephant*). The number of words articulated per second was calculated.

Speed of counting was measured using cards previously used for the counting span test. The time taken for children to count the number of red spots on five cards was recorded, and any errors of counting were noted. The number of dots counted per second was calculated.

Processing Speed

Visual number matching and cross-out tasks. These tests were based on the Woodcock-Johnson tests of Cognitive Ability, but with stimuli derived by the experimenter. The visual number matching task comprised 30 rows of 6 digits, with 2 digits in each row being identical (for example, 7 1 2 6 1 6). The child was instructed to circle the identical digits in each row, and to work as quickly and as accurately as possible. The performance measure in this case was the time taken to complete all 30 rows of digits. The cross-out task consisted of 12 rows of a geometric figure at the left-hand side of the row and 19 similar figures to the right. An example of this may be a triangle containing a dot. The 19 alternative figures are all triangles but contain various internal objects, for example, a square, an addition sign, a diamond shape. The child's task was to cross out the 5 figures of the 19 that were identical to the target stimulus presented on the left side of the page. Again, performance was measured by the time taken to complete all 12 rows. The visual number matching and cross-out tasks were chosen as measures of processing speed because they were initially devised to assess the processing speed factor in the theory of fluid and crystallized intelligence (Cattell, 1963), and have been shown to be good measures of processing speed in previous studies (e.g., Kail & Hall, 1994).

Perceptual motor speed. This was measured by the use of a pegboard, with children being required to transfer 10 pegs from one side of the board to 10 holes in the other side of the board as quickly as possible. The time taken to complete this task with both the right and left hand was recorded.

Long-Term Memory

Speed of number and letter identification. This involved the child's being presented with a number or letter in the center of the computer screen and instructed to name the letter or number as quickly as possible. This was achieved through the use of a voice response key attached to the computer. Timing for identification of the stimulus began when it was presented on the screen and stopped when the child responded with the answer by means of the voice key. The numbers used were 1 to 20 presented in Arial font. All the letters of the alphabet were used, presented in Arial font, except the letter "a" which was presented in Century Gothic ("ɑ"). Letters and numbers appeared in the same random order for each child. For the purposes of statistical analysis, any incorrect responses were later excluded.

Sequencing ability. The child was asked to complete a range of sequences which would provide a comparison to the sequence task of number counting. As well as being asked to count from 1 to 20, children were also asked to count higher numbers which are not so well learned (31 to 50), and asked to say the letters of the alphabet in sequence. To be certain that performance on this task was not simply a measure of repetition ability, the children were also asked to put a range of numbers in numerical order and words in alphabetical order. For example, the child was presented with six cards each displaying a different number and the child was asked to put them in numerical order. In the alphabetical ordering task, the child was again presented with six cards, each displaying a different word, which the child was asked to put in alphabetical order, with alphabetical order being described as the order with which the first letter of each word appears in the alphabet. To make this more obvious to the child, the first letter of each word was written in a different color. Familiar words were used which could be easily read by the child. Each child was asked to complete three trials of six items for both the numerical and alphabetical ordering tasks. Accuracy of the counting and alphabet sequences was taken to be the number of items remembered in the correct sequence order before the first mistake was made. No credit was given for any items placed into the correct sequence after one mistake had already been made. Accuracy of the numerical and alphabetical order tasks was measured as the number of items placed in the correct alphabetical or numerical order. For purposes of comparison between the tests, all scores were converted to percentages.

Single-digit addition. In order to assess the child's ability to solve simple arithmetical questions, arithmetical problems were presented to each child, with stimuli being presented in the center of the computer screen (all stimuli presented in Arial font). Addition questions from 1 + 1 = through to 9 + 9 = were presented over four blocks, with blocks being counterbalanced to account for any practice effects or teaching that may have occurred between each testing session. However, tie questions (e.g., 2 + 2, 3 + 3) and ''+ 1'' questions were later excluded from the analysis due to there being very little difference in performance on these questions. These questions are of less interest, as previous studies have shown that answers to these questions are often highly automated and frequently retrieved directly from long-term memory for individuals of all abilities. Children were instructed to say the answer to the presented addition questions verbally into the microphone (linked to the voice key). Time taken to reach the answer, accuracy, and strategy use were recorded for every arithmetical question. The child was instructed to use any strategy for reaching the answer that they wished (for example, counting using physical objects (children were provided with counting blocks to use if they so wished), counting on fingers, verbal counting, direct memory retrieval, or any other feasible strategy). If the strategy use was not immediately obvious, the experimenter recorded the most likely strategy based on the child's behavior during the solving of that particular problem, and then questioned the child as to how they had solved the problem. This procedure is given credence by a number of studies which have demonstrated that children can accurately describe problem-solving strategies in arithmetic, if they are asked immediately after the problem has been solved (Siegler, 1987; Siegler & Shrager, 1984). Because one of the major interests of the study was to examine the extent of direct memory retrieval in simple arithmetic problem-solving, trials involving counting (finger counting and verbal counting) were combined to form a counting measure of arithmetic problem-solving which was compared to direct memory retrieval.

RESULTS

Differences between the groups on the various measures were first analyzed by way of 2 × 2 analysis of variance, testing for significant differences between the high- and low-ability mathematics groups and for sex differences. Sex differences were not found on any of the experimental measures, and so only the one-way analyses comparing high- and low-ability mathematics groups will be reported. To be certain that any differences found between the two mathematics ability groups were due to mathematics ability alone, analysis of covariance (ANCOVA) was also carried out for each task, controlling for differences between the groups in reading ability. In all cases the covariate was performance on the BAS Word Reading Test. All results from these analyses are reported in Table 1.

Single-Digit Addition

Strategy use. For each addition question presented, the strategy used by the child to answer the question was recorded at the time of testing. These strategies were split into two main strategies, counting (finger and verbal counting) and direct long-term memory retrieval. One-way ANOVA was carried out comparing groups (low- versus high-ability mathematicians) on their frequency of use of direct memory retrieval in solving arithmetic problems. Significant differences were found between high- and low-ability mathematicians, $F(1,66) = 38.92$, $p < .001$. ANCOVA did not remove this result with significant differences still being present between the groups once differences in word reading ability had been controlled for, $F(1,65) = 20.37$, $p < .001$ (see Table 2). Children in the high-ability mathematics group used direct memory retrieval to solve addition problems significantly more frequently than children in the low-ability mathematics group.

Error frequency. The number of errors made by each group when using counting strategies only was calculated. Only this strategy was considered because very few errors were made by any child when using memory retrieval. A one-way ANOVA comparing groups (low versus high ability mathematicians) revealed significant differences between the groups, $F(1,66) = 18.78$, $p < .0001$. ANCOVA did not remove this result, with differences between the groups in frequency of counting errors made still being apparent once differences between the groups in word reading ability had been controlled for, $F(1,65) = 4.47$, $p < .05$. Observation of the mean number of errors made shows that low-ability mathematicians made significantly more counting errors than high-ability mathematicians (see Table 2).

Strategy times. For each strategy, the mean time taken to produce correct responses to arithmetical questions using that particular strategy was calculated. A two-way repeated-measures ANOVA was carried out with one within-subjects factor, strategy (counting versus direct memory retrieval), and one between-subjects factor, group (low versus high mathematicians). This

TABLE 1
Performance Characteristics of Low- and High-Ability Mathematicians on Screening Tests and Experimental Measures (*SD*)

	Low ability		High ability		ANOVA			ANCOVA		
Measure	*M*	Adjusted *M*	*M*	Adjusted *M*	*df*	*F*	*p*	*df*	*F*	*p*
Group mathematics	86.1 (6.1)	88.02	105.5 (6.5)	103.6	1, 64	149.77	<.001	1, 65	86.28	<.001
BAS word reading	43.5 (8.4)		55.5 (9.5)		1, 64	30.17	<.001			
Counting span	3.3 (1.1)	3.5	3.6 (1.2)	3.3	1, 66	.72	*ns*	1, 65	.47	*ns*
Digit span	4.3 (0.9)	4.6	5.2 (1.2)	4.9	1, 66	10.75	<.01	1, 65	1.18	*ns*
Word span	3.7 (0.6)	3.8	4.3 (0.7)	4.2	1, 66	16.15	<.001	1, 65	3.78	*ns*
1 Syll speech rate[a]	2.3 (0.4)	2.4	2.9 (0.6)	2.8	1, 66	25.36	<.001	1, 65	11.68	<.001
3 Syll speech rate[a]	1.6 (0.4)	1.6	1.8 (0.3)	1.8	1, 66	3.21	*ns*	1, 65	1.18	*ns*
Counting rate[a]	2.5 (0.5)	2.5	2.9 (1.0)	2.9	1, 66	3.77	*ns*	1, 65	2.86	*ns*
Cross-out task[b]	185.4 (40.1)	180.7	151.4 (45.0)	156.1	1, 66	10.73	<.01	1, 65	3.90	=.053
Visual matching[b]	210.7 (39.3)	208.6	171.9 (41.7)	174.0	1, 66	15.41	<.001	1, 65	8.37	<.05
Pegboard[b]	10.0 (1.4)	9.9	9.3 (1.5)	9.4	1, 66	4.65	<.05	1, 65	4.54	<.05
Letter ID[c]	1.1 (0.2)	1.1	0.9 (0.2)	1.0	1, 66	11.26	<.01	1, 65	2.84	*ns*
Number ID[c]	0.9 (0.2)	0.9	0.7 (0.1)	0.8	1, 66	19.65	<.001	1, 65	5.04	<.05
Alphabet[d]	40.2 (17.7)	46.3	80.3 (26.3)	74.2	1, 42	24.23	<.001	1, 41	8.17	<.01
Numerical order[d]	65.9 (29.3)	66.3	88.1 (20.9)	87.7	1, 42	8.39	<.01	1, 41	4.86	<.05
Alphabetical order[d]	47.0 (29.1)	55.0	70.0 (24.7)	61.8	1, 42	7.96	<.01	1, 41	.52	*ns*

[a] Items per second.
[b] Total time (seconds).
[c] Mean time (seconds).
[d] Percentage correct.

TABLE 2
Mean Frequency of Strategy Use, Mean Frequency of Counting Errors, and Mean Time Taken to Answer Simple Addition Questions by Low- and High-Ability Mathematicians (*SD*)

Group	Counting	Memory retrieval	Counting (adjusted means)	Memory retrieval (adjusted means)
Low Frequency (%)	95.42 (7.04)	4.57 (7.04)	94.07	5.93
High	74.48 (17.73)	25.50 (17.76)	75.83	24.14
Low % errors made	30.51 (25.22)		26.00	
High	10.22 (11.67)		14.73	
Low Time (correct answers only)	9.60 (5.66)	3.22 (0.69)	9.24	3.18
High	5.21 (1.82)	2.46 (0.64)	5.55	2.49

revealed significant main effects of group, $F(1,46) = 15.90$, $p < .0001$, and strategy, $F(1,46) = 185.66$, $p < .0001$, along with a significant interaction between group and strategy, $F(1,46) = 15.77$, $p < .001$. This result remained significant after differences in word reading ability had been controlled for through ANCOVA, with main effects of group, $F(1,45) = 12.55$, $p < .001$, and strategy, $F(1,46) = 185.66$, $p < .0001$, being found, along with an interaction between group and strategy, $F(1,46) = 11.47$, $p < .001$. Newman–Keuls tests analyzing the adjusted means revealed that although high-ability mathematicians were faster at reaching the correct answer using both counting and memory retrieval, they were only significantly faster when using counting strategies ($p < .01$), with the difference between the groups in memory retrieval time proving not to be reliable. Both ability groups were significantly faster at using direct memory retrieval than counting to reach correct answers, ($ps < .01$) (see Table 2).

To summarise the analyses of between-groups differences, the low-ability mathematics group showed significantly poorer performance on measures of speed of number identification, processing speed (i.e., visual number matching and perceptual motor speed), one-syllable speech rate, and sequencing ability (alphabet and numerical ordering), even when differences in reading ability between the groups had been controlled for. No differences were found between the low- and high-ability mathematics groups on any of the short-term memory span measures once differences in reading had been controlled for. Examination of the simple addition questions revealed that high-ability mathematicians used direct memory retrieval of arithmetic facts significantly more frequently than low-ability mathematicians, although when this strategy was used there was no difference between the two ability groups in the time taken to retrieve the answer. Analysis of the use of computational strategies (finger and verbal counting) revealed that low-ability mathematicians made significantly more errors using this strategy than high-ability mathematicians, and that they were slower to reach the correct answer.

Correlational Analyses

For the purposes of correlational analysis, all raw scores on the experimental tests were converted to standard scores with a mean of zero and a standard deviation of one. A number of composite measures were formed by combining together experimental variables theoretically thought to be measuring the same substrate and taking the average standard score of the measures, with reliability analysis being performed to confirm these theoretical beliefs. All the correlations reported below are those between mathematics ability and the reported variables, with the reliability coefficient being that of all the variables which were combined to form the composite variable. These were *speed of item identification* (mean letter identification time, $r(66) = -.44$, and number identification times, $r(66) = -.54$, standardized $\alpha = .74$), *short-term memory* (mean counting span, $r(66) = .28$, digit span, $r(66) = .50$ and word span, $r(66) = .51$, standardized $\alpha = .78$), *sequencing* (mean percentage of alphabet, $r(42) = .48$, numerical order, $r(42) = .36$, and alphabetical order, $r(42) = .39$, sequenced correctly, standardized alpha = .65), *processing speed* (cross-out task, $r(66) = -.42$, visual number matching, $r(66) = -.49$, and pegboard, $r(66) = -.23$, standardized $\alpha = .71$), and *speech rate* (counting rate, $r(66) = .14$, one-syllable speech rate, $r(66) = .51$, and three-syllable speech rate, $r(66) = .22$, standardized $\alpha = .66$). All correlation coefficients reported above are those between mathematics ability and the reported variables. As can be seen, all variables that were combined to form a new composite variable had similar correlations with mathematics ability, and therefore no spurious correlations were formed by combining these variables. Correlation coefficients revealed a number of composite variables which correlated significantly with mathematics ability (see Table 3). Indeed, all the experimental measures of item identification, sequencing ability, speech rate, speed of processing, and short-term memory were significantly correlated with mathematics ability. However, once differences in word reading ability had been controlled for through partial correlation (controlling for performance on the BAS Word Reading Test), sequencing ability and speech rate were no longer significantly related to mathematics ability. Performance variables found to be significantly correlated with mathematics once differences in word reading ability had been controlled for were *speed of item identification* (speed of identifying numbers and letters), $r(65) = -.32$, $p < .01$, *processing speed* (time), $r(65) = -.39$, $p < .01$, and *short-term memory*, $r(65) = .25$, $p < .05$.

In order to gain insight into how these cognitive processes may affect the component mechanisms underlying arithmetic, data from the arithmetic questions were also entered into the correlational analysis, these being the frequency of memory retrieval to arithmetic problems, the time taken to reach correct answers both in counting and direct memory retrieval, and the number

TABLE 3

Correlation Coefficients between Test Measures (below Principal Diagonal) and Partial Correlation Coefficients Controlling for Reading Ability (Performance on the BAS Word Reading Test, above Principal Diagonal)

	1	2	3	4	5	6	7	8	9	10
1. Short-term memory span	—	.17	.02	−.18	−.01	.25*	.12	−.03	−.18	−.30*
2. Speech rate	.29*	—	−.28*	−.09	.04	.21	.34**	−.17	−.03	−.14
3. Item identification (time)	−.24*	−.38**	—	.34**	.06	−.32**	−.32**	.29*	.12	.02
4. Processing speed (time)	−.33**	−.18	.46**	—	−.35**	−.39**	−.25*	.10	.49**	.23*
5. Sequencing ability	.30	.21	−.26	−.47**	—	.23	.33*	−.27	.13	−.15
6. Group mathematics test	.50**	.34**	−.54**	−.51**	.54**	—	.51**	−.46**	−.69**	−.43**
7. Memory retrieval (frequency)	.31**	.42**	−.47**	−.36**	.50**	.63**	—	−.26*	−.22	−.13
8. Counting (time)	−.20	−.26*	.42**	.22	−.42**	−.56**	−.38**	—	.78**	.24
9. Memory retrieval (time)	−.29*	−.06	.24	.54**	−.08	−.68**	−.32*	.80**	—	.29
10. Counting errors	−.48**	−.27*	.28*	.39**	−.41**	−.62**	−.33**	.38**	.39**	—

Note. Correlation coefficients: $df = 66$ in all cases apart from sequencing ability where $df = 42$, and memory retrieval (time) where $df = 46$. Partial correlation coefficients: $df = 65$ in all cases apart from sequencing ability where $df = 41$, and memory retrieval (time) where $df = 45$. Where sequencing ability and memory retrieval (time) correlate, $df = 32$ for correlation and $df = 31$ for partial correlation.

$^{*}p < .05$.

$^{**}p < .01$.

of counting errors made. Partial correlations controlling for performance on the BAS Word Reading Test revealed that frequency of memory retrieval was significantly correlated with a number of the performance variables, these being speech rate, $pr(66) = .34$, $p < .01$, item identification, $pr(66) = -.32$, $p < .01$, processing speed, $pr(66) = -.25$, $p < .05$, and sequencing ability, $pr(41) = .33$, $p < .05$. The time taken to correctly solve arithmetic questions using counting strategies was found to be significantly correlated with speed of item identification, $pr(66) = .29$, $p < .05$, with the time taken to retrieve correct answers from long-term memory being significantly correlated with processing speed, $pr(45) = .49$, $p < .01$. The number of counting errors made was significantly correlated with short-term memory. $pr(66) = -.30$, $p < .05$, and processing speed, $pr(66) = .25$, $p < .05$.

Fixed-Order Multiple Regression Analyses

Specific links between predictor variables and mathematics ability were further tested in fixed-order multiple regression analyses, assessing whether the variables speed of item identification, short-term memory, and processing speed accounted for further significant unique variance in mathematics ability, once the variance associated with word reading ability had been removed (BAS Word Reading was always entered on the first step of each regression equation). The outcomes of the regression analyses are summarized in Table 4. The first thing to note is the substantial amount of variance accounted for by word reading ability (45%). A significant unique relationship between mathematics ability and processing speed was found however, with processing speed accounting for a further 8% ($p < .01$) of variance in mathematics ability after the variance associated with reading ability had been accounted for. No variables entered on later stages of the regression equation after processing speed were found to add a significant amount of unique variance, suggesting that processing speed accounts for all the variance associated with short-term memory and speed of item identification. Indeed, even when processing speed was entered on the final step of the regression equation, it was still found to add a significant amount of unique variance which was not accounted for by other variables in the regression equation. Short-term memory appears to be the least predictive of mathematics ability, compared to processing speed and speed of item identification. All of the performance variables entered in the regression equation after short-term memory were found to be contributing a significant amount of unique variance in predicting mathematics ability. Therefore, speed of processing makes a contribution towards predicting differences in mathematics ability that is independent of the contribution made by speed of item identification and short-term memory.

DISCUSSION

The results clearly show that children with arithmetical difficulties have particular problems on a number of tasks designed to assess cognitive pro-

TABLE 4
Summary of Multiple Regression Analyses (Outcome Measure: Mathematics Ability as Measured by the Group Mathematics Test)

Order of entry in regression equation	(R^2)	(R^2 change)	*df*	*F*	*p*
1. BAS word reading	.45	.45	1, 66	54.45	<.0001
Contribution of item identification and processing speed, controlling for BAS reading and STM					
2. STM	.48	.03	2, 65	4.39	<.05
3. Item identification	.54	.06	3, 64	7.93	<.01
4. Processing speed	.58	.04	4, 63	5.23	<.05
3. Processing speed	.55	.07	3, 64	9.64	<.01
4. Item identification	.58	.03	4, 64	3.65	*ns*
Contribution of item identification and STM, controlling for BAS reading and processing speed					
2. Processing speed	.53	.08	2, 65	11.69	<.01
3. STM	.55	.02	3, 64	2.62	*ns*
4. Item identification	.58	.03	4, 63	3.65	*ns*
3. Item identification	.56	.03	3, 64	3.00	*ns*
4. STM	.58	.02	4, 63	3.29	*ns*
Contribution of processing speed and STM, controlling for BAS reading and item identification					
2. Item identification	.51	.06	2, 65	7.28	<.01
3. Processing speed	.56	.05	3, 64	7.09	<.01
4. STM	.58	.02	4, 63	3.29	*ns*
3. STM	.54	.03	3, 64	5.05	<.05
4. Processing speed	.58	.04	4, 63	5.23	<.05

Note. Item identification indicates the speed of number and letter identification; processing speed, mean time to complete cross-out, visual number matching and pegboard tasks; STM, short-term memory span.

cesses. Initially, analyses of variance revealed that children in the low-ability mathematics group performed significantly worse than children in the high-ability mathematics group on measures assessing short-term memory span, speed of item identification, speech rate for one-syllable words, processing speed, and sequencing ability. However, to get a true account of the cognitive processes specifically underlying mathematics ability, it was necessary to control for differences between the groups in word reading ability. Controlling for reading-ability differences through analyses of covariance eliminated previously found differences between the groups on a number of the performance variables, including all of the short-term memory measures, the cross-out task (matching geometric shapes), and speed of letter identification. However, significant differences were still apparent between the groups in speed of number identification, processing speed (visual number matching and perceptual motor speed), one-syllable speech rate, and sequencing ability. Partial correlation coefficients, also controlling for differences in word reading ability, revealed a number of the composite measures to be related to mathematics ability. These included processing speed, speed of item identification, and short-term memory. Fixed-order multiple regression analyses, however, revealed speed of processing to be the best predictor of mathematics ability, subsuming all the variance that speed of item identification and short-term memory accounted for in mathematics ability.

One of the main differences found between groups, once reading ability had been controlled for, was the speed of identifying numbers, which constitute representations which must be retrieved from long-term memory. Measures of strategy usage for the arithmetical problems presented in this study also showed that children in the low-ability mathematics group were less likely to use direct memory retrieval to solve arithmetical questions. Furthermore, when this strategy was used, poor mathematicians were slower than good mathematicians in retrieving the answer. It has been proposed in previous studies that factors associated with long-term memory, such as familiarity of the stimuli and the strength of representations, have an influence on short-term memory, such that items which can be identified and retrieved from long-term memory more rapidly will aid short-term recall (Gathercole & Adams, 1994; Henry & Millar, 1991; Hulme et al., 1991). Additionally, studies investigating children's counting strategies point out that children with arithmetical difficulties have poor working memory resources leading to poor or incomplete representations of numbers and numbers facts in long-term memory (Geary, 1990; Geary & Brown, 1991; Hitch & McAuley, 1991; Siegler & Shrager, 1984). This study is supportive of this view as it was found that speed of item identification, involving retrieval of information from long-term memory, was significantly correlated with short-term memory span. Children with arithmetical difficulties were indeed significantly poorer than children in the other ability group on measures of short-term memory, but not after differences in reading skill had been controlled for.

Another factor found to be strongly correlated with mathematics ability was processing speed. It was further found that low-ability mathematicians tended to be slower to complete the cross-out task (comparing geometric shapes) than the other ability group, although this difference was not significant after differences between the groups in reading ability had been controlled for. Low-ability mathematicians were significantly slower at completing the visual number matching and pegboard tasks than the high-ability group. Correlational analyses showed a composite measure of processing speed to be significantly related to mathematics ability once differences in reading ability had been partialled out, that is, as mathematics ability increased, time taken to complete the processing speed tasks decreased. Furthermore, multiple regression analyses revealed processing speed to be the best predictor of mathematics ability. Garnett and Fleischner (1983) and Geary (1993) proposed that the major problem for children with arithmetical difficulties is associated with the slow execution of operations, particularly with regard to long-term memory access. This can be interpreted in several ways. On the one hand, children with arithmetical difficulties may simply be slower in general information processing, in the way that other children may be slow at running. Alternatively, these children may have specifically failed to automate basic arithmetical operations. If this is so, performance on more complex mathematical tasks is also likely to suffer, as performance on such tasks is contingent upon the fluency of carrying out the simple operations underlying them. Lack of automaticity also provides a link to the finding that children who were poor at mathematics were slow to identify numbers, and that they used direct memory retrieval of number bonds much less frequently than children in the other ability groups. Low-ability mathematicians may be slow to automate numbers and number bonds, which may be due to lack of experience and familiarity with the subject area (Hitch & McAuley, 1991). Frequency of use of direct memory retrieval was found to be partially correlated with numerous performance measures, including speech rate, speed of item identification, processing speed, and sequencing ability. It would therefore appear that the development of mature, efficient strategies for solving simple arithmetic questions, such as direct memory retrieval, is dependent on the establishment of a firm basis of intellectual development, whereby performance becomes automated and information can be processed efficiently. Time taken to retrieve arithmetic answers from long-term memory was also correlated with processing speed, again suggesting this variable measure represents a fundamental deficit in children's arithmetical difficulties.

If lack of automaticity is simply due to lack of practice, then the prospects for remediation are good. If there is no underlying cognitive deficit for children with arithmetical difficulties, then frequent practice of the basic skills, such as counting and simple arithmetic, may provide a simple but highly effective approach for teaching such children, which should enable children to quickly catch up with their peers. Geary and Brown (1991) noted that as children had more experience with numbers and number bonds, fewer computational errors were made and the child relied more frequently on direct memory retrieval of arithmetic facts. Therefore, such difficulties may simply represent a developmental delay. However, Howell, Sidorenko, and Jurica (1987) showed that drill and practice of arithmetic facts, while having some short-term benefits on mathematical ability, did not have any long-lasting effects, and failed to lead to any improvements in strategy development. They found that teacher intervention was much more beneficial, as the specific difficulties of each child could be addressed. Memory retrieval deficits may represent a more serious difficulty. Geary and Brown noted that retrieval deficits, such as slowness, do not disappear, and that this may represent a much more fundamental deficit. In the study reported here, mathematics ability was significantly correlated with various measures of processing speed, some of which did not contain number components, such as speed of identifying letters, speed of matching shapes (cross-out task), and speed of motor functioning. Given these findings, it may be that children with arithmetical difficulties do indeed have a more centrally limiting cognitive deficit in the speed of executing operations, rather than simply a developmental delay in the automatization of numbers and number facts. However, the major differences between the two ability groups appeared to be on tasks containing number components such as speed of number identification and visual number matching. Therefore, the possibility of a very specific deficit relating to the automatization of numbers and arithmetic facts in long-term memory can not be ruled out.

Sequencing ability has not been extensively examined in studies of children's arithmetic, with most studies simply ensuring the child is able to count from 1 to 20. In this study, it was found that children with arithmetical difficulties were able to count accurately from 1 to 20 and were also able to count higher value numbers. However, they tended to be less accurate at sequencing letters of the alphabet and were significantly less accurate at placing items in numerical order. Partial correlation coefficients showed that sequencing ability was not significantly correlated with mathematics ability. One reason for children to be able to successfully count in sequence from 1 to 20 may be that this is a such a well-learned sequence, whereas the numerical ordering task requires the child to order numbers out of context where repetition ability cannot be relied upon. If the child relies on counting to solve arithmetic problems, then this sequence of numbers will be used each time an arithmetic question is encountered. It may that children with arithmetical difficulties are less fluent in their counting, which may impede their speed of arithmetical functioning, although in the present study low-ability mathematicians were not impaired in speed of counting. It may be necessary in future studies to concentrate more specifically on assessing the child's counting knowledge in a similar fashion, as reported in Geary et al. (1992). While children may be able to count in sequence, more needs to be known about the child's understanding of counting principles.

Findings associated with the speech-rate measures are inconclusive. There were significant differences between the high- and low-ability mathematics groups on the measure of one-syllable speech rate, although these differences were no longer significant after differences in reading ability had been controlled for, However, one drawback of both the one- and three-syllable speech-rate tasks was that familiar words were used, which may involve retrieval from long-term memory. Indeed, partial correlation coefficients did reveal that speech rate was significantly correlated with speed of item identification, a measure concerned with retrieving known representations from long-term memory. One consideration for future studies would be the use of nonwords to measure speech rate, which would reduce the confounding effect of long-term memory processes. However, the lack of significant differences between the two ability mathematics groups on counting-rate task cannot be overlooked. It has been proposed in previous studies that children with arithmetical difficulties may have particular problems in the fluency of number counting. This study found that children with lower mathematical ability were not significantly different in their speed of counting dots, suggesting that lack of fluency in counting is not a problem. One important finding concerning counting is with regard to the number of counting errors made. Low-ability mathematicians made significantly more counting errors than high-ability mathematicians, suggesting that there is some underlying problem that leads these children to make mistakes while counting. Clearly, this is not a problem involving the speed of counting. Partial correlation coefficients revealed that the number of counting errors made was correlated with short-term memory span and processing speed. Therefore, one explanation for these increased counting errors may be that children with arithmetical difficulties are slow at processing information, meaning their efficiency of manipulating information in short-term memory may be reduced. This supports the arguments of Case (1985) and Kail (1992) who propose that speed of processing is an important variable in performance and intellectual development.

Given the abundance of studies showing children with arithmetical difficulties to have some form of short-term memory deficit, it can be questioned why no such clear-cut deficit was found in this study, independently of reading skill. Once differences in reading ability had been controlled for, there were no differences between the two ability groups on any of the short-term memory measures. Hulme and Roodenrys (1995) suggest that associations between cognitive impairments and deficits in short-term memory skills should be interpreted cautiously. They point out that there has been little direct evidence for direct causal links between limitations of short-term memory and other impairments of cognitive development and suggest that weaknesses of short-term memory should be considered in conjunction with other cognitive weaknesses when trying to interpret problems associated with cognitive skills such as reading and arithmetic. Furthermore, previous studies which have claimed to examine *specific* arithmetical difficulties have used lenient selection criteria in classifying children as having a specific deficit, and in many cases poor arithmetical skills have been accompanied by relatively poor reading skills (e.g., Geary et al., 1991; Hitch & McAuley, 1991; Siegel & Ryan, 1989). Therefore, it is difficult to ascertain for certain whether previously found results are, in fact, the difficulties associated with specific arithmetic difficulties, or whether they are the outcome of generally poor academic skills, incorporating both reading and arithmetic difficulties.

As shown in this study, an investigation purely of short-term memory skills would not have revealed the true nature of the deficits underlying children's arithmetical difficulties. While there clearly is involvement from short-term memory in children's arithmetical difficulties, as shown by the significant correlations between mathematics ability and short-term memory after controlling for word reading ability, this needs to be considered in conjunction with other factors, such as the ability to identify numbers, which are dependent upon retrieving information from long-term memory. Furthermore, speed of processing obviously represents an element of major importance in the explanation of children's arithmetical difficulties. This study found that children with arithmetical difficulties were slow in the speed of executing operations, such as the speed of identifying numbers, speed of matching numbers and shapes, speed of perceptual-motor performance, and the speed of executing arithmetical procedures. This finding could be linked to lack of familiarity with the subject area. On the one hand, this could simply represent a developmental delay, particularly in the automatization of basic arithmetic facts. Alternatively, and more seriously, it could represent a more fundamental speed-of-processing deficit, which may be specific to arithmetic, or a general deficit which may also underlie often associated reading difficulties. There is clearly a need in studies of children's arithmetical difficulties to address the issues raised by this research, rather than simply relying on an explanation in terms of a short-term memory deficit.

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