The Maximum Dimensional Fault-Free Subcube Allocatable in Faulty Hypercube

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Abstrust

The maximum dimensional subcube located in faulty hypercubes is studied in this paper. Most parallel algorithms can be formulated with the dimension n of the hypercube being a parameter of the algorithm. The reconfiguration problem in a hypercube reduces to finding the maximum dimensional fault-free subcube in the hypercube, that is, helps in achieving graceful and the most effective degradation of the system. That paper presents the maximum number of faults on an n-cube on which there always exists at least one fault-free $(n-2)$*-subcube.*

key words: *Hypercube, Fault Tolerance, Allocation, The Maximum Dimensional Fault-Free Subcube*

1 Introduction

Different but related approaches to hypercube fault-tolerance have been reported. Chang and Bhuyan [1] presented a modified subcube partitioning strategy to construct fault-free subcubes in a hypercube with faults. The technique selects a regular $(n-1)$-cube and then the faulty node in the selected $(n-1)$-cube is replaced with its corresponding nonfaulty node in the other $(n-1)$-cube. They showed that $\lceil n/2 \rceil$ faults can be tolerated in the worst case by the modified subcube partitioning technique while maintaining a fault-free $(n-1)$-cube. Yang and Raghavendra [2] presented a distributed scheme for the reconfiguration of an embedded binary tree into a hypercube with faults. In this scheme, the faulty node in the selected binary tree is replaced with a nonfaulty node which is out of the tree and at Hamming Distance 1. Horng and Kleinrock [3] proposed a set of techniques to restore the regularity of a Boolean n-cube network in the presence of node failures. The shortest path between any node pair in the modified fault-free network holds the "shortest" property also in the n-cube. Bruck et al. [4] studied a technique to obtain tolerant partitions by which partitioned m-cube S is guaranteed such that S contains a connected component of $2^{m-1}+1$ or more nonfaulty nodes.

Most parallel algorithms can be formulated with the dimension n of the hypercube being a parameter of the algorithm [5]. Since a subcube is a subset of a hypercube which preserves the properties of hypercube, the reconfiguration problem in a faulty hypercube reduces to find the maximum dimensional fault-free subcube. B.Becker and H.U.Simon [5] presented simple procedures to find a maximal dimension of a fault-free subcube. F.Özgüner and C.Aykanat [6] also presented a procedure to find the maximum dimensional fault-free subcube where the principle of inclusion-exclusion is used. This procedure can always give the maximum dimension d, the number of fault-free d-subcubes, and the complete set of fault-free d-subcubes. Another algebraic technique, called ATARIC [7], was reported. ATARIC uses two algebraic operators # and \$ to identify the maximum dimensional fault-free subcube. These presented procedures are all based on the given addresses of faulty nodes. In this paper, we will try to formulate the maximum dimension of a fault-free subcube located in a faulty hypercube when the number of faults is given.

It will be assumed throughout this paper that all faults are static and are known. Both nodes and edges may be faulty. However, we will only consider node faults, as an edge fault can be tolerated by assuming that one of the node's incidence upon it is faulty. It will be assumed that faulty nodes can neither perform computation nor communication.

2 Any Dimensional Subcube Allocation

The n-cube network consists of 2^n nodes, each of which are numbered from 0 to 2^n-1 by n-bit binary numbers $a_{n-1}a_{n-2}\cdots a_0$, called the address of the node. Any two nodes on an n-cube are adjacent iff their addresses differ by exactly one bit. A subcube of dimension m, denoted by an m-subcube, is addressed by a string of n symbols drawn from the set $\{0,1,*\}$, where $*$ is *a* don't care symbol. Coordinate values " 0 " and " 1 " can be referred to as the fixed or bounded coordinates and " $*$ " as a free coordinate. An m-subcube on an n-cube has $(n-m)$ bounded coordinates and m free coordinates. For example, in a 4-cube, nodes 0000,0001,0010 and 0011 form a 2-cube addressed by 00$**$.

[Property 1]: The number of faults on an $n(\geq 3)$-cube on which there always exists at least one fault-free $(n-2)$-subcube is not over n.

Proof: We first assume any index represented in the binary system with n bits. Let $\tilde{F}$ be a set of indices which includes 4 different kinds of ordered pairs $(1,1),(1,0),(0,1)$ and $(0,0)$ for every bit pair i and j $(i \neq j)$ in n (≥ 3) bits. Then, $\tilde{F}$ is given in the following equation:

$$\tilde{F} = \{(a_{n-1}a_{n-2}a_{n-3}\cdots a_2a_1a_0), (a_{n-1}\overline{a}_{n-2}\overline{a}_{n-3}\cdots\overline{a}_2\overline{a}_1\overline{a}_0), (\overline{a}_{n-1}a_{n-2}\overline{a}_{n-3}\cdots\overline{a}_2\overline{a}_1\overline{a}_0), \cdots (\overline{a}_{n-1}\overline{a}_{n-2}\overline{a}_{n-3}\cdots\overline{a}_2a_1\overline{a}_0), (\overline{a}_{n-1}\overline{a}_{n-2}\overline{a}_{n-3}\cdots\overline{a}_2\overline{a}_1a_0)\}$$

$\tilde{F}$ means that there exists no fault-free $(n-2)$-subcube when $\tilde{F}$ is assumed to be a set of faulty nodes on n-cube. The existence of $\tilde{F}$ proves Property 1. **Q.E.D.**

The hypercube we consider is assumed to be $n \geq 3$, which is satisfied with the property mentioned in the above proof.

Next, we will try to extend an fault-free $(n-2)$-subcube allocation up to an arbitrary m-subcube allocation.

[Property 2]: The number of faults on an $n(\geq 3)$-cube on which there always exists at least one fault-free $(n-3)$-subcube is not over $(2n-1)$.

Proof: In this proof, in order to evaluate fault-free $(n-3)$-subcubes, the condition of fault-free $(n-2)$-subcube to exist on n-cube is utilized. We will divide an n-cube into two $(n-1)$-subcubes and discuss the minimum number of faults such that there exists no fault-free $(n-3)$-subcube in an $(n-1)$-cube. Namely, we will show that $2n$ faults on an n-cube are sufficient to exist exactly one fault in every $(n-3)$-subcube.

In Property 1, we have introduced a set $\tilde{F}$ of faults such that there exists no fault-free $(n-2)$-subcube on an n-cube. We will apply $n-1 \leftarrow n$ in $\tilde{F}$ to obtain a set $\tilde{F}_{n-1}$ of faults such that there exists no fault-free $(n-3)$-subcube in an $(n-1)$-cube. Then, $\tilde{F}_{n-1}$ is given in the following equation:

$$\tilde{F}_{n-1} = \{(a_{n-2}a_{n-3}\cdots a_1a_0), (a_{n-2}\overline{a}_{n-3}\cdots\overline{a}_1\overline{a}_0), (\overline{a}_{n-2}a_{n-3}\cdots\overline{a}_1\overline{a}_0), \cdots (\overline{a}_{n-2}\overline{a}_{n-3}\cdots a_1\overline{a}_0), (\overline{a}_{n-2}\overline{a}_{n-3}\cdots\overline{a}_1a_0)\}$$

We obtain $|\tilde{F}_{n-1}| = n$. Thus, if each $(n-1)$-subcube includes at least n faults, then there not always exist one fault-free $(n-3)$-subcube on n-cube. In other words, $2n$ is the number of faults such that there exists at least one fault in every $(n-3)$-subcube on an n-cube.

According to Property 1, we will discuss the number of fault-free disjoint $(n-3)$-subcubes on an n-cube when the number of faults is $2n$ or more. Assume that there are $2n$ faults on an n-cube. On the condition that the number of fault-free $(n-3)$-subcubes is zero on the n-cube, we can consider only one case such that one $(n-1)$-subcube includes n faults and the other includes n faults. That is, by applying Property 1 to an $(n-1)$-cube, we obtain that there not always exists one fault-free $(n-3)$-subcube in the both $(n-1)$-cubes which has at least n faults in each. Therefore, Property 2 is proven. **Q.E.D.**

[Property 3]: The number of faults on an n-dimensional hypercube on which there always exists at least one fault-free $(n-i-2)$-subcube is not over $2^i(n-i+1)-1$.

Proof: We will prove that $2^i(n-i+1)$ is the number of faults such that there not always exists fault-free $(n-i-2)$-subcube on an n-cube. By using Properties 1 and 2, we will obtain the number of faults such that all disjoint $(n-i-2)$-subcubes in each one of 2^i disjoint $(n-i)$-subcubes are able to be not fault-free.

The case of $i=0$: An $(n-i=n)$-subcube is an n-cube. From Property 1, we obtain that $(n+1)$ is the number of faults such that all $(n-i-2=n-2)$-subcubes are able to be not fault-free on the n-cube. Then, the address set of faulty nodes is $\tilde{F}$.

The case of $i=1$: In the proof of Property 2, we obtain that n is the number of faults such that all disjoint $(n-i-2=n-3)$-subcubes are able to be not fault-free in each $(n-i=n-1)$-subcube. Then, the address set of faulty nodes is obtained in the following discussion. The addresses of any 2^1 $(n-1)$-subcubes are obtained by rewriting address $(a_{n-1}a_{n-2}\cdots a_{j+1}a_ja_{j-1}\cdots a_1a_0)$ to $(**\cdots*1*\cdots**)$ and $(**\cdots*0*\cdots**)$, where $*$ means don't care. Let $n \leftarrow n-1$ be in $\tilde{F}$ and apply $\tilde{F}$ to $(n-1)$ bits with $*$. We obtain an address set of faulty nodes such that all disjoint $(n-3)$-subcubes are able to be not fault-free in an $(n-1)$-subcube $(**\cdots*1*\cdots**)$. We can perform the same discussion in the case of another $(n-1)$-subcube $(**\cdots*0*\cdots**)$. The total number of faults in these two $(n-1)$-subcubes is clearly twice n.

The case of $i \geq 2$: Let's evaluate the addresses of faulty nodes such that all disjoint $(n-i-2)$-subcubes are able to be not fault-free in any $(n-i)$-subcube. In the case of $i=1$, we could induce the address set of faulty nodes from $\tilde{F}$ by taking $n-1 \leftarrow n$. In the case where all disjoint $(n-i-2)$-subcubes are able to be not fault-free in any $(n-i)$-cube which is set i bits to fixed values, the address set of faulty nodes is induced by taking $n-i \leftarrow n$. Since the number of faulty nodes is $(n-i+1)$ in each $(n-i)$-subcube, then we obtain $2^i(n-i+1)$. **Q.E.D.**

Let us remember that the minimum number of faults on an $n(\geq 3)$-cube on which there always exists no fault-free $(n-m)$-subcube is the minimum number E of indices such that every E m-bit binary numbers composed of m bits in each of E indices can cover all 2^m binary numbers.

3 $(n-2)$-Dimensional Subcube Allocation

In this section, we will focus on fault-free $(n-2)$-subcubes and try to bring the number $2^i(n-i+1)-1$ presented in Property 3 close to the optimum value.

Let $\hat{F}$ be the minimum set of indices which includes 4 different kinds of ordered pairs $(1,1),(1,0),(0,1)$, and $(0,0)$ for every bit pair i and j $(i \neq j)$in $n(\geq 3)$ bits. Let $\tilde{F}_k$ be $\tilde{F}$ when $n=k$.

$$\tilde{F}_3 = \{(a_2a_1a_0), (a_2\overline{a}_1\overline{a}_1), (\overline{a}_2a_1\overline{a}_0), (\overline{a}_2\overline{a}_1a_0)\}$$

Since $|\hat{F}| \geq 4$, then $\tilde{F}_3 = \hat{F}$ when $n=3$. We can calculate $\hat{F}$ when $n = k$ in an exhaustive method. Now, we estimate the time complexity to calculate $\hat{F}$ with $|\hat{F}| = l$ in this method. The calculation procedure is as follows : Select l indices which make a new combination in a set of 2^n indices, and judge whether there exist 4 different kinds of ordered pairs or not for every bit pair in the l indices. Therefore, the upper bound of the time complexity can be given as follows :

$$\binom{2^n}{l} \cdot 2l \cdot \binom{n}{2}$$
$$\approx \left\{(2^n/2)^{2^n}/(l/2)^l((2^n-l)/2)^{(2^n-l)}\right\} \cdot 2l \cdot \{(n-1)n/2\}$$
$$\approx \left\{(2^n/l)^l\right\} \cdot 2l \cdot \left\{n^2\right\}$$

Then, the order of the upper bound of the time complexity is $O(2^{ln} \cdot l^{1-l} \cdot n^2)$. The time complexity to calculate $\hat{F}$ becomes tremendous as n becomes very large, but $\hat{F}$ can be calculated within the realistic range, that is, $n \leq 15$. The calculated values of $\hat{F}$ when $n \leq 15$ are shown in Table 1. When $n = 16$, the fact $\hat{F} \neq 7$ is calculated.

Next, we will formalize an approximate value $\bar{F}(\geq \hat{F})$ of $\hat{F}$. Let $\hat{F}_k$ and $\bar{F}_k$ be $\hat{F}$ and $\bar{F}$ when $n = k$, respectively. $\bar{F}_k$ can be calculated by using $\bar{F}_{k'}(k > k')$. Since $|\hat{F}_4| = 5$ is known in an exhaustive method, we get $\bar{F}_4 = \tilde{F}_4 = \hat{F}_4$. J.Bruck, et al. [8] said in Corollary 8 that $\hat{F}_4 = 5$. For $n \geq 5$, we will calculate $\bar{F}_k$ without $\hat{F}_k$. For the expressional simplicity, we express one index by one row, and $\bar{F}_k$ with $|\bar{F}_k| = l$ by a matrix with l rows and k columns. $\bar{F}_k$ can be expressed by using two $\bar{F}_{k'}$'s, three $\bar{F}_{k'}$'s, and six $\bar{F}_{k'}$'s as shown in Fig.1 (a), (b), and (c), respectively. The point to express $\bar{F}_k$ shown in Fig.1 (a), (b), and (c) is as follows : Since there always exist all kinds of ordered pairs for every bit pair in one $\bar{F}_{k'}$ and there always exist only 2 ordered pairs $(0,0)$ and $(1,1)$ for every bit pair whose 2 bits are in different $\bar{F}_k$'s each other, we need the minimum number of indices to make 2 ordered pairs $(1,0)$ and $(0,1)$ for every bit pair whose 2 bits are in different $\bar{F}_k$ each other. These minimum numbers of required indices are 2, 3, and 4 when the numbers of basic $\bar{F}_{k'}$'s are 2, 3, and 6, respectively, as shown in Fig.1(a), (b), and (c). When $mk' \neq k$, in order to set $n = k$, eliminate any bits from every index with mk' bits. The important point is to select the combination lead to $|\bar{F}_k| =$ minimum. The expressions of $\bar{F}_k$ shown in Fig.1 (a), (b), and (c) will be referred to as types $2\bar{F}_{k'}, 3\bar{F}_{k'}$, and $6\bar{F}_{k'}$, respectively.

Let us calculate $\bar{F}_6$. $|\bar{F}_6| = 7$. By using type $2\bar{F}_3$, we get $\bar{F}_6$ with six 6-bit indices. $|\bar{F}_6| < |\bar{F}_6|$. $\bar{F}_5$ with six 5-bit indices can be obtained by eliminating any one bit from $\bar{F}_6$. Let us next calculate $\bar{F}_9$. By using type $3\bar{F}_3$, we get $\bar{F}_9$ with seven 9-bit indices. $|\bar{F}_9| < |\bar{F}_9|$. $\bar{F}_7$ with 7 indices and $\bar{F}_8$ with 8 indices can be eliminating any 2 bits and 1 bit, respectively, from every index in $\bar{F}_9$. Though $\bar{F}_8$ can be also obtained by using type $2\bar{F}_4$, both types $3\bar{F}_3$ and $2\bar{F}_4$ require 7 indices. Any $\bar{F}_k$ can obtained by extending this method. On condition that the number of required indices must be as small as possible, though single type $2\bar{F}_3, 3\bar{F}_3, 6\bar{F}_3, 2\bar{F}_4$, or $3\bar{F}_4$ is adequate to express any $\bar{F}_n$ for $n \leq 18$, a hierarchical design by using several different types can be also introduced.

From the above discussion, $|\bar{F}_n|$ can be formalized. In the case where type $mF_3(m = 2, 3, 6)$ is used, since $2^{\alpha_1} \cdot 3^{\alpha_2} \cdot 6^{\alpha_3}(\geq n) = 3 \cdot 2^{\alpha_1} \cdot 3^{\alpha_2 - 1} \cdot 6^{\alpha_3}(\alpha_2 \geq 1)$, then types $2F_3, 3\bar{F}_3$, and $6F_3$ must be used $\alpha_1, (\alpha_2 - 1)$, and α_3 times, respectively. This fact leads to $|\bar{F}_n| = 4 + 2\alpha_1 + 3(\alpha_2 - 1) + 4\alpha_3$. On the other hand, in the case where type mF_4 is used, since $2^{\alpha_1} \cdot 3^{\alpha_2} \cdot 6^{\alpha_3}(\geq n) = 4 \cdot 2^{\alpha_1 - 2} \cdot 3^{\alpha_2} \cdot 6^{\alpha_3}(\alpha_1 \geq 2)$, then types $2F_4, 3F_4$, and $6F_4$ must be used $(\alpha_1 - 2), \alpha_2$, and α_3, respectively. This fact leads to $|\bar{F}_n| = 5 + 2(\alpha_1 - 2) + 3\alpha_2 + 4\alpha_3$. The final formula of $|\bar{F}_n|$ is defined as follows:

For α_1, α_2, and α_3 given from $3 \leq n < 2^{\alpha_1} \cdot 3^{\alpha_2} \cdot 6^{\alpha_3}$,

$$|\bar{F}_n| = min(4 + 2\alpha_1 + 3\alpha_2 + 4\alpha_3 \text{ (for } \alpha_2 \geq 1),$$
$$5 + 2(\alpha_1 - 2) + 3\alpha_2 + 4\alpha_3 \text{ (for } \alpha_1 \geq 2))$$

$|\bar{F}_n|$ obtained from the above equation is presented in Table 1.

From the above discussion, we can present the following theorem without the proof.

[Theorem 1]: The number of faults on an $n(\geq 3)$-cube on which there always exists at least one fault-free $(n-2)$-subcube is not over $|\bar{F}_n|$.

We will present the other theorem. This theorem depends on the following easy lemmas.

[Lemma 1]: For all $n \geq 4$, given at most 1 faulty

Table.1 The number of required indices.

n	$\|\hat{F}_n\|$	$\|\bar{F}_n\|$	employed type
3	4	4	
4	5	5	
5	6	6	$2\bar{F}_3$
6	6	6	$2\bar{F}_3$
7	6	7	$3\bar{F}_3$ or $2\bar{F}_4$
8	6	7	$3\bar{F}_3$ or $2\bar{F}_4$
9	6	7	$3\bar{F}_3$
10	6	8	$6\bar{F}_3$ or $3\bar{F}_4$
11	7	8	$6\bar{F}_3$ or $3\bar{F}_4$
12	7	8	$6\bar{F}_3$ or $3\bar{F}_4$
13	7	8	$6\bar{F}_3$
14	7	8	$6\bar{F}_3$
15	7	8	$6\bar{F}_3$
16	(8)	8	$6\bar{F}_3$
17	(8)	8	$6\bar{F}_3$
18	(8)	8	$6\bar{F}_3$

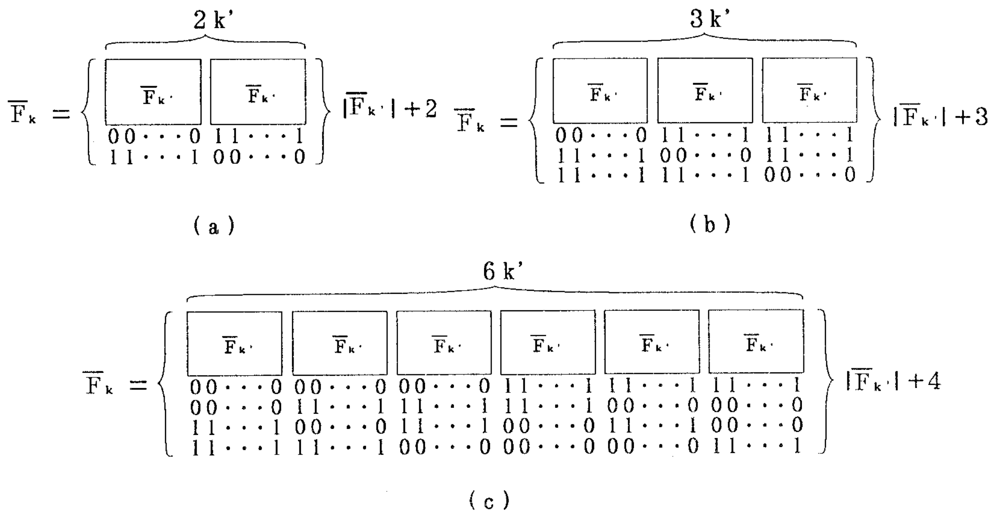

Fig.1 3 types (a) $2\bar{F_{k'}}$, (b) $3\bar{F_{k'}}$, and (c) $6\bar{F_{k'}}$.

node in an $(n-1)$-subcube, there exist two disjoint fault-free $(n-2)$ and $(n-3)$-subcubes which can take any dimensions.

Proof: Obvious.

[Lemma 2]: For all $n \geq 4$, given at most $\lfloor|\hat{F_n}|/2\rfloor$ faulty nodes in an $(n-1)$-subcube, there exist two disjoint fault-free $(n-3)$-subcubes.

Proof: Obvious.

The above two lemmas lead to that, given at most 1 and $\lfloor|\hat{F_n}|/2\rfloor$ faulty nodes in an $(n-1)$ and the other disjoint $(n-1)$-subcubes respectively, there exist two disjoint fault-free $(n-2)$-subcubes.

[Theorem 2]: For all $n \geq 4$, given any set of F consisting of $|\hat{F_n}| - \lfloor|\hat{F_n}|/2\rfloor$ or fewer faulty nodes on an n-cube, there exist at least two fault-free disjoint $(n-2)$-subcube.

Proof: $|\hat{F_n}|$ is the minimum number of faults on an n-cube on which there always exists at least one fault on every disjoint $(n-2)$-subcube. Given $|\hat{F_n}|$ faulty nodes in an n-cube, the distributions of the faulty nodes are classified into three types : Types 1, 2, and 3 are that the n-cube has zero, one, and two or more fault-free disjoint $(n-2)$-subcubes respectively. Since the last type satisfies this theorem, let us prove this theorem in the case of Types 1 and 2.

Type 1 : On condition that an n-cube has no fault-free $(n-2)$-subcube, the n-cube has two $(n-1)$-subcubes on which $|\hat{F_n}|$ faults must be well balanced in the faulty nodes distribution, that is, one $(n-1)$-subcube is occupied by $\lceil|\hat{F_n}|/2\rceil$ faults and the other by $\lfloor|\hat{F_n}|/2\rfloor$ faults. Therefore, if $\lfloor|\hat{F_n}|/2\rfloor$ faults are removed, then the distributions of the remaining $(|\hat{F_n}| - \lfloor|\hat{F_n}|/2\rfloor)$ faulty nodes can be classified into the following two cases : Every two disjoint $(n-1)$-subcubes have one and $\lfloor|\hat{F_n}|/2\rfloor$ faulty nodes respectively, and every two disjoint $(n-1)$-subcubes have zero and $\lceil|\hat{F_n}|/2\rceil$ faulty nodes respectively. In the former case, this theorem can be proven by Lemmas 1 and 2. In the latter case, this theorem can be proven by the result that one $(n-1)$-subcube becomes fault-free.

Type 2 : On condition that an n-cube has only one fault-free $(n-2)$-subcube, the n-cube has two $(n-1)$-subcubes on which $|\hat{F_n}|$ faults must be also well balanced in the faulty nodes distribution. Therefore, if $\lfloor|\hat{F_n}|/2\rfloor$ faults are removed, there always exist two disjoint $(n-1)$-subcubes which have at most one and at most $\lfloor|\hat{F_n}|/2\rfloor$ faults respectively. In this case, the theorem can be also proven by Lemmas 1 and 2.

Q.E.D.

Fig.2 shows a 5-cube which possesses two disjoint fault-free 3-subcubes.

Next, from a different angle, we will formalize a different approximate value $F'(\geq \hat{F})$ of $\hat{F}$.

Let F' be also a set of indices which include 4 different kinds of ordered pairs $(1,1),(1,0),(0,1)$, and $(0,0)$ of 2 different bits i and j for every pair i and j in $n(\geq 3)$ bits, as shown in Fig.3 (a) and (b) where F' is shown by a matrix in the same way as $\bar{F}$. In this figure, F' has α indices whose bit length is ${}_{\alpha-1}C_j$ where $1 \leq j \leq \lceil(\alpha-1)/2\rceil - 1$ in (a), $\lfloor(\alpha-1)/2\rfloor + 1 \leq j \leq \alpha - 1$ in (b). A set of indices in the square solid line in Fig.3 (a) always includes three kinds of ordered pairs $(1,0),(0,1)$, and $(0,0)$ for every bit pair, but not always includes $(1,1)$. Therefore, a special index $(11\cdots1)$ is prepared for the lacked ordered pair $(1,1)$. On the other hand, a set of indices in the square solid line in Fig.3(b) always includes three kinds of ordered pairs $(1,0),(0,1)$, and $(1,1)$ for every bit pair, but not always includes $(0,0)$. A special index $(00\cdots0)$ is prepared for the lacked ordered pair $(0,0)$. Notice that F' in the case $j=1$ conforms to $\bar{F}$, and that there exists a bit complement relation between two F''s in Fig.3 (a) and (b). The optimum F'

is F' defined by the minimum α. We will present the following property for F' shown in Fig.3 (a).

[Property 4]: The optimum F' is given by α on condition that $_{(\alpha-1)}C_{(\lceil(\alpha-1)/2\rceil-1)}$ is minimum but equal or greater than n.

Proof: The optimum F' is given by the minimum α which satisfies the condition $_{\alpha-1}C_j \geq n$. The minimum α which makes $_{\alpha-1}C_j$ maximum is given on condition that $|(\alpha-1)/2-j| =$ minimum. **Q.E.D.**

For $n \leq 18, |F'| = |\hat{F_n}|$. Therefore, F' is the minimum set of indices which includes 4 different kinds of ordered pair $(1,1),(1,0),(0,1)$, and $(0,0)$ for every bit pair in $n \leq 18$.

4 Conclusion

In this paper, the maximum dimension of fault-free subcube located in a faulty hypercube when the number of faults was given was formulated.

This paper is leaving the formalization of the maximum number of faults located on an n-cube on which there always exists at least one fault-free $m(< n-2)$-subcube. The time complexity to calculate the maximum number in an exhaustive method is tremendous as n becomes large, then the formalization of the maximum number is significant.

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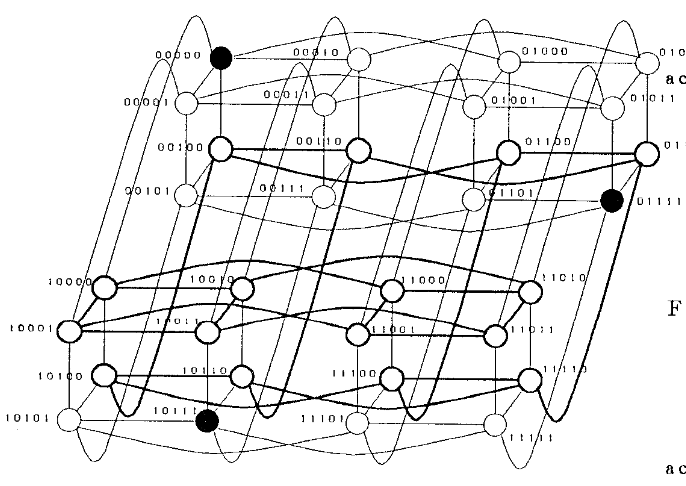

Fig.2 A 5-cube which possesses two disjoint fault-free 3-subcubes.

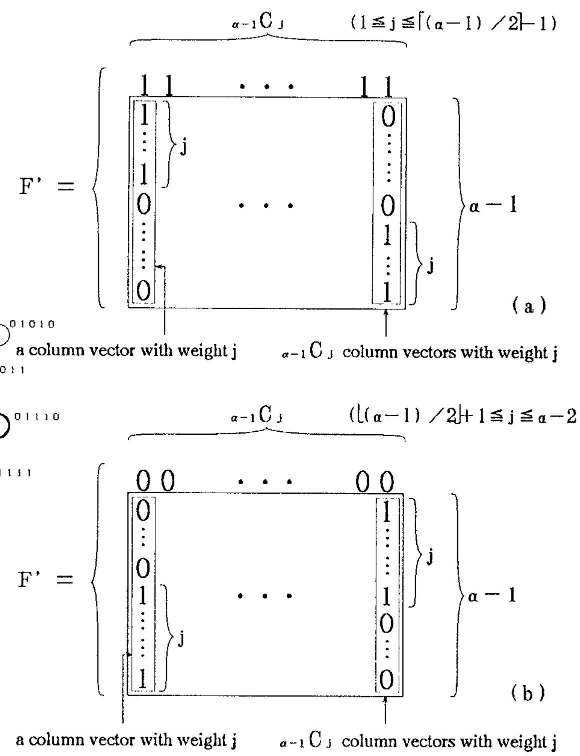

Fig.3 A method to make F'.