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Some examples of negative feedback in the Earth climate system

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Abstract: Temporal variability of daily time series for total solar irradiance at the top of the atmosphere, the Microwave Sounding Unit (MSU) based global, hemispherical and zonal average temperature for the lower troposphere and stratosphere together with 5 surface air temperature data, measured at various meteorological stations have been studied by means of the structure function. From the growth rate of the structure function in the time interval between 32 and 4096 days it follows that the variability of the series represents an anti-persistent (AP) behavior. This property in turn shows a domination of negative feedback in the physical system generating the lower tropospheric temperature variability. Distribution of the increments over various ranges and correlations between them are calculated in order to determine the quantitative characteristics describing temporal variability.

Keywords: Fluctuations, random processes, noise and Bm
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1 Introduction

Measurable variables (e.g air temperature) characterizing the climate have been archived at least two decades on a global scale, and for more than a century at several meteorological stations [1]. Temporal variability of such time series contains valuable information about the present climate. Generally, these series may contain also a deterministic part but always contain random components. The methods for analysis of random processes are accordingly necessary.

Quantitative characterization of random processes is inseparably connected to the

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terms stationarity and non-stationarity. For practical classification of finite series a class of broadly (weakly) stationary processes is useful (e.g [2]): Series $X(t)$ where $t = 0, 1, 2, \ldots$ is stationary if its mean value is constant and its auto-covariance function is invariant under translation. A physical system generating stationary random time series should have a dominating negative feedback because its mean value remains constant during a sufficiently long sample. The same is obviously not true for non-stationary series and several studies in recent years have shown that non-stationarity should be expected if handling geophysical time series (see [3] for discussion).

The non-stationary series can be divided into two classes according to the increments, i.e with stationary or non-stationary increments. The first class is attractive for the present study because many important geophysical time series show a similar behavior. From the non-stationary cases, those series are better studied which are scale invariant. Spectral density $p(f)$ for a scale invariant process has a power-law behavior in a considerable frequency scale $1/L_2 < f < 1/L_1$ [2], [4]

$$p(f) \propto f^{-\beta}, \tag{1}$$

where $L_1 < L_2$ are the corresponding time intervals.

The value of β contains information about the degree of stationarity. Using the Wiener-Khinchine theorem about Fourier transform duality, the spectral criterion for stationarity can be established [4]: $\beta < 1$. The increment series of a scale invariant series with the spectral exponent β are also scale invariant with the spectral exponent $\beta - 2$ [4]. From this it follows that scaling processes with $1 < \beta < 3$ are non-stationary with stationary increments. For the scale-invariant processes with stationary increments the structure function

$$D(\tau) =< [X(t+\tau) - X(t)]^2 >\propto \tau^{2H}, \tag{2}$$

where $\beta = 2H + 1$ and $<>$ stand for ensemble averaging e.g. [5]. $1 < \beta < 3$ implies $0< H <1$. The exponent H (often called the Hurst exponent) enables one to divide the non-stationary series with stationary increments into three classes according to the range of H: $0.5 < H < 1$, $H = 0.5$, and $0 < H < 0.5$. The latter class representing anti-persistence (AP) [2] plays the main role in this paper.

The duality relationship $\beta = 2H + 1$ does not hold for plain spectra if $\beta < 1$. But due to the fact that for stationary series $D(\tau) \to$const, when $\tau \to \infty$ [5], $D(\tau)$ can be used to quantify (potential) non-stationarity by means of H paying no direct attention to the value of β. This approach automatically puts the stationary processes into the AP class but that does not affect our analysis. The main goal here is to make sure whether the H estimates appear to be less than 0.5.

An empirical estimation of H from time series turns out to be important for obtaining information about the dominating feedback sign in the physical system over the available time scale.

A variety of methods have been introduced in various scientific papers (see e.g. [6], [7] for overview). Most popular are power spectrum, structure function and R/S analysis. These methods have been often applied for analyzing various geophysical time series

([8],[9],[10],[11]. They differ in accuracies [7] and can cause a remarkable bias in the results. For the present goal the accurate values for H are not of primary importance, but only those belonging to the class with AP property (H <0.5), is crucial. Thus, we can easily use one method i.e $D(\tau)$ here in order to explain various situations.

For geophysical series $D(\tau)$ is often not a monotonic function of τ. Random components are generally superimposed upon cyclic courses determined by the periodic (seasonal and daily) changes in solar forcing. Such mixtures can always be observed in geophysical data. It brings about an undulation in $D(\tau)$ near some mean value. The situation does not prevent the estimation of average $D(\tau)$ growth rate over the interesting τ interval. One can estimate a dependence between log$D(\tau)$ and log τ by means of a linear equation log $D(\tau) = 2H^* \log \tau + b$ using least-squares over the appropriate τ interval. Generally, large values of τ are important for climate studies. The approximation result may be given as $H^* \pm \Delta H^*$, where ΔH^* the half-width of the confidence interval of H^* at some appropriate significance level. Actual approximations are carried out in the following sections.

Geophysical time series are generally mono-scaling (i.e H = const) during a limited time interval only. An obvious scale break (i.e a change in empirically fitted H value) for various surface air temperature series has been detected between 10 and 30 days (provisionally $10 < \Lambda < 30$) [8]. Recently, a similar scale break interval has been found also for the total solar irradiance at the top of the atmosphere (TOA) and the global mean tropospheric temperature time series [11]. The region $\tau < \Lambda$ is not of interest for the present study. Several daily satellite based data enable one to study a sufficiently longer scale up to 4096 days. That scale is closer to that commonly connected to the climate scale (i.e some decades [12]). The existing temperature archives enable us to compare the H estimates on the basis of the series with different spatial averaging.

The main tasks of the present study are:

- To estimate H over the region $32 < \tau < 4096$ (days) by means of the structure function. The latter estimates the growth of the time series increment's variance as a function of the increment range. It has an important role among the tools of non-stationarity analysis, corresponding to the role of correlation function in the theory of stationary processes [5]. It is especially useful in analyzing experimental series with largely unknown scaling properties.
- Using the H estimates to ascertain the cumulative feedback sign dominating in the Earth climate system for the particular variable. In the present study the term *feedback* is used in the sense of total reaction of the variable to customary forcing in the Earth climate system. Such an understanding is unavoidable in statistical analysis of meteorological time series because, as a rule, they are affected by many forcing types including the seasonal and daily cycles in solar radiation. In climatology the term *feedback* is usually connected to the corresponding feedback loop, e.g *ice-albedo feedback* [13]. For the whole climate system this means that one has to consider many feedbacks at the same time. According to Hansen et al [14] : "Climate feedbacks are internal reactions of the climate system to (natural or anthropogenic) climate

change". If the last two words in it were replaced by a weaker condition "system's state change", i.e the climate scale abandoned, the definition would be in reasonable accord with that used in the present study, although "the state" is presently characterized on the basis of a single variable (temperature), only.

The H values calculated on the basis of satellite data show the reaction of the climate system to all forcing occurred during the years 1978 – 2004, or during considerably longer intervals for the surface air temperature records.

- The initial series are non-stationary. This means that their distribution functions do not exist (e.g [5]). Thus, using sample moments to characterize the series is in vain. Statistical characteristics of the (stationary) increments are more informative in that case. In the present paper we show that the scale break at Λ is closely connected to changes in various statistical properties of the time series increments. Estimation of sample frequency distributions and correlations between the increments is carried out to get more information for describing the feedback.

The temporal variability of daily series has not been well analyzed (e.g. [15]. Mainly, the variability has been studied on the basis of monthly data (e.g [8],[16], [17],[18],[19]). The current study is a sequel to our earlier work [11]. Some modifications have been introduced: Firstly, H is estimated using the structure function. The latter directly reveals a leading role of correlations between the increments. The exponent is estimated on the basis of an updated version which is also about 3 years longer than that used in [11]. The updated data enables one to analyze also hemispheric and zonal mean anomaly series in addition to the global ones.

2 Data

Air temperature datasets correspond to various averaging scales, from particular station measurements to largely averaged Microwave Sounding Unit (MSU) measurements from satellites. Satellite data have been collected mainly during the last two decades and they present an important source for climate research. In the present study both counterparts of the climate making process – the main forcing (total solar irradiance) and response at different averaging scale – air temperature and outgoing long wave radiation (OLR) are analyzed.

Table 1 contains primary information about the analyzed time series. More detailed description about their construction can be found from the cited literature.

A composite record of the sun's total irradiance (I in Wm^{-2}) at the TOA compiled from measurements made by five independent space-based radiometers since 1978 and adjusted for drifts in the radiometric data by Fröhlich and Lean [20] are available on-line. Missing daily values are linearly interpolated for structure function calculations.

Global temperatures have been monitored by satellite since 1979 with the Microwave Sounding Units (MSU) flying on the National Oceanic and Atmospheric Administration's (NOAA) TIROS-N series of polar-orbiting weather satellites [21],[22]. Data of the thermal emissions of radiation by molecular oxygen at 4 frequencies near 60 GHz from

Variable	Location	Label	Time
Irradiance	at the TOA	I	1978 - 2004(May)
OLR	at the TOA	OLR	1974 - 1999
Temperature	MSU Troposphere	T_{TR}	1978 (Nov) - 2004(Dec)
”	MSU Stratosphere	T_{ST}	1978 (Nov) - 2004(Dec)
”	Central England	T_{CE}	1772 - 2001
”	Ristna, Estonia	T_{Ri}	1950 - 2000
”	Verhoyansk, Russia	T_{Ve}	1926 - 1995
”	Kluchi, Russia	T_{Kl}	1931 - 1995
”	Ashgabat, Turkmenistan	T_{As}	1951 -1995

Table 1 Primary information about the daily data used.

nine separate satellites have been combined to provide a global record of temperature fluctuations in the lower troposphere (the lowest 8 km of the atmosphere) have shown that the MSU calibrations have been very stable, with a precision of monthly satellite measurements of 0.02° Celsius for the global mean [22]. The global, hemispheric and zonal daily averaged temperature anomalies for lower troposphere (6-8 km deep) –T_{TR} and stratosphere (covering an altitude range of about 15 - 19 km) – T_{ST}, are used in the present study. Observation time intervals are presented in Table 1.

Five surface air temperature time series measured in meteorological stations are used. The longest existing continuous daily temperature record is that for Central England, the triangle between Bristol, Manchester and London (see [1] for details). The continuous registration started in 1772. The other used daily temperature data belong to different climate regions and have various annual amplitudes. Two records are measured at Russian stations – Verhoyansk, (67.55° N and 133.38° E) and Kluchi (56.3° N and 160.8° E), one from Estonian station Ristna (58.91° N and 22.07° E) and one from Turkmenistan station Ashghabat (38.0° N and 58.4° E).

Air temperature and OLR series have significant annual amplitude. In order to eliminate the influence of that strong variability we need to study the variability of anomalies. Their variance measures the deviation in respect to the seasonal cycle, thus, measuring the possibility of climate change during the period under study. The global MSU based atmospheric temperature data is originally presented in (daily) anomalies. The anomalies are also calculated for the station data used. Averaged values for each calendar day (including February 29) are used to compute them.

3 Quantifying non-stationarity

The value for a time series X_t at the moment t can be represented by

$$X_t = \sum_{i=0}^{\infty} x_{t-i}, \tag{3}$$

where $x_t = X_t - X_{t-1}$ is the corresponding (in the present study daily) increment during the step t. Temporal variability of a non-stationary series X_t can be studied on the basis of the growth rate of variance for the increment

$$X_{t+\tau} - X_t = x_{t+1} + \ldots + x_{t+\tau}, \tag{4}$$

where $t = 0, 1, \ldots, T - \tau$, as a function of τ. Properties of that function are determined by the increments x_t.

The appropriate function (structure function or variogram) can be expressed as:

$$\begin{aligned} D(\tau) &= \frac{1}{T-\tau}\sum_{i=1}^{T-\tau}(X_{i+\tau} - X_i)^2 \\ &= \frac{1}{T-\tau}\sum_{i=1}^{T-\tau}(x_{i+1} + \ldots + x_{i+\tau})^2 \\ &= \tau C(0) + 2[(\tau-1)C(1) + (\tau-2)C(2) + \ldots + C(\tau-1)] \\ &= \tau[C(0) + 2\sum_{i=1}^{\tau-1}(1 - i/\tau)C(i)], \end{aligned} \tag{5}$$

where $C(i)$ stands for auto-covariation of the (anomaly) increments x_t at the lag i,

$$C(i) = \frac{1}{T-i}\sum_{j=1}^{T-i} x_j x_{j+i}.i = 0, 1, 2, ..., \tag{6}$$

The non-decreasing function $D(\tau)$ [5] may or may not grow together with τ. The answer depends on the behavior of $U(\tau)$:

$$U(\tau) = C(0) + 2\sum_{i=1}^{\tau-1}(1 - i/\tau)C(i). \tag{7}$$

3.1 Some important special cases

Next we look at some special cases of $D(\tau)$ behavior if τ grows over a sufficiently wide range.

1. Statistically, the simplest model is $X_t = a_t$, where a_t are independent identically distributed random variables with zero mean and a common variance $\sigma^2 = const$ (white noise). In this case $C(0) = 2\sigma^2$, $C(1) = -\sigma^2$ and $C(i) = 0$ for every $i > 1$, thus, giving $D(\tau) = 2\sigma^2$ for any $\tau > 0$, and $H = 0$. This is an example from the class of strictly stationary processes. The corresponding situation in actual analysis may occur if any estimated confidence interval $H^* \pm \Delta H^*$ covers zero.

2. Let $U \rightarrow$ const $< \infty$ if $\tau \rightarrow \tau_{cr}$. In this case $D(\tau)$ starts growing in proportion to τ if $\tau > \tau_{cr}$ (i.e $H = 0.5$). The case is more general than the classical random walk (i.e with independent increments which implies that $C(i) = 0$ for every $i > 0$). Not all of the correlations must be zero, but only those from certain (finite) lag. Short memory processes lead to that case.

Random walk (or Brownian motion for continuous time) is a well-known process [23]. In the present situation, its property relevant for climatology is, that in the one-dimensional case its trajectory is oscillating around the initial state with a random period.

This means that no arbitrarily long trend exists in any sample path. The model is believed to be useful in representing the global mean stratospheric temperature in scales up to the average solar activity cycle [11].

3. If $U \to \infty$ then positive correlations dominate between the increments of the trajectory X_t over arbitrarily wide τ region. In case of the growth rate $D(\tau) \propto \tau^{2H}$ where 0.5< H <1 we have a persistent scale invariant process (e.g [2]). Positive feedback is dominating in the physical system generating a persistent time series.

4. If $U(\tau)$ is decreasing over the interval $\tau_0 < \tau < \tau_1$ so that $D(\tau) \propto \tau^{2H}$, where 0< H <0.5, the process is called anti-persistent (AP) [2]. The physical system generating an AP series tends to eliminate deviations, showing a negative feedback in sum.

3.2 $D(\tau)$ behavior for some simple models

In order to illustrate the growth rate for $D(\tau)$ with τ, four simple models for the increments $x(t)$ are used. The results for $D(\tau)$ calculated up to τ=360 years (i.e $\log_2\tau$=18) are shown in Figure 1.

The first one corresponds to fractional Brownian motion (fBm) with H=0.05 (i.e the increments x_t are independent identically N(0,1)-distributed), calculated using the midpoint replacement algorithm (see [24] for details). Here N(0,1) stands for normal distribution with zero mean and unit standard deviation. N(0,1)-distributed pseudo-random variables are generated using the codes published in [25]. Figure 1 shows a permanent growth of $D(\tau)$ proportionally to τ^{2H} independent of τ, a classical example of mono-scaling.

Crucial effect of a periodic process in development of $D(\tau)$ is described by the second example presenting an ordinary sine wave, having the increments $x_t = \cos(2\pi t/365)$. Yearly period is chosen in order to get a better comparison with the actual series. The process causes a rapid increase of $D(\tau)$ initially. If τ becomes larger than half of the period the growth stops. Note scale break after label "a" in Figure 1 between 128 and 256 days. $D(\tau)$ keeps undulating around the value reached for larger τ values. Calculating the corresponding H* value over that larger τ interval, it would appear approximately zero.

The third one has increments in terms of a sum of periodic and random component

$$x_t = 0.21\cos(\frac{2t\pi}{365}) + 0.18\epsilon_t, \tag{8}$$

where ϵ_t are independent N(0,1)-distributed random variables. The amplitudes are adjusted to be comparable with the contemporary solar forcing. The amplitude for the first term is consistent with approximately 24 (Wm^{-2}) annual range for global average total solar irradiance at the TOA. The latter corresponds to that for annual range due to the eccentricity of the earth's orbit [12]. The other coefficient in the second term is in order to get the noise term with variance coinciding with that for the daily increments of solar irradiance at the TOA in case of the average earth-sun distance. The mean daily increment for total solar irradiance at the TOA has the variance $\sigma^2 = 0.032$ Wm^{-2} [11].

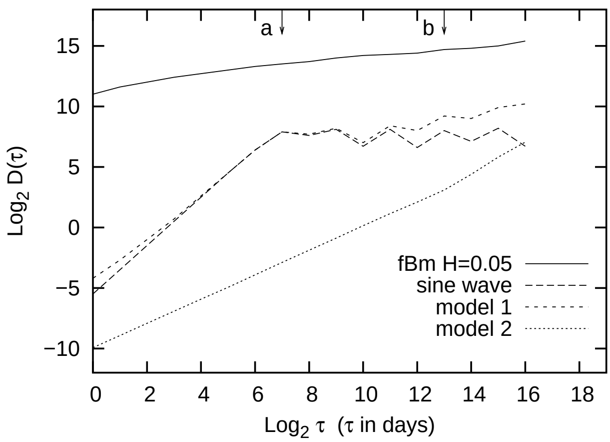

Fig. 1 Structure function behavior for some well-determined models. Essential scale breaks labeled by a (4 months) and b (22 years) are commented in the text.

This model (labeled as model 1) has amplitudes which are similar to those for total solar irradiance at the TOA. But due to white noise it evidently overestimates the role of noisy component in the increments. The actual increments (e.g those for total solar irradiance) are correlated i.e. dependent [11].

Adding the white noise component to the increment of the basic sine wave (formula (8)) causes the scale break at the previous τ value but $D(\tau)$ nevertheless keeps growing with τ. The growth rate depends upon the ratio of standard deviation of noise to the amplitude of the periodic component. Due to the additivity, the noise component becomes dominant and τ increases. For the situation, modelled by formula (8) the growth rate is characterized with $H^* \approx 0.13$. It is important to add that an additive approach (function + noise) is not really valid. Actually the noise is dynamical i.e it generates the feedback and thus, changes the model output. In case of an additive noise, the noise component starts to dominate if τ becomes sufficiently large. This means that there will be a scale where $H^* \rightarrow 0.5$. Due to a strong AP, the outcome is probably different for actual geophysical time series.

For practical reasons the study of methods of estimating H should be interesting. It is obvious that a trivial case of a deterministic straight line having the slope $a \neq 0$ leads to $H^* = 1$. The fourth example (model 2) is constructed in order to explain the effect of a low trend overshadowed by white noise, i.e. the increments following the equation

$$x_t = 0.000055 + 0.18\epsilon_t, \qquad t = 1, 2, \ldots. \tag{9}$$

Equation (9) is designed using experimental information about increase of the CO_2 concentration in the atmosphere. World Data Center of Greenhouse Gases (WDCGG) website (gaw.kishou.go.jp/wdcgg.html) tells that the concentration of CO_2 in the earth atmosphere grows at a nearly constant rate 1.6 ppm/year. Houghton et al [26] predict that the direct result due to CO_2 concentration doubling (from 320 to 640 ppm) in OLR is about 4 Wm^{-2}. Assuming the persistence of the present growth rate, it would take 320/1.6=200 years to reach that value. The corresponding daily increment in OLR would

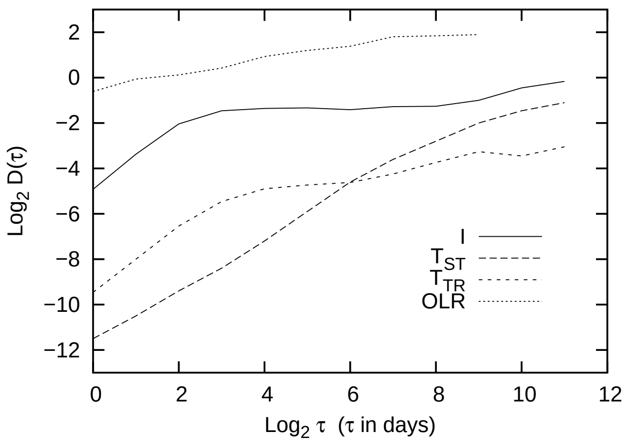

Fig. 2 Structure function behavior for global climatic series, τ in days.

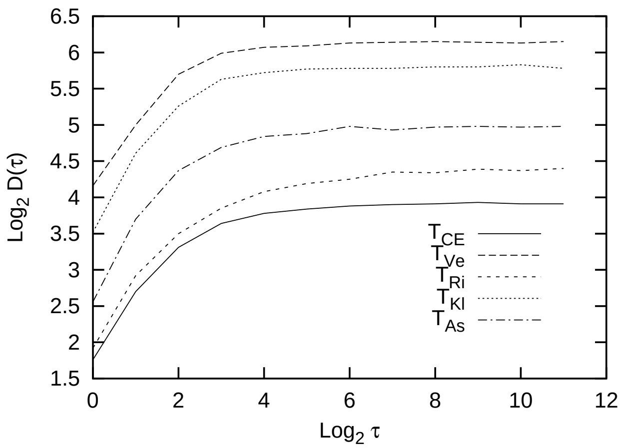

Fig. 3 Structure function behavior for station data, τ in days.

be $4/(200\times365)=0.000055$ Wm^{-2}. This effect should be compared with the variability in the total solar radiation flux density at the TOA. Starting from $t = 0$, the budget has a linear trend with a daily increment of 0.000055 Wm^{-2}. The value corresponds to the daily increase in the atmospheric CO_2 concentration in ppm. We are interested in the time interval needed by the trend component in order to take the lead, i.e then $D(\tau)$ has a break from random walk to a persistent fBm. The break labelled 'b' in Figure 1 is located somewhere between 22 and 44 years (i.e in between 2^{13} and 2^{14} days). This means that in an essentially simplified climate system the leading role of the increase in greenhouse gases might be achieved.

4 $D(\tau)$ behavior for actual climate time series

Figure 2 illustrates the behavior of $D(\tau)$ for the global average OLR, T_{TR} and T_{ST} series in logarithmic scale and Figure 3 shows the same function for the surface air temperature anomalies based on observations in 5 stations.

Figures 2 and 3 show a rapid growth of $D(\tau)$ for $\tau < 8$ days which is similar for

Variable	Unit	n (in days)	τ interval	$H^* \pm \Delta H^*$
I (TOA)	Wm^{-2}	9452	16-4096	0.06±0.01
T_{ST} Glob	°C	9358	32–4096	0.363±0.03
T_{ST} SH	°C	9358	32–4096	0.171±0.013
T_{ST} NH	°C	9358	32–4096	0.179±0.011
T_{TR} Glob	°C	9358	32–4096	0.123±0.020
T_{TR} SH	°C	9358	32–4096	0.071±0.015
T_{TR} NH	°C	9358	32–4096	0.109±0.016
T_{CE}	°C	84006	32–4096	0.0045±0.0018
T_{Ri}	°C	18262	32–4096	0.0134±0.0040
T_{As}	°C	16436	32–4096	0.0063±0.0023
T_{Kl}	°C	23376	32–4096	0.0023±0.0016
T_{Ve}	°C	25346	32–4096	0.0016±0.0016

Table 2 Calculated $H^* \pm \Delta H^*$ values for one solar activity cycle.

different scale temperature data (global tropospheric and local surface air temperature). The same growth rate is found for the total solar irradiance at the TOA. The fast $D(\tau)$ growth rate up to the one month scale for several surface air temperature records has been thoroughly discussed previously in [8] and [11]. The empirical scaling exponents appear to be in good accord with that for the turbulent energy dissipation spectral density in synoptic scale (e.g [5]).

For $\tau > 32$ (days) the growth rate of $D(\tau)$ decreases remarkably. Temporal variability in that region is less studied than that in the synoptic scale.

The time scale is known as *local spectral plateau* [8] and extends over the time interval from one month to several years. The name is related to the fact that $\beta \approx 0$ (equation 1) for the surface air temperature spectral density. The term may lead to a conclusion that the temperature series should be rather stationary in that scale. Using the structure function enables us to estimate H directly and to ascertain the anti-persistence for several temperature series of various spatial averaging. The situations where the spectral and structure function based stationarity criterion may not lead to the same result for any particular time series sample is not striking. In the present analysis the stationarity is treated as a special case of the AP, and its detailed determination makes no sense for the study.

The satellite measured global average series enables us to estimate the τ interval up to 4096 days. For a doubling of τ the number of measurements is evidently too low. Thus, the same interval is used in order to compare H estimates for all the analyzed series. The results averaged over the interval $32 < \tau < 4096$ are shown in Table 2. Results for the hemispheric mean series are also included.

$D(\tau)$ is calculated also for the tropospheric and stratospheric zonal mean temperature series. The MSU based archive contains daily data for 67 wide zones of 2.5° over the belt from −82.5° to 82.5°. H^* values over the same τ interval for the different zones are shown

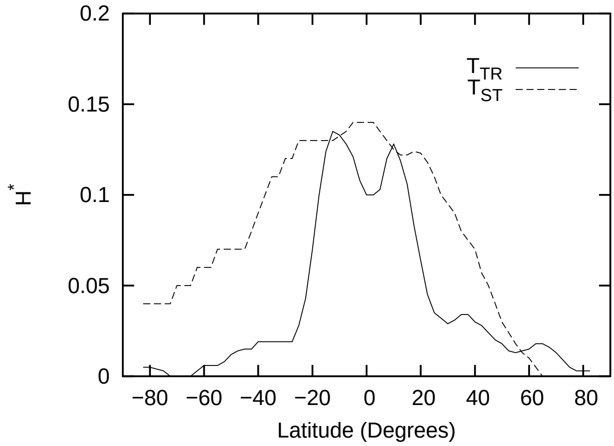

Fig. 4 H^* values for the zonal mean tropospheric and stratospheric temperature series over the τ interval 32 – 4096 days.

in Figure 4. There appears to be a remarkable zonal dependence of both temperatures in terms of non-stationarity. Over the Tropics the H^* values are close to those for the hemispheric mean series (i.e $0.1 < H^* < 0.15$), whereas over the mid-latitudes the zonal series behave similar to the analyzed station temperature series, i.e. they have low H^* values. A remarkable hemispheric asymmetry prevails for both series. The stratospheric series show stronger non-stationarity over the mostly water covered Southern Hemisphere. Behavior of the tropospheric zonal data is converse.

Figures 2, 3, and 4 together with Table 2 show that the fitted H^* values decrease together with decreasing spatial averaging scale of the initial data. The decrease of H^* shows strengthening of negative feedback together with diminishing spatial averaging.

The largest H^* value occurs for the global averaged stratospheric temperature , showing weakly correlated increments. Even a larger exponent is estimated in [11] by different methods using the initial series shorter by three years. The question whether the considerable difference in that estimate is attributable to method remains open until more data is gathered. The alternative is that there might occur another scale break corresponding to the solar activity cycle ([11]). The current (and longer) dataset supports that possibility.

A remarkable difference occurs also between the H^* estimates for the global average T_{TR}. The current result (Table 2) is based on the updated version (Dec 2004) of the anomaly series and the previous [11] result on an older version (from the year 2001). The $D(\tau)$ based exponent estimate is remarkably smaller than that obtained earlier ($0.25 < H^* < 0.35$) by means of regression on the periodogram and R/S analysis. In addition to the different version of the initial data a certain part of the difference is evidently due to the different methods used. As long as the final result, i.e a dominance of AP prevails, no more detailed comment is needed. An interested reader can find some explanations in papers treating the long memory estimation methods [6], [7].

$D(\tau)$ for the global OLR record grows slowly with τ over all the available scale, giving an average value of $H^* = 0.13$. The result is in good accord with the corresponding H^* value for I and the mean tropospheric temperature. The similarity is expected because the tropospheric temperature plays an essential role in the production of OLR. The lack of any break in the $D(\tau)$ behavior for OLR data is at least partly due to the use of 5-day averages instead of daily values. An averaging operation essentially smooths the resulting $D(\tau)$ curve provided that any scale break exists inside that scale. There is another real possibility that no remarkable scale break to show the end of the synoptic scale exists in the global OLR record, because the latter is generated via the interacting surface and atmospheric radiative fluxes.

If $\tau \geq 32$ days, $D(\tau)$ becomes nearly saturated for the station surface air temperature records or keeps growing very slowly (for I and T_{TR}) up to maximum calculated τ values. The confidence intervals $H^* \pm \Delta H^*$ generally show weak non-stationarity for the studied local temperature series. This means that $D(\tau)$ is growing very slowly with τ, hence, larger deviations can be expected in these scales. One of them (for Verhoyansk) almost covers zero showing that the necessary condition for stationarity (i.e $H = 0$) holds in that scale. To get more information about the temporal variability in various climatological time series more detailed analysis of their increments is necessary.

5 Distribution of the increments

The calculated $D(\tau)$ growing rates show that a scale break exists for most of the series. Thus, some statistical properties for the increments might change if the increment range increases from daily to monthly and beyond.

The daily increment distributions have been studied. The Kolmogorov-Smirnov criterion is used to test whether such increment samples can be considered as those from normally distributed collectives . The criterion (e.g [27]) is based on a calculation of the difference between theoretical $F_1(x)$ and empirical $F_2(x)$ probability distribution functions, $\Delta_i = \mid F_1(x_i) - F_2(x_i) \mid$, where $i = 1, 2, \ldots, M$, and M is the number of bins in the histograms. In case of reasonably high number of observations ($n >$100), the critical difference at $(1 - \alpha)\times 100$ % significance level can be found by the asymptotic formula

$$\Delta^* = \sqrt{\frac{-ln(0.5\alpha)}{2n}} \tag{10}$$

If $\max\Delta_i \leq \Delta^*$ the difference may be neglected at the corresponding significance level. In the present study the 99% level is used.

5.1 A tendency to normalize while τ increases

The T_{CE} data enables us to study daily temperature increments during 230 years. Comparison of the anomaly and daily increment frequency distributions are shown in Figure 5, the normal density curve N(0,1.84) is also presented. The daily increments appear to have a remarkably abnormal distribution having higher peak and fatter tails than the normal distribution with the same mean and standard deviation. The same kind of deviation has been found earlier for global variables T_{TR} and I [11].

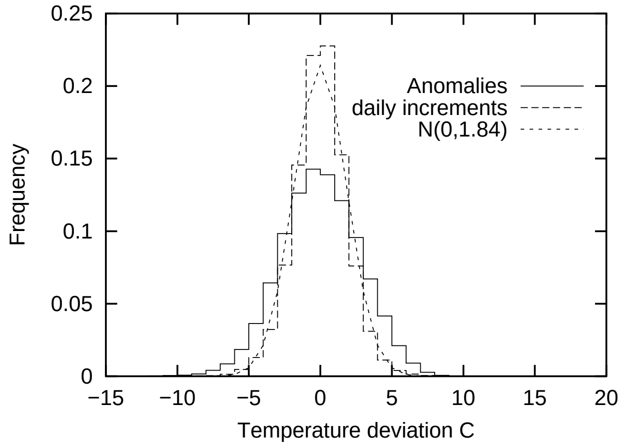

Fig. 5 Frequency distribution for the global T_{CE} daily anomaly and increments.

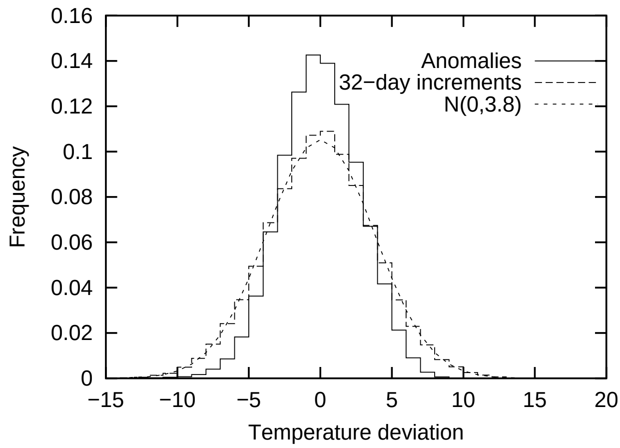

Fig. 6 Frequency distribution for the global T_{CE} daily anomaly and its 32-day increment.

Fat tails indicate to abnormal frequency of large deviations for the short-term increments. Due to negative feedback the corresponding frequency should decrease while τ increases. There may appear a range $\tau = \Lambda_1$ from which the increments have (approximately) normal distribution. Λ_1 is likely inside the region where $D(\tau)$ grows slowly. As an example, the frequency distribution for $x_t^{(32)}$ (i.e 32-day increments) was collected over the available 230 year sample and shown in Fig 6.

Applying Kolmogorov-Smirnov test shows no conflict with the conclusion that the 32-day increment sample has the distribution N(0,3.8). Actual maximum deviation is 0.0049, whereas the 99% limit equals to 0.0056.

Deciding on the basis of that particular sample, an evident abnormality for short-range increments can be transformed to approximate normality if the increment range is sufficiently modified. One can define an efficient self-normalizing (or feedback) scale Λ_1 on that basis. Abnormal short-range ($\tau < 8$ days) increment distributions will be transformed by the climate system to approximately normal ones during the period Λ_1. In the region of nearly saturated $D(\tau)$, the variance for increments is also approximately constant. The situation may mean that (for some stations) there exists a τ region where all the temperature increments have approximately one and the same distribution. Next we show that an extensive analysis is needed to get the answer.

The analyzed series have different strength of non-stationarity on the basis of H. Local meteorological station series seem to be only weakly non-stationary. Calculated $x_t^{(\tau)}$ histograms for five station series show different behavior. In 4 occasions the results show that the scale Λ_1 really exists, but not unique for all the data. For several data a significantly longer interval appears to be necessary in order to get approximately normally distributed increment sample. For Verhoyansk station data 128-day increments, for Ashghabat 256-day one and for the Ristna dataset even 512-day increments are necessary in order to reach a satisfactory approximation. For one station, Klutchi no satisfactory scale was found.

The self-normalization of the increments appears to be quite an unstable i.e., sample dependent, property. Analysis of time series with larger H^* leads to less satisfactory results. For I, T_{ST} data also no satisfactory Λ_1 value was found. All the samples studied appeared to be abnormal up to very long range. The negative result in case of I is probably caused by the varying variance of the irradiance increment record. Figure 3 in [11] showing running standard deviation for the increments of total solar irradiance (I) at the TOA supports that possibility. T_{ST} behavior is essentially different from that of T_{TR}. The daily increments for T_{ST} in case of at least one shorter sample appeared to be approximately normally distributed [11]. The current record is significantly abnormal at the 98 % level. There is a tendency for increasing abnormality while τ increases in the record of T_{ST} increments (not shown). This means that the increment distribution transformation problem while τ increases requires a more detailed investigation, which is rather outside of the scope of the current work.

6 Correlations between the increments

Small H values indicate a domination of negative correlations between the increments. To confirm that one can calculate the corresponding correlations directly on the basis of the increment records. Let the increments (as functions of τ) be $x_t^{(\tau)} = X_{(t+1)\tau} - X_{t\tau}$. Autocorrelations for series $x_t^{(\tau)}$ $t = 1, 2, \ldots n/\tau$, where n is the total number of observations in days, over a large τ interval may be calculated for the available global and local time series. Correlation between the consecutive time periods of the non-overlapping time intervals is crucial in order to determine the feedback sign. Thus, the coefficient $r(1) = C(1)/C(0)$ as a function of τ, for the increment range $1 \leq \tau \leq 200$ days are shown in two figures. The values of $r(1)$ obtained for three global series I, T_{ST}, and T_{TR} up to the increment range 200 days are shown in Fig 7, and for three station temperature series are shown in Fig. 8.

Figure 7 describes exactly what is expected on the basis of the previously calculated H estimates, i.e a remarkable difference in the correlations between I, T_{TR} and T_{ST} increments.

There appears to be a stable negative correlation ($-0.55 < r(1) < -0.4$) between

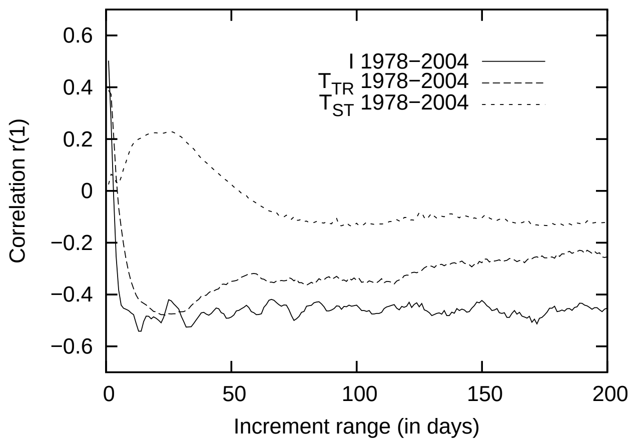

Fig. 7 Autocorrelation $r(1)$ between the consecutive increments of various range in global series.

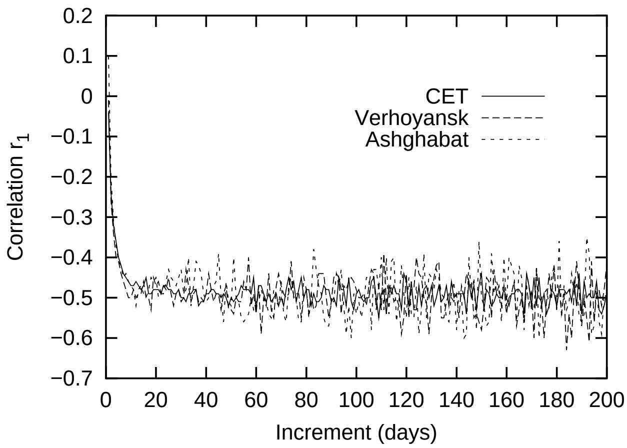

Fig. 8 Autocorrelation $r(1)$ between the consecutive temperature increments over various ranges.

the I increments of interval $5 < \tau < 200$ days. This means that if during one step the anomaly tends to increase, then in the next one it has a tendency to decrease and *vice versa*, showing that the negative feedback has a short-range. The feature corresponds to a rapid fluctuation of anomalies around the mean value.

Small positive correlations can be observed between short-range increments (up to one month) in the stratospheric temperature. If the increment range increases, the consecutive increments become more and more uncorrelated. The corresponding line in Figure 2 also shows the same (i.e $H^* \approx 0.5$) behavior. Thus, the global average stratospheric temperature increments appear to have the most independent behavior in comparison with other MSU based data.

The increments for T_{TR} show strong negative correlation over that range. The (negative) correlation weakens slowly as the increment range increases. This means that the consecutive mean tropospheric temperature increments during longer time intervals may also become uncorrelated.

Figure 8 shows the corresponding coefficient for three station surface air temperature increments. All these series show a behavior of $r(1)$ very similar to that for I: the coefficient is fluctuating near -0.5 starting from about 20 day increments up to 200 days. The situation extends up to 1000 day increments for T_{CE} (not shown). Longer ranges are not used for the correlation calculations. This means that there appears to be a stronger negative feedback between the station temperatures than between the MSU based spatially averaged series. Zonal and hemispheric average MSU variables show the $r(1)$ behavior similar to that for T_{TR} in global case (not shown).

7 Summary and conclusions

We have analyzed several time series describing the earth climate system variability in order to quantify their non-stationarity and show a domination of negative feedback in the physical systems generating them. Total solar irradiance at the TOA characterizes the main driving force and the temperature series averaged in various spatial scale describe the global or local response of the climate system of the earth during the available observation periods. The analysis is carried out by means of the structure function $D(\tau)$. The latter is an efficient tool to quantify the non-stationarity of the series comparing the growth rate of $D(\tau)$ with that for τ^{2H} where $0 < H < 1$.

If a high amplitude periodic term pertains to the time series the varying signs of correlations in $U(\tau)$ (formula 7) become crucial to introduce an AP behavior for the trajectory if τ becomes larger than half the period. Total solar irradiance at the TOA is exactly such type of forcing due to eccentricity of the earth's orbit. The feature introduces the seasonal cycles also into the response series. Total solar irradiance (better known as *solar constant*) *per se* is a low variance AP random process with the (current) mean value 1366 Wm^{-2}. This means that, there are at least two mechanisms in support of the AP behavior - annual course and short-range variations in I. In order to eliminate the strong AP source, the anomaly records (i.e the seasonal cycles removed) have been analyzed. In principle, the behavior of the anomaly series can be different.

Results obtained for the anomaly series for $\tau > 32$ show that:

- All the H estimates for the interval $32 < \tau < 4096$ satisfy the condition $0 < H^* < 0.5$. As a result $D(\tau) \propto \tau^{2H}$ while $\tau > \Lambda$ where $H < 0.5$, showing the dominating AP in the temporal variability. This means that an overall negative feedback dominates in that part of the earth climate system which rules the air temperature anomalies in lower troposphere. The global temperature series appear to contain stronger non-stationarity than the station series on the basis of the H^* (Table 2) obtained. The estimated H values for the station data are close to zero. This shows the possibility that, the necessary criterion for stationarity ($H = 0$) may be valid for a scale longer than 4096 days. Further tests are certainly useful for a better understanding of the climate variability. If any of the local series appeared to be stationary in the scale $\tau > \Lambda_x$ (evidently, $\Lambda_x > 4096$ days) then one should have an experimental basis to discuss the local scale of a stable climate. Otherwise, the climate remains always changing as stated in [14], for example.
- All the studied series show a domination of AP and negative feedback confirming

the results from the previous studies [11] and [28]. This means that, on average, the earth's climate system may be better balanced than theoretically believed. The result shows an opposite sign for the cumulative feedback in comparison with that generally obtained in theoretical studies like [26]. Theoretically, the cumulative feedback is generally calculated as a sum of the feedbacks estimated for the main feedback loops (ice-albedo, water vapor, cloud etc) [13]. A reaction of the non-linear climate system is probably more complicated than predicted by current theory. There are two possible reasons for this. Referring to equation (5) one can assume that the high amplitude cyclic (seasonal and daily) forcing can play a crucial role in damping the deviations. Another possible reason is connected to the variability of I *per se*. The standard deviation of its daily increments at the TOA is remarkable (0.18 Wm^{-2} [11]) and its AP behavior produces an additional mechanism for damping the deviations.

- A non-stationarity of the temperature anomalies means that the increments must be studied in order to characterize the series satisfactorily. The distributions for short-range (e.g daily) increments are evidently abnormal showing higher peak and fatter tails than normal distribution with the same variance. The distributions for the station temperature series under investigation essentially change while the increment range τ increases from 1 (day) to Λ and beyond. There exists a tendency of normalization for the distributions while the increment range increases, provided that the corresponding $D(\tau)$ is nearly saturated in that τ region. The tendency appears to be sensitive to the station location. An extensive study is necessary to test if such a sensitivity can be useful for a better characterization of various climate types.

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