Discrimination properties of invariants using the line moments of vectorized contours

Georg Lambert and Joachim Noll
Darmstadt University of Technology
Control System Theory and Robotics Dept.
Landgraf-Georg-Str.4, D-64283 Darmstadt, Germany
E-mail: lamb@irt1.rt.e-technik.th-darmstadt.de

Abstract

In this paper a new approach for image analysis in real time based on the vectorized contours of a scene is presented. Taking advantage of the fast and efficient determination of the line moments, invariants with respect to translation, scaling and rotation are derived. Four different sets of rotational invariants are introduced and their performance is examined on two examples. Moreover, a quality measure for class discrimination of feature sets is presented and investigated. Using this quality measure as a cost function, heuristic search strategies and genetic algorithms are employed for the feature selection. Thus, high dimensional feature spaces are reduced significantly without losing relevant image information. The performance of both the full and the reduced data sets is investigated on a set of noisy patterns and on a set of letters.

1. Introduction

Geometric moments are very powerful for various image analysis tasks and therefore, considerable research effort has gone into their examination (e.g. [9]). Area moments of the form

$$m_{pq}^{(2)} = \sum_{(x,y) \in D} x^p y^q f(x, y)$$

have been proven to be the best suitable features for characterizing objects or whole images. Equation (1) represents the discretized version of the continuous area moments for digital images. $f(x, y)$ is the discrete gray level and $D$ an arbitrary domain of definition. The exponents $p$ and $q$ determine the order of the moments. According to the uniqueness theorem ([5]) the set of all moments $m_{pq}^{(2)}$ with $p, q = [0, 1, \ldots, \infty]$ is determined uniquely by the discrete gray level function $f(x, y)$ and vice versa. Thus, choosing a sub-set of $L$ moments up to a specific order $(p + q) \leq N$ is a common technique for feature selection. Nevertheless, theoretical investigations about the information content in the various moments have been performed only by very few researchers (e.g. [1]).

The system depicted in Figure 1 is developed as a new approach for image analysis in real time. The underlying idea is paraphrased by the following hypothesis:

**Hypothesis:** The relevant information of a gray level image is represented by the gray level transitions.

Extracting the edge contours, is equivalent to compressing the image data while preserving the relevant information. A real time vectorizer extracts the contours and approximates them by polygons. Exploiting this technique, the amount of data is reduced tremendously compared to the original image data. Given the contours, the continuous area moments of the form

$$m_{pq}^{(2)} = \int \int_{(x,y) \in D} x^p y^q f(x, y) dA$$

$$p, q = [0, 1, \ldots, \infty]$$

Figure 1. A new approach to real time image analysis based on contour processing.
2. Line moments of objects

Figure 2 shows the polygons of a sample scene extracted by the hardware contour extractor. Each object contains several edge fragments which do not have to be connected to each other. The contours between two edge points of the object are modeled by straight lines. Unlike most other techniques, the calculation of the line moments requires neither the existence of a continuous area nor a closed contour around each object. This is a significant benefit of this approach.

The correct segmentation of the contour fragments is the only inevitable prerequisite for the recognition of an object in the image plane. A variety of algorithms can be applied to the segmentation of edge fragments. A very efficient algorithm is presented in [2].

For contours approximated by polygons the resulting moment is computed as the summed moments of all \( n \) line fragments \( m_{pq}^{(1)} = \sum_{i=1}^{n} D_i \). Hence Equation (3) can be analytically solved, because the contour moments \( D_i \) are calculated by integrating along straight lines. Thus, a pixel-based computation can be avoided leading to very fast algorithms.

3. Invariants of line moments

Invariance with respect to translation is easy to fulfill using the central line moments instead of line moments given by Equation (3). Analogous to the central area moments the central line moments are given by

\[
\xi_{pq}^{(1)} = \int_C (x(s) - x(s))^{p} (y(s) - y(s))^{q} f(s) ds
\]

with \( \bar{x} \) and \( \bar{y} \) being the coordinates of the linear center of gravity. The central line moments can be calculated very fast recursively as shown in [8].

Invariance with respect to scaling can not be achieved following the approaches for the area moments because the area moments \( \mu_{pq}^{(1)} \) and line moments \( \mu_{pq}^{(1)} \) do not have the same dimensions. Therefore, the following two scale-invariant line moments are introduced in [8]

\[
\mu_{pq}^{(1)} = \int_C \left( \frac{x(s)}{l_c} \right)^{p} \left( \frac{y(s)}{l_c} \right)^{q} f(s) ds
\]

or

\[
\mu_{pq}^{(1)} = \int_C \left( \frac{x(s)}{l_p} \right)^{p} \left( \frac{y(s)}{l_p} \right)^{q} \frac{d\bar{s}}{l_p}
\]

where \( \gamma_1 = p + q + 1 \) and \( \gamma_2 = \frac{p+q+1}{3} \).

Since the rotation of area objects and contour objects in the image plane is given by the same rotation matrix the approaches presented in [1], [3], [10] and [11] to achieve rotational invariance can be applied to line moments. This motivates to investigate the following sets of moments and invariants:

- Hu line moment polynomials (HLM) and Hu line moment invariants (HLMI)
- Zernicke line moments (ZLM) and Modified Zernicke line moment invariants (MZLM) as well as Teague-Zernicke line moment invariants (TZLMI)
- Complex line moments (CLM) and Complex line moment invariants (CLMI).

In order to compute the rotational invariants fast and efficiently, only those of the above moments are examined that

\[
\int_C (x(s) - x(s))^{p} (y(s) - y(s))^{q} f(s) ds
\]
can be expressed as a linear combination of the geometric line moments. The definitions of the Zernicke moments ([10]) and Complex moments ([11]) change for the Zernicke line moments, Equation (7), and the Complex line moments, Equation (8):

$$A_{nL}^{(1)} = \frac{n + 1}{\pi} \int_{C} [V_{nL}(s)]^* f(s) \, d\bar{s}$$  \hspace{1cm} (7)$$

$$C_{pq}^{(1)} = \int_{C} (x(s) + iy(s))^p (x(s) - iy(s))^q f(s) \, d\bar{s}$$  \hspace{1cm} (8)$$

The three types of line moments HLM, ZLM and CLM can be expressed in terms of the geometric line moments $\mu_{p,q}^{(1)}$ using the equations given in [11] for area moments. Thus a very efficient calculation of the HLM, ZLM and CLM as well as their corresponding invariants can be realized.

The moments and invariants were implemented up to a maximum moment order of 7. This results in 32 HLMI ([5]), 32 MZLMI ([3]), 32 TZLMI ([10]) and 78 CLMI ([11]) as object features. Since the magnitude of the invariants grows exponentially with the underlying moment order normalization is essential to obtain good classification results. Except for the MZLMI (where the choice of the exponent $p$ with $0 < p \leq 1$ as suggested in [3] avoids this effect), all other invariants show this undesirable property.

One approach to normalize the invariants is the normalization to a Gaussian $(0,1)$ distribution. This certainly neglects the fact that the features belong to different classes. Hence normalizing the class specific feature distributions should be the aim. Since the correspondence between features and classes is unknown in advance, normalizing the moments depending on their order is the only practicable way. The following normalizations are applied:

$$I_{p-r,r}^{*} = \sqrt{I_{p-r,r}}$$  \hspace{1cm} (9)$$

$$A_{nL}^{*} = \sqrt{|A_{nL}|}$$  \hspace{1cm} (10)$$

$$|C_{pq}|^{*} = r^{*} \sqrt{|C_{pq}|}$$  \hspace{1cm} (11)$$

### 4. Assessment of image features

In this section a new measure is introduced to numerically assess the quality of feature distributions in high dimensional feature spaces. This measure will serve to select the subset of significant features for a given task to permit the use of common classifiers. Since in our application the amount of invariants is huge it is absolutely necessary to apply automatic feature selection to reduce feature spaces.

Ideal feature distributions consist of compact and well-separated class specific clusters. Certainly in most applications the features of different classes overlap each other, hence the separability is decreased. To numerically assess the quality of features we need a measure which is sensitive to class overlaps.

The question of separability and overlap of data sets has been treated comprehensively in [6] and [7] where a quality measure $q_0$ is introduced. This measure has two main disadvantages. First, small overlapping regions are not detected because of arithmetic averaging. Second, the distance between features of different classes is not taken into account correctly.

Therefore we propose the following measure:

$$q_{0}^{*} = \sqrt{\frac{N}{\sum_{j=1}^{N} \left(\frac{1}{q_{ij}}\right)^{\alpha}}}$$

$$q_{0j}^{*} = \frac{\sum_{i=1}^{k} q_{iN_{j}} + k}{\sum_{i=1}^{k} n_{i}^{*} + k}$$

$$n_{i}^{*} = 1 - \left(\frac{d_{NN_{j}}}{d_{NN_{j}}}\right)^{\beta}$$

$$q_{iN_{j}}^{*} = \begin{cases} n_{i}^{*} & \omega_{j} = \omega_{i} \\ -n_{i}^{*} & \omega_{j} \neq \omega_{i} \end{cases}$$

$$N: \text{number of sample points}$$

$$k: \text{number of neighbors considered}$$

$$q_{iN_{j}}^{*}: \text{quality measure of the examined point } x_{i} \text{ and its i-th neighbor } N_{N_{j}}$$

$$d_{NN_{j}}: \text{euclidian distance between } x_{i} \text{ and } N_{N_{j}}$$

$$d_{NN_{j}}: \text{euclidian distance between } x_{i} \text{ and the farest neighbor } N_{N_{j}}$$

$$n_{i}^{*}: \text{weight on the position of the i-th neighbor } N_{N_{j}}$$

$$\omega_{j}: \text{class of the examined data point } x_{i}$$

$$\omega_{i}: \text{class of the i-th neighbor } N_{N_{j}}$$

The parameter $\alpha$ adjusts the punishment on sample points $x_{i}$ in the close vicinity of the examined point $x_{i}$ belonging to different classes.

The harmonic average (Equation (12)) makes even small overlaps of the class regions affect the quality measure $q_{0}^{*}$ significantly controlled by parameter $\beta$. 

![Figure 3. Quality measures $q_0$ and $q_0^*$ for different values of $\alpha$ and $\beta$ with $k = 10$.](image-url)
Figure 3 illustrates the two quality measures $q_0^*$ from [7] and $q_0^*$ presented here as a function of relative area of class overlap for different parameters $\alpha$ and $\beta$ with $k = 10$ neighbors considered. The two dimensional sample data set consists of two classes with the data points being equally distributed in squares. The overlap of the class specific distributions is varied from 0% to 100%. Figure 3 shows the sensitivity of the quality measure $q_0^*$ for small class overlaps. The larger $\beta$, the faster the quality measure $q_0^*$ decays. The fluctuations of the measure $q_0^*$ for larger area overlaps occur due to features of different classes in very small distances. This behavior has nothing to do with instability and underlines the sensitivity of the measure.

5. Feature selection and results

In this section the quality measure $q_0^*$ introduced in the previous section is exploited to diminish the entity of image features to those features containing the significant information of the scene. Reducing the number of invariants is inevitable to increase the evaluation speed further. Moreover, the interclass discrimination may even be improved as shown for two examples in this section.

The only way to select the optimal subset from a set of $M$ elements is to examine the quality measure $q_0^*$ for all $\binom{M}{H}$ possibilities with $H$ being an integer number ($0 \leq H \leq M$), hence in total $2^M - 1$ possibilities. This problem is NP-complete implying that even for small $M$ the computational costs are extremely high. The class of NP-complete problems has been addressed by many researchers recently and several approaches have been examined to achieve suboptimal solutions:

- Heuristic approaches (e.g. Sequential Backward or Sequential Forward Selection)
- Simulated Annealing
- Genetic Algorithms

In this section the heuristic sequential backward search algorithm and genetic algorithms are used to reduce the number of image features. The sequential backward search investigates the performance reducing the dimension of the feature space sequentially by one until it is one dimensional. In each step the feature with the minimum quality measure $q_0^*$ is excluded from the subset. The theory of genetic algorithms is treated in [4].

Figure 2 shows a sample scene consisting of patterns of four different classes. In Table 1 the quality measures $q_0^*$ and the number of invariants are listed for all four types of invariants investigated in this paper. The first two columns contain the number of invariants $N_I$ and the quality measure $q_0^*$ for the full set of invariants, the next two columns contain the same information for the reduced set of invariants using the sequential backward search and the last two columns for the reduced set of invariants using the genetic search. For this set of pattern data all invariant types provide a very good separability. The size of the reduced invariant set is the smallest for both the MZLMI and the TZLMI.

Although the quality measure of the CLMI is the highest, the large set of invariants is not diminished significantly. Therefore, either the MZLMI or the TZLMI yield the most efficient reduction of the feature set without losing relevant image information. Figure 4 visualizes the 4 class regions (for details refer to [6], [7].) The second example is a scene composed of two alphabets (Figure 5), hence 26 different classes have to be distinguished. The quality measures $q_0^*$ and the number of invariants are listed in Table 2 for all four invariant types.

The first two columns contain the number of invariants and the quality measure $q_0^*$ for the full set of invariants, the next two columns contain the same information for the reduced set of invariants (sequential backward search). On the

<table>
<thead>
<tr>
<th>invar.</th>
<th>$N_I$ full set</th>
<th>$q_0^*$</th>
<th>$N_I$ seq. back.</th>
<th>$q_0^*$</th>
<th>$N_I$ genetic</th>
<th>$q_0^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HLMI</td>
<td>32</td>
<td>0.9817</td>
<td>19</td>
<td>0.9966</td>
<td>16</td>
<td>0.9988</td>
</tr>
<tr>
<td>MZLMl</td>
<td>32</td>
<td>0.9835</td>
<td>16</td>
<td>1.0000</td>
<td>14</td>
<td>0.9999</td>
</tr>
<tr>
<td>TZLMl</td>
<td>32</td>
<td>0.9871</td>
<td>14</td>
<td>0.9992</td>
<td>14</td>
<td>0.9994</td>
</tr>
<tr>
<td>CLMI</td>
<td>78</td>
<td>0.9926</td>
<td>63</td>
<td>0.9999</td>
<td>44</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

Table 1. Number of invariants and performance for the pattern given by Figure 2
Table 2. Number of invariants and performance for the letters given by Figure 5

<table>
<thead>
<tr>
<th>invar.</th>
<th>N_t (full set)</th>
<th>q^*_0</th>
<th>N_t (seq. back.)</th>
<th>q^*_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>HLMI</td>
<td>32</td>
<td>0.3733</td>
<td>26</td>
<td>0.4112</td>
</tr>
<tr>
<td>MZLMI</td>
<td>32</td>
<td>0.4342</td>
<td>11</td>
<td>0.4587</td>
</tr>
<tr>
<td>TZLMI</td>
<td>32</td>
<td>0.3877</td>
<td>10</td>
<td>0.4292</td>
</tr>
<tr>
<td>CLMI</td>
<td>78</td>
<td>0.3974</td>
<td>32</td>
<td>0.4270</td>
</tr>
</tbody>
</table>

set of alphabetic data the MZLMI permit the best separability for both the full and the reduced invariant set. The size of the reduced invariant set is the smallest for both the MZLMI and the TZLMI, but the classification quality of the TZLMI is much lower. Thus, the MZLMI are the best suitable for the reduction of the feature set without losing significant image information. Figure 6 visualizes the 26 classes for the full set of MZLMI (left side) and the reduced set (11 features, sequential backward search). In the reduced set, the classes form more compact regions. However, no final conclusions about the separability of the classes can be drawn from Figure 6 because of the information loss due to the mapping procedure.

Nevertheless all 26 classes of letters are classified correctly using the reduced set of MZLMI and a Bayes normal distribution classifier.

6. Conclusions

It has been shown that evaluating the line moments of contours extracted from gray level images is a computationally efficient and highly selective approach in image analysis. In analogy to area moments geometric line moments, Zernicke line moments and Complex line moments can be derived. In order to reduce the number of invariants calculated out of the different moment types automatic feature selection has been applied. Classification of the selected feature subsets has shown the selectivity of the proposed approach.

References