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Informative voting and condorcet jury theorems with a continuum of types

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Abstract. We consider a model of information aggregation in which there are two possible states of the world and agents receive private signals from the set of probability measures over the binary state space – the unit interval. For a reasonably general set of signal densities, a unique symmetric Bayesian Nash equilibrium in responsive strategies exists and voting is informative in this equilibrium. Asymptotic analysis shows that society makes the correct decision almost surely as population size grows. In contrast to findings of Feddersen and Pesendorfer (1998) in the finite signal space case and Duggan and Martinelli (1999) in an alternative model in which the signal space is a continuum, this result holds for unanimity rule. The key to the efficiency of unanimity rule is that there are perfectly informative (or at least nearly perfectly informative) signals. A corollary to the asymptotic efficiency result is that for all rules the collective performs better than a single agent's dictatorship for large but finite populations. This need not be true for arbitrary population sizes.

1 Introduction

We consider models in which there are two possible states of the world. Agents each receive a private signal drawn from the set of probability measures over the binary state space – the unit interval. There is a unique symmetric Bayesian

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Nash equilibrium in responsive strategies. In this equilibrium voting is informative. Additionally, for all anonymous and monotonic voting rules (termed q-rules) as population size grows the collective makes the correct decision almost surely.[1] The main contribution of this paper is to show that this informational efficiency holds even for unanimity rule. Moreover, we isolate a necessary condition for informational efficiency under unanimity rule when the state-conditional signal densities satisfy continuity and monotone likelihood ratio assumptions. The condition amounts to assuming that the likelihood ratio of the signal densities is unbounded at the lower boundary of the support. When conditional densities are themselves monotone this condition is equivalent to assuming that the private signal could take on a value that occurs with positive density when the state is innocent and 0 density when the state is guilty. This type of signal is interpreted as perfectly informative. We also show that society may make the correct decision with lower probability than an individual's dictatorship for finite populations, but that for sufficiently large populations the collective has a higher probability of making the correct decision.[2]

Intuitively, the difference between equilibrium behavior when the state space is binary and the signal space is a continuum relative to the binary state and signal models (Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1998), is that, here, the conjecture that one is pivotal only provides weak information about the private signals of the other agents; i.e., whether their signals are higher or lower than a cutpoint. In contrast, the agent has precise knowledge of her own private signal. Thus, when the signal space is a continuum the agent is never willing to disregard her signal completely, as is the case with binary signals. That is, there are always sufficiently low (or high) signals that will cause her to vote against a large but finite super-majority. Alternatively, in the binary signal model Austen-Smith and Banks (1996) (hereafter A-S&B) show that conditional on being pivotal the agent is willing to ignore her own signal as she is able to infer the exact value of everyone else's signals.

In the current model unanimity rule is shown to aggregate information efficiently in the limit because there is always the possibility that an agent will receive a perfectly (or at least nearly perfectly) informative signal. When the true state is guilty the probability that all agents get signals higher than the cutpoint converges to unity since the cutpoint converges to zero at a sufficient rate as population size grows. In contrast, when the true state is innocent the probability that at least one agent receives a signal lower than the cutpoint approaches unity. This is different from the result of Feddersen and Pesendorfer

[1] See Austen-Smith and Banks (1999) for a discussion of q-rules. For the current paper it is sufficient to think of q-rules as rules which require at least q votes in favor of conviction for conviction to occur.

[2] The model is related to a common value auction (Milgrom and Weber 1990, 1991) in the sense that agents receive private types from a continuum and the expected value of an outcome to an agent is partially dependent on both her own information and the information of others.

(1997) where the percentage of informative voters vanishes. In the current paper all voters remain informative but the probability vanishes that any voter receives a sufficiently low signal to induce a vote to acquit. The rate of this convergence is sufficiently fast (slow) when the state is guilty (innocent) so that, in the limit, no one (at least one person) votes to acquit with probability one.

The analysis of information aggregation through voting rules dates to Condorcet (1785) who showed that the probability that a majority would make the correct decision tends to unity as the number of members of the group tends to infinity. In models in which there is a correct, or best action, and agents are imperfectly informed about which choice is best there are two well studied types of theorems.[3] Condorcet Jury Theorems of the first type (CJT1) make statements of the form: the probability that the group makes the correct decision is higher than the probability that a single person would make the correct decision. Condorcet Jury Theorems of the second type (CJT2) make statements of the form: as population size grows to infinity, the group makes the correct decision with probability one.

Early work on Condorcet Jury theorems was statistical in nature and began with the assumption that agents voted based purely on their private information, so an agent was characterized by a probability of voting correctly. With varying degrees of generality, these works rely on informative voting – voting to acquit if and only if an agent receives a private signal of guilty – to invoke a law of large numbers on the sample frequency of correct votes (Condorcet 1785; Miller 1986; Young 1988; Grofman and Bernard et al. 1988; Ladha 1992, 1993; Berg 1993; Berend and Paroush 1998).

Following the seminal work of Austen-Smith (1990) identifying the incentive to vote strategically in environments with asymmetric information, A-S&B demonstrate that if voters are strategic the Condorcet Jury theorems may not hold. In the A-S&B framework, conditional on being pivotal, a voter may infer the signals received by other voters, and her optimal response can be to ignore her own signal. Thus, it is not always rational for an agent to vote informatively if all other players vote informatively. The absence of an informative equilibrium means that a Condorcet Jury theorem is not guaranteed. This result stands at odds with the statistical literature that assumed informative voting.

Several papers have reestablished the validity of Condorcet Jury Theorems with strategic voting. Feddersen and Pesendorfer (1998) (hereafter denoted F&P) demonstrate that all q-rules, other than unanimity, efficiently aggregate information asymptotically; thus establishing CJT2's. They also find that the probability of convicting the innocent is bounded away from 0 under unanimity. McClennan (1998) and Wit (1998) take a different approach. Witt shows that when mixed strategies are allowed the A-S&B model has at least one equilibrium in which a Condorcet Jury Theorem holds. In a more general approach, McClellan proves that for a game of common interest, if it is the

[3] A third type of Condorcet Jury Theorem addressing whether the probability of the correct decision is increasing in population size, has also received attention.

case that informative voting yields a statistical CJT2, then when mixed strategies are allowed an equilibrium exists in which a CJT2 attains.

Others have extended the basic game form. Myerson (1997) demonstrates jury theorems in a model with uncertainty about the population size and a countable number of signals. Under an alternative framework that allows for mistrials, Coughlan (2000) shows that unanimity is more informationaly efficient than any other q-rule. In this framework, Coughlan also shows that a Condorcet Jury Theorem need not hold for any q-rule. Meirowitz (1998) considers an extension of Coughlan's model in which votes are repeatedly taken until a super-majority favors conviction or acquittal. In this framework a CJT2 exists for unanimity rule. No other rule has this property for all parameterizations of prior probabilities of guilt and signal accuracies.

In contrast to the above models in which preferences are at least partially aligned, another related strand of research focuses on the aggregation of both information and preferences. Feddersen and Pesendorfer (1996, 1997) consider voting in an asymmetrically informed electorate with heterogenous preferences and establish a type of CJT2 (termed full information equivalence) in which, asymptotically, voting yields the outcome that would be reached by the heterogenous population with perfect information.

None of the these models treat the signal space as uncountable. In current and independent work Duggan and Martinelli (1999) (hereafter denoted D&M) consider a general model which has as a special case the model of F&P. For a model of aligned preferences with a binary state space D&M show that with a continuum of possible private signals and conditional distributions that satisfy a monotone likelihood ratio property, all q-rules except unanimity yield a CJT2. Again, the probability of making a mistake is bounded away from 0 under unanimity.

This paper assumes that agents receive private signals from the set of probability measures over the binary state space (thus the signal space is the unit interval). The model of D&M is more general in its analysis of rules other than unanimity.[4] Accordingly, the proof of some independently attained results are suppressed as the reader's time is better spent considering the more powerful non unanimity analysis of D&M.[5] We assume, however, that there is a possibility (although not a positive probability) of an agent being essentially perfectly informed. That is, there is a positive probability that an agent receives a signal arbitrarily close to a perfectly informative signal. With unanimity rule, this assumption ensures that when the true state is guilty, eventually no agents vote to acquit; and when the true state is innocent, eventually at least one agent votes to acquit.

The structure of the remainder of the paper is as follows. In Sect. 2 we

[4] Technically speaking, the current model is not a special case of the D&M model because the boundary assumptions of the two models are inconsistent. This difference only matters when addressing the efficiency of unanimity.

[5] Proofs of these results are available in a previous version of the current paper, (Meirowitz 1999).

present the model. In Sect. 3 we establish the existence and uniqueness of informative voting equilibrium (Proposition 1). We then turn to the asymptotic analysis and establish the existence of a CJT2 for any non unanimous q-rule (Proposition 2). We then derive a CJT2 for unanimity (Proposition 3) and show the necessity of the boundary assumption (Proposition 4). We conclude the analysis by illustrating the failure of CJT1's in finite populations and show that a CJT1 holds for sufficiently large populations (Proposition 5). In Sect. 4 we conclude.

2 The model

2.1 The game

A finite set of players $N = \{1, 2, \ldots, n\}$ is to make a collective decision over the choice set $X = \{a, c\}$, where a and c correspond to acquit or convict. The unknown state is $s \in \{I, G\}$ with the interpretation of innocent or guilty. Individuals have identical preferences over the choice $x \in X$ and state s. Preferences are represented by the utility function:

$$u_i(x, s) = \begin{cases} 1 & \text{if } x = c \text{ and } s = G \text{ or } x = a \text{ and } s = I \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N. \tag{1}$$

At the beginning of the game, players have a common prior probability $\pi \in (0, 1)$ that $s = G$. Each player simultaneously receives a private signal $\sigma_i \in [0, 1]$ having the state conditional density $f(\sigma_i|s)$. Knowing only their own private signal, players simultaneously cast a vote $v_i(\sigma_i) \in \{a, c\}$. We use the notation v to denote the profile of votes for all players and v_{-i} to denote the profile of votes for all players other than player i. We refer to the positive integers as $\mathbb{Z}_{++}$, the rationals as $\mathbb{Q}$, the integer part of a number y as $\lceil y \rceil$, and the cardinality of a set A as $|A|$. The social choice is made by a particular q-rule. That is, if $|i \in N : v_i = c| \geq q$, then the social choice is c; otherwise it is a for $q \in \left\{\mathbb{Z} : \frac{n}{2} < q \leq n\right\}$. Since we consider a sequence of games, letting population size tend to infinity, it is convenient to parameterize a q-rule by the percentage of votes needed for a choice of $x = c$. Let $r \in (0.5, 1] \cap \mathbb{Q}$ characterize a q-rule. The interpretation is that for a fixed population size, n, a given rule, r, yields the minimal number of votes required to convict of $q(r) = \lceil rn \rceil$. The choice $x \in \{a, c\}$ may be written as a function $x(v; r, n)$ of the vote profile, for a given rule, r, and population size, n.

We model the uncertainty as follows. Nature selects a realization of the random variable $\omega \in \Omega := \{I, G\} \times \prod_{n=1}^{\infty} [0, 1]^n$. A sequence (indexed by n) of ω-measurable functions is the mappings $\psi^n : \Omega \to \Psi^n$, where $\Psi^n := \{G, I\} \times [0, 1]^n$ for $n \in \mathbb{Z}_{++}$. For a fixed n, this random variable is written $\psi^n(\omega) = (s, (\sigma_1, \sigma_2, \ldots, \sigma_i, \ldots, \sigma_n))$; which consists of the unknown draw of the state variable $s \in \{G, I\}$ and the vector of private signals $\sigma^n = (\sigma_1, \sigma_2, \ldots, \sigma_i, \ldots, \sigma_n)$. Signal profiles σ^n are conditionally independent and individual signals σ_i are conditionally independent and identically distributed with the state conditional density $f(\sigma_i|s)$ for $s \in \{I, G\}$. These assumptions imply that the triangular array of individual signal draws is independent in n. That is, conditional on the state s, the elements of the sequence $\{\sigma^1, \sigma^2, \ldots, \sigma^n, \ldots.\}$ are independently distributed. Thus, conditional on s, the values of the individual signals in σ^n convey no information about the individual signals of σ^{n+1}. Under these assumptions the parameter π and conditional densities $f(\sigma_i|G)$ and $f(\sigma_i|I)$ completely characterize the law of ω and, thus, characterize the law of the sequence of random variables $\{\psi^n(\omega)\}_{n=1}^{\infty}$ which is a measurable function of ω.

The specification of Ω may seem unnecessary and cumbersome. However, in order to prove that the correct decision is chosen almost surely, it is necessary for the phrase *almost surely* to have meaning. This requires that we are precise about the random variable and its law.

While the assumption of state conditionally independent vectors σ^n is not the most natural way to model a sequence of jury games where population size tends to infinity, it yields more powerful results. Specifically, if we assume that there is one sequence of signals $\{\sigma_i\}_{i=1}^{\infty}$ and consider a sequence of games (indexed by n) where in each game only the first n private signals are realized, establishing the almost sure convergence in Proposition 3 is not straightforward. Under the assumption that there is just one sequence $\{\sigma_i\}_{i=1}^{\infty}$ the problem is that the choice $x(v; r, n)$ from a game of n jurors is not state conditionally independent of the choice $x(v; r, n+1)$ from a game of $n+1$ jurors. However, under the assumption that there is a sequence $\{\sigma^n\}_{n=1}^{\infty}$ the choices $x(v; r, n)$ and $x(v; r, n+1)$ are state conditionally independent.

2.2 Informational environments

It is necessary to impose additional restrictions on $\langle f(\sigma_i|G), f(\sigma_i|I)\rangle$. While not completely innocuous, these restrictions seem to embody a reasonable notion of well-behaved informative signals. Moreover, they ensure that the equilibria have desirable continuity properties that simplify the analysis. The restrictions are of three types: continuity, monotonicity, and boundary behavior.

The assumption that conditional densities are continuous makes the analysis simpler. To ensure that private signals convey information about the payoff relevant state, s, we assume that the conditional densities satisfy a monotone likelihood ratio property (denoted MLRP) that $\dfrac{f(\sigma_i|I)}{f(\sigma_i|G)}$ is strictly decreasing. We also consider a stronger requirement that conditional densities satisfy a monotone density property (termed MDP) that $f(\sigma_i|G)$ is strictly increasing and $f(\sigma_i|I)$ is strictly decreasing. Under the stronger MDP when the state is $G(I)$, higher signals are more (less) likely than lower signals. Under MLRP higher signals are stronger indications of guilt. The boundary assumptions are that $f(0|G) = 0$, $f(0|I) > 0$. This should be interpreted as saying that given the state is I there is positive density that an agent will receive a signal which is only possible when the state is I. These perfectly informative signals occur with 0-probability because $f(\cdot|s)$ is a conditional density. However, with positive probability an agent will receive a signal arbitrarily close to perfectly informative. More precisely, $\lim_{\varepsilon\downarrow 0} p(G \mid \sigma_i \in [0,\varepsilon)) = 0$ and $\lim_{\varepsilon\downarrow 0} p(I \mid \sigma_i \in [0,\varepsilon)) = 1$, where $p(G \mid \sigma_i \in [0,\varepsilon)) = \dfrac{\pi \int_{[0,\varepsilon)} f(\sigma_i|G)\, d\sigma_i}{\pi \int_{[0,\varepsilon)} f(\sigma_i|G)\, d\sigma_i + (1-\pi) \int_{[0,\varepsilon)} f(\sigma_i|G)\, d\sigma_i}$ is a version of conditional probability.

The motivation for the boundary assumption is the idea that in any situation there is the possibility that an agent may observe something that is only consistent with one of the two states. For example, in a senate committee choosing whether to censure a member; one of the senators may know with certainty that the accused was not in the location of the alleged crime. In such a case, no number of individuals that believe the member to be guilty will sway the well-informed senator's belief. We consider two sets of priors and state conditional distributions. The smaller set is termed permissible.

Definition 1. *The permissible set* Θ^P *consists of informational environments* $\theta = \langle \pi, \langle f(\sigma_i|G), f(\sigma_i|I) \rangle \rangle$ *where* $f(\sigma_i|G)$ *is strictly increasing and continuous on its support* $[0,1]$ *and* $f(\sigma_i|I)$ *is strictly decreasing and continuous on its support* $[0,1]$ *(monotone density property), and the boundary behavior is as follows:* $f(0|G) = 0$, $f(0|I) > 0$, $f(1|G) > 0$, $f(1|I) = 0$.

One consequence of the monotone density assumption is that $F(\sigma_i|G)$ is strictly convex and $F(\sigma_i|I)$ is strictly concave, where $F(\sigma_i|s)$ is a conditional distribution function. A second consequence is that the distribution of signals conditional on guilty strictly first-order stochastically dominates the distribution of signals conditional on innocence.

We also consider a superset of permissible environments.

Definition 2. *The weakly permissible set* Θ^W *consists of informational environments* $\theta = \langle \pi, \langle f(\sigma_i|G), f(\sigma_i|I) \rangle \rangle$ *where* $\dfrac{f(\sigma_i|I)}{f(\sigma_i|G)}$ *is strictly decreasing on* $[0,1]$ *(monotone likelihood ratio property), the conditional densities are continuous on* $[0,1]$*, and the boundary behavior is as follows:* $f(0|G) = 0$, $f(0|I) > 0$, $f(1|G) > 0$, $f(1|I) = 0$.

Since MDP implies MLRP, $\Theta^P \subset \Theta^W$. Under the weaker MLRP the boundary assumption is stronger than necessary for a CJT2 under unanimity. As shown in Corollary 1 the necessary boundary assumption is $\lim_{\sigma_i \downarrow 0} \dfrac{f(\sigma_i|I)}{f(\sigma_i|G)} = \infty$. Under the monotone density condition this limiting condition coincides with the boundary assumptions $(f(0|G) = 0, f(0|I) > 0)$. We interpret the weaker unbounded likelihood ratio property as the presence of nearly perfectly informative signals.

D&M conduct their analysis on a set of distributions which is neither a superset nor a subset of permissible, or weakly permissible densities. They assume that:

1. conditional distribution functions are absolutely continuous with respect to Lebesgue measure.
2. conditional distributions have piecewise continuous densities with a likelihood ratio that is decreasing.
3. the densities have common support and are strictly positive on the support.
4. $\lim_{\sigma_i \to 0} \frac{f(\sigma_i|I)}{f(\sigma_i|G)} > \frac{\pi}{1-\pi} > \lim_{\sigma_i \to 1} \frac{f(\sigma_i|I)}{f(\sigma_i|G)}$.

Clearly, the permissible and weakly permissible conditional densities satisfy 1 and 2. Moreover, the boundary conditions imply that condition 4 is satisfied. Condition 3 may appear to not be satisfied, as the boundary conditions imply that the conditional densities are not positive on the boundary of the support, but since D&M define the support as an open set the condition is satisfied. In the asymptotic analysis D&M impose an additional condition that the likelihood ratios are bounded, which only matters for the analysis of unanimity rule. In all but the results regarding the efficiency of unanimity rule, the set of weakly permissible environments may be viewed as a subset of those considered by D&M.

Given the symmetry of the model, we may characterize a specific permissible (weakly permissible) voting situation with the pair $\gamma = \langle r, \theta \rangle \in \Gamma^P :\equiv \{(0.5, 1] \cap \mathbb{Q}\} \times \Theta^P$ $(\Gamma^W \equiv \{(0.5, 1] \cap \mathbb{Q}\} \times \Theta^W)$ This characterization is independent of the number of voters. A voting game (γ, n) has voting situation γ and population n. We only consider n's for which nr is an integer. This assumption is purely technical, but it simplifies the notation. Note that since $\Theta^P \subset \Theta^W$, we have $\Gamma^P \subset \Gamma^W$.

2.3 *The equilibrium concept*

We seek symmetric Bayesian Nash equilibria. To save on space we use the acronym BNE for Bayesian Nash equilibria. Since the signal space is larger than the strategy space a notion of informative voting is not completely transparent. The term informative voting tends to be used in models where the signal space and the state space are the same size. The most straightforward extension is to say voting is informative when players vote to convict if they receive a signal higher than some cut-point $\hat{\sigma}$, and vote to acquit if they receive a signal lower than $\hat{\sigma}$. Under the assumption of symmetry the cut-point will be the same for all players and need not be indexed by player.

Definition 3. *Given a voting game (γ, n) with $\gamma \in \Gamma^W$, we say that voting is informative iff there exists a cutpoint $\hat{\sigma} \in (0, 1)$ s.t. $\forall i \in N$, $\sigma_i < \hat{\sigma}$ implies $v_i = a$, and $\sigma_i \geq \hat{\sigma}$ implies $v_i = c$.*

Letting ρ_i be an arbitrary probability that the true state is G, equation (1) implies that we can express expected utility (under the belief ρ_i) as:

$$E_{\rho_i} u_i(x) = \begin{cases} \rho_i & \text{if } x = c \\ 1 - \rho_i & \text{if } x = a \end{cases} . \tag{2}$$

We now discuss the form that ρ_i may take if voting is informative (and thus symmetric). We use the notation $\{N_a^i, N_c^i\}$ to denote a partition of $N \backslash i$, according to how the players are voting. Thus, $|N_a^i|$ denotes the number of individuals other than i, casting vote of type a. Under informative voting $N_a^i = \{j \in N \backslash i : \sigma_j < \hat{\sigma}\}$ and $N_c^i = \{j \in N \backslash i : \sigma_j \geq \hat{\sigma}\}$. Under an informative voting profile the fact that the conditional distributions satisfy MDP or MLRP, and thus have full support, implies that every profile of votes occurs with positive probability. This implies that the vote cast by each agent affects the outcome (i.e. the agent is pivotal) with positive probability, and thus in a BNE with informative voting agents will vote as if they are pivotal. For a given r, this implies that voters will vote as if $|N_c^i| = rn - 1$ and $|N_a^i| = n(1 - r)$. Given this information and an agent's private signal, the posterior probability of guilty is

$$b_i(G|\sigma_i, \hat{\sigma}, \pi, n, r)$$
$$= \frac{\pi f(\sigma_i|G) F(\hat{\sigma}|G)^{n(1-r)} (1 - F(\hat{\sigma}|G))^{rn-1}}{\pi f(\sigma_i|G) F(\hat{\sigma}|G)^{n(1-r)} (1 - F(\hat{\sigma}|G))^{rn-1} + (1-\pi) f(\sigma_i|I) F(\hat{\sigma}|I)^{n(1-r)} (1 - F(\hat{\sigma}|I))^{rn-1}}. \quad (3)$$

The interpretation of (3) is that an individual with prior π, private signal σ_i, the knowledge that $rn - 1$ agents received signals higher than $\hat{\sigma}$ and $n(1 - r)$ agents received signals lower than $\hat{\sigma}$ would believe via Bayes' rule that the probability of guilty is $b_i(G|\sigma_i, \hat{\sigma}, \pi, n, r)$. This and Eq. (2) (by setting $\rho_i = b_i$) imply that agent i prefers $v_i = c$ iff $b_i(G|\sigma_i, \hat{\sigma}, \pi, n, r) \geq \frac{1}{2}$. Given this we can define an informative (and therefore symmetric) BNE in a rather simple manner.[6]

Definition 4. *Given a game (γ, n) with $\gamma \in \Gamma^W$, a voting profile v^* is an informative BNE iff*

$$v_i^*(\sigma_i) = \begin{cases} c & \text{iff } \sigma_i \geq \hat{\sigma} \\ a & \text{otherwise} \end{cases}$$

for all $i \in N$, where $\hat{\sigma} \in (0, 1)$ solves $b_i(G|\hat{\sigma}, \hat{\sigma}, \pi, n, r) = \frac{1}{2}$.

To see that this definition involves a BNE, note that if every player other than i is using the specified strategy $v_i^*(\sigma_i)$ then an agent's action is only decisive if $|N_c^i| = rn - 1$ and $|N_a^i| = n(1 - r)$. But, given this information the agent believes $s = G$ with probability $b_i(G|\sigma_i, \hat{\sigma}, \pi, n, r)$. The fact that $b_i(G|\hat{\sigma}, \hat{\sigma}, \pi, n, r) = \frac{1}{2}$, and the assumptions we have imposed on permissible environments implies that $\sigma_i > (<) \hat{\sigma}$ implies $b_i(G|\sigma_i, \hat{\sigma}, \pi, n, r) > (<) \frac{1}{2}$. This, and (2) imply that i using $v_i^*(\sigma_i)$ is a best response. So an informative BNE consists of a cutpoint and a profile of strategies that involve voting to convict iff conditional on one's private signal and the knowledge that everyone else is voting according to the cutpoint strategy, a pivotal voter prefers conviction to acquittal iff she receives a private signal higher than the cutpoint.

[6] Note that we have defined informative to require that voting strategies are symmetric. Also appealing to weak dominance offers nothing in the current setting, because under any informative profile, there is positive probability that agent i is pivotal, and thus she will vote as if pivotal in a BNE.

3 Results

In this section we present 4 main results. Proposition 1 characterizes the set of BNE in responsive strategies for all permissible voting games. All such games have a unique symmetric BNE in responsive strategies and in this equilibrium voting is informative. We then present a CJT2 for non-unanimity rules. We then establish a CJT2 for the special case of unanimity rule. Moreover, we show that if the conditional densities are continuous and satisfy the MLRP then unboundedness of the likelihood ratio at 0 is necessary for a CJT2 under unanimity rule.

3.1 Equilibrium

The notion of BNE presented above is such that establishing existence hinges on establishing that a cutpoint $\hat{\sigma}$ for which $b_i(G|\hat{\sigma}, \hat{\sigma}, \pi, n, r) = \frac{1}{2}$, exists. The existence of such a cutpoint follows from the continuity of $b_i(G|\sigma_i, \hat{\sigma}, \pi, n, r)$ in the arguments $(\sigma_i, \hat{\sigma})$ and the intermediate value theorem. Moreover, the monotonicity of permissible conditional densities implies that for each permissible environment and rule r the cutpoint is unique.

It is instructive to think of the cutpoint as a function of the rule, r, the size, n, and the environment, θ, of the form $\hat{\sigma} : \{(0.5, 1) \cap \mathbb{Q}\} \times \mathbb{Z}_+ \times \Theta^W \to (0, 1)$, which we denote as $\hat{\sigma}(r, n, \theta)$. The fact that $\hat{\sigma}(r, n, \theta)$ is single-valued is easily established.

Proposition 1. *For all voting games (γ, n) with $\gamma \in \Gamma^W$, there is a unique symmetric BNE in which $v_i(\sigma_i)$ is responsive (i.e. not constant). This equilibrium is an informative BNE and the cut point is implicitly defined by the equation $b_i(\hat{\sigma}, \hat{\sigma}, \pi, N_a, N_c) = \frac{1}{2}$.*

The proof is available in either D&M or Meirowitz (1999).

3.2 Non unanimity – Large population informational efficiency

In the binary signal case establishing that voting is informative is often sufficient for establishing the informational efficiency (in the sense of a CJT1 or CJT2) of the aggregation rule.[7] In the current model the notion of informative voting is weaker than in the binary signal case and this equivalence is not obvious.

To establish a CJT2 we must define a few additional terms. Recall that ω is the underlying random variable so we may express the law of individual voting in an informative equilibrium as $p\{\omega : v_i(\omega) = c\} = p\{\omega : \sigma_i(\omega) \geq \hat{\sigma}(r, n, \theta)\}$. Since it is clear which terms are stochastic we suppress the ω and denote the event $\{\omega : \textit{something occurs}\}$ as $\{\textit{something occurs}\}$ to shorten the notation.[8]

[7] See for instance the first model considered in A-S&B.

[8] We have not included an additional index on the individual signals to denote which σ^n it is a component of. This is not problematic as the distribution of σ_i is the same regardless off which σ^n it is a component of.

The correct decision is made iff the event $\{x(v;r,n) = c \text{ iff } s = G\}$ occurs. This occurs iff $\{|i : v_i = c| > q \text{ iff } s = G\}$. This occurs with probability

$$p^*(\hat{\sigma}(r,n,\theta)) = \pi p\left(\left\{\frac{|i:\sigma_i \leq \hat{\sigma}(r,n,\theta)|}{n} \leq 1-r\right\}|G\right) + (1-\pi)p\left(\left\{\frac{|i:\sigma_i \leq \hat{\sigma}(r,n,\theta)|}{n} \geq 1-r\right\}|I\right). \quad (4)$$

The fact that individual signals are independent conditional on the state implies that (4) is equivalent to

$$p^*(\hat{\sigma}(r,n,\theta)) = \pi \sum_{i=q}^{n} \binom{n}{i} F(\hat{\sigma}(r,n,\theta)|G)^{n-i}(1-F(\hat{\sigma}(r,n,\theta)|G))^{i} + (1-\pi)\left(1-\sum_{i=q}^{n}\binom{n}{i}F(\hat{\sigma}(r,n,\theta)|I)^{i}(1-F(\hat{\sigma}(r,n,\theta)|I))^{n-i}\right). \quad (5)$$

In (5) and what follows the notation $F(\hat{\sigma}(r,n,\theta)|s)$ refers to the state conditional distribution $F(\cdot|s)$ evaluated at $\hat{\sigma}(r,n,\theta)$ which is the probability that an individual's signal is less then the specified cutpoint given s. A CJT2 is a statement of the form: for a given r and $\theta \in \Theta^W$, $p(\{x(v;r,n) = c \text{ iff } s = G, \textit{eventually}\}) = 1$.[9] Equivalently, the result may be stated – the limiting decision is almost surely correct.[10] We first turn to the non unanimity rule CJT2.

Proposition 2. *For all* $\theta \in \Theta^W$ *and* $r \in (0.5,1) \cap \mathbb{Q}$ *a CJT2 holds; that is,* $p(\{x(v;r,n) = c \textit{ iff } s = G, \textit{ eventually}\}) = 1$.

The proof of this result is a consequence of Theorem 3 in D&M and is also found in Proposition 2 of Meirowitz 1999. The intuition of this latter proof is that if $\hat{\sigma}(r,n,\theta)$ is well behaved the problem of proving a CJT2 reduces to verifying that $1-r \in (F(\hat{\sigma}(r,n,\theta)|G), F(\hat{\sigma}(r,n,\theta)|I)]$ eventually. An important observation is the fact that $\hat{\sigma}(r,n,\theta)$ is not homogenous of degree 0 in n. This presents the possibility that as n goes to infinity the graph of the critical solution $(r,\hat{\sigma}(r,n,\theta))$ may run outside of the lens defined by the set

$$\{(z,y) \in [0,1]^2 \mid y > F(z|G)\} \cap \{(z,y) \in [0,1]^2 \; s.t. \; y \leq F(z|I)\}.$$

However, for all weakly permissible environments it may be shown algebraically that the critical relationship $1-r \in (F(\hat{\sigma}(r,n,\theta)|G), F(\hat{\sigma}(r,n,\theta)|I)]$ holds for all n.

[9] The term eventually refers to the fact that eventually n is large enough so that the relevant statement is true. The phrases for all but a finite number of n's, and eventually are equivalent.

[10] A related result is a weak CJT2 stating that $p^*(\hat{\sigma}(r,n,\theta)) \to 1$ as $n \to \infty$. This weaker result follows from the almost sure convergence proved in Proposition 2, and the weaker convergence is used below in the proof of Proposition 3.

3.3 Unanimity – Large population informational efficiency

A question that has received much attention is the relative efficiency of unanimity rule ($r = 1$) (F&P and D&M). This interest is not surprising given that American juries operate under a form of unanimity rule. The current model uses the same interpretation of unanimity rule that F&P and D&M use. Specifically, the rule considered here requires a unanimous vote to convict. If at least one vote to acquit is cast the outcome is acquit. This is in contrast to the commonly used rule that requires a unanimous decision to convict or acquit. Coughlan (1997) considers a model with this rule. One justification for the current treatment is that it is equivalent to the rule requiring unanimity to convict or acquit, if the outcome of a hung jury is equivalent to that of an acquittal.

We find that for unanimity rule a CJT2 holds for weakly permissible (and therefore permissible) environments. Under the MLRP the boundary assumption of unbounded likelihood ratios is necessary to establish a CJT2 for unanimity rule. First a few preliminaries. Clearly proposition 1 applies to the case of unanimity rule. Thus, we may implicitly characterize the path of $\hat{\sigma}(r, n, \theta)$. In equilibrium $\hat{\sigma}(r, n, \theta)$ solves $b_i(G|\hat{\sigma}, \hat{\sigma}, \pi, n, r) = \frac{1}{2}$. We suppress the r and θ arguments in $\hat{\sigma}(\cdot, \cdot, \cdot)$ for the remaining analysis. With the appropriate substitution of $r = 1$ in the exponents of (3), the equation $b_i(G|\hat{\sigma}, \hat{\sigma}, \pi, n, 1) = \frac{1}{2}$ yields the following implicit definition of $\hat{\sigma}(n) := \hat{\sigma}(1, n, \theta)$, which holds for all n:

$$\left(\frac{(1-\pi)f(\hat{\sigma}(n)|I)}{\pi f(\hat{\sigma}(n)|G)}\right)\left(\frac{(1-F(\hat{\sigma}(n)|I))}{(1-F(\hat{\sigma}(n)|G))}\right)^{n-1} = 1. \tag{6}$$

A useful result is that $\hat{\sigma}^{\infty} := \lim_{n\to\infty} \hat{\sigma}(n) = 0$.

Lemma 1. *If conditional signal distributions satisfy MLRP, the BNE cutpoint, $\hat{\sigma}(n)$, defined implicitly by (6) converges to 0, as $n \to \infty$.*

Proof. Assume conditional signal distributions satisfy MLRP. By inspection of (6), the continuity of $f(\cdot|s)$, and the fact that $\hat{\sigma}(n) \in [0, 1]$ it is clear that $\hat{\sigma}(n)$ has a limit. By MLRP we know that $F(\hat{\sigma}(n)|I)) \geq F(\hat{\sigma}(n)|G))$ with the inequality strict unless $\hat{\sigma}(n) \in \{0, 1\}$. This implies that $\lim_{n\to\infty}\left(\frac{(1-F(\hat{\sigma}(n)|I))}{(1-F(\hat{\sigma}(n)|G))}\right)$ < 1 or $\hat{\sigma}^{\infty} \in \{0, 1\}$. If the former is true then $\left(\frac{(1-F(\hat{\sigma}(n)|I))}{(1-F(\hat{\sigma}(n)|G))}\right)^{n-1} \to 0$, so that (6) implies $\left(\frac{(1-\pi)f(\hat{\sigma}(n)|I)}{\pi f(\hat{\sigma}(n)|G)}\right) \to \infty$. But this implies that $\hat{\sigma}^{\infty} \in \{0, 1\}$. Since $\hat{\sigma}^{\infty} = 1$ implies that $\left(\frac{(1-\pi)f(\hat{\sigma}(n)|I)}{\pi f(\hat{\sigma}(n)|G)}\right) \to 0$ which by (6) implies that $\left(\frac{(1-F(\hat{\sigma}(n)|I))}{(1-F(\hat{\sigma}(n)|G))}\right)^{n-1} \to \infty$, which contradicts that fact that $\lim_{n\to\infty}\left(\frac{(1-F(\hat{\sigma}(n)|I))}{(1-F(\hat{\sigma}(n)|G))}\right) \leq 1$, it must be the case that $\hat{\sigma}^{\infty} = 0$. ■

We now present the CJT2 for unanimity. In the proof we first establish a weak CJT2 involving convergence in probability to the correct decision and then apply Kolmogorov's 0-1 law to extend this to a strong CJT2.

Proposition 3. *For all $\theta \in \Theta^W$ (and thus for all $\theta \in \Theta^P$) when $r = 1$ a CJT2 holds; that is, $p(\{x(v; r, n) = c \text{ iff } s = G, \text{ eventually}\}) = 1$.*

Proof. We proceed in steps, first establishing a weak CJT2, and then extending convergence in probability to almost sure convergence.

Step 1. The boundary assumption implies that $\lim_{\sigma_i \downarrow 0} \frac{(1-\pi) f(\sigma_i) | I)}{\pi f(\sigma_i | G)} = \infty$. This, Lemma 1 and (6) imply that $\lim_{n \to \infty} \left(\frac{(1 - F(\hat{\sigma}(n)|I))}{(1 - F(\hat{\sigma}(n)|G))} \right)^{n-1} = 0$.

Step 2. Construct the indicator,

$$1_a(n) \equiv \begin{cases} 1 & \text{if } \sigma_i < \hat{\sigma}(n) \ \exists i \in N \\ 0 & \text{otherwise} \end{cases}.$$

For $r = 1$, $x(v; r, n) = c$ iff $1_a(n) = 0$. To establish a weak CJT2 it is sufficient to show that $p(1_a(n) = 0 \,|\, I) \to 0$ and $p(1_a(n) = 0 \,|\, G) \to 1$. Note that $p(1_a(n) = 0 \,|\, s) = [1 - F(\hat{\sigma}(n)|s)]^n$. For all n we have

$$0 \leq [1 - F(\hat{\sigma}(n)|I)]^n \leq \left(\frac{(1 - F(\hat{\sigma}(n)|I))}{(1 - F(\hat{\sigma}(n)|G))} \right)^n. \tag{7}$$

– For $s = I$: (7) and the conclusion of Step 1 imply that $[1 - F(\hat{\sigma}(n)|I)]^n \to 0$.
– For $s = G$, (7), the fact that $[1 - F(\hat{\sigma}(n)|I)]^n \to 0$ and the conclusion of Step 1 imply that $\lim_{n \to \infty} (1 - F(\hat{\sigma}(n)|G))^n = 1$, establishing the weak CJT2.

Step 3. From above we have $p(1_a(n) = 0 \,|\, I) \to 0$ and $p(1_a(n) = 0 \,|\, G) \to 1$. Fix s. By assumption the triangular array of signals $\{\sigma^1, \ldots, \sigma^n, \ldots\}$ are independent. This, the fact that the events $\{1_a(n) = 0, \text{ eventually}\}$ and $\{1_a(n) = 1, \text{ eventually}\}$ are in the tail sigma-algebra, and Kolmogorov's 0-1 law imply that these events occur with degenerate probability (Durrett 1996). Thus, $p(1_a(n) = 0 \,|\, I) \to 0$ implies that $p\{1_a(n) = 0, \text{ eventually}|I\} = 0$ and $p(1_a(n) = 0 \,|\, G) \to 1$ implies $p\{1_a(n) = 0, \text{ eventually}|G\} = 1$. Thus, the result is established. ■

The intuition behind Proposition 3 is that the equilibrium cutpoint, $\hat{\sigma}(n)$, converges to 0 at a sufficient rate to insure that conditional on $s = G$ the probability of at least one agent getting a signal lower than the cutpoint vanishes and conditional on $s = I$ the probability of at least one agent getting a signal lower than the cutpoint approaches unity. It is clear in the proof that the boundary assumptions are used in Step 1. An interesting question is whether the boundary assumptions (the existence of perfectly informative signals) are necessary for a CJT2 under unanimity rule. In the next proposition we provide a partial converse to Proposition 3.

Proposition 4. (i) *If conditional signal distributions are continuous and satisfy the MDP, then a necessary condition for a CJT2 under unanimity rule* $(r = 1)$ *is that* $f(0|G) = 0$.

(ii) *If conditional signal distributions are continuous, satisfy the MLRP, and either (I)* $f(0|I) \neq 0$ *or (II)* $f(0|G) \neq 0$*, then a necessary condition for a CJT2 under unanimity rule* $(r = 1)$ *is that* $f(0|G) = 0$ *and* $f(0|I) > 0$.[11]

Proof. (i) By way of contradiction assume: (i) $f(0|G) > 0$, (ii) that the monotonicity and continuity properties are satisfied, and (iii) that we have a CJT2 for $r = 1$. Since MDP implies MLRP, Lemma 1 implies $\hat{\sigma}^{\infty} = 0$. We use this fact to attain a contradiction.

Step 1. We cannot have $f(0|I) = 0$ because $f(\cdot|I)$ is non negative and strictly decreasing.

Thus, $f(0|G) > 0$ and $f(0|I) > 0$.

Step 2. The result of step 1 and $\hat{\sigma}^{\infty} = 0$, implies that $\lim_{n\to\infty}\left(\frac{(1-\pi)f(\hat{\sigma}(n)|I)}{\pi f(\hat{\sigma}(n)|G)}\right) \in (0, \infty)$. This and (6) imply that

$$\lim_{n\to\infty}\left(\frac{(1 - F(\hat{\sigma}(n)|I))}{(1 - F(\hat{\sigma}(n)|G))}\right)^{n-1} \neq 0. \tag{8}$$

But the CJT2 for $r = 1$ implies that $p(1_a(n) = 0 \mid I) \to 0$ and $p(1_a(n) = 0 \mid G) \to 1$ This implies that (a) $\lim_{n\to\infty}(1 - F(\hat{\sigma}(n)|I))^{n-1} = 0$ and (b) $\lim_{n\to\infty}(1 - F(\hat{\sigma}(n)|G))^{n-1} = 1$. But (a), (b) and (8) are inconsistent-contradiction.

(ii) It is sufficient to again establish that the only relevant case is $f(0|G) > 0$ and $f(0|I) > 0$

- We cannot have $f(0|I) = 0$ and $f(0|G) > 0$, as this implies that $\frac{f(0|I)}{f(0|G)} = 0$ contradicting the fact that the likelihood ratio is strictly decreasing and non-negative on the support.
- We cannot have $f(0|I) = 0$ and $f(0|G) = 0$ by assumption.

Thus the only relevant case is $f(0|G) > 0$ and $f(0|I) > 0$ and the logic of part (i) applies. ■

The difference between parts (i) and (ii) in Proposition 4 is the only case of a different result coming from MDP than MLRP. The problem with MLRP is that it does not rule out $f(0|G) = 0$, $f(0|I) = 0$ and $\lim_{\sigma_i \downarrow 0}\frac{f(\sigma_i|I)}{f(\sigma_i|G)} = \infty$ from holding simultaneously. In this case we would also have a CJT2 for unanimity rule illustrating that the boundary conditions are not necessary for a CJT2 under unanimity rule with the MLRP. There are surely conditional densities

[11] The statement of (ii) may seem awkward, as we claim that if (II) $f(0|G) \neq 0$ then a necessary condition is $f(0|G) = 0$. This peculiarity stems from the fact that under continuity we cannot have a CJT2 for unanimity with $f(0|G) \neq 0$.

that satisfy the three conditions above by having $\frac{df(\sigma_i|I)}{d\sigma_i} > 0$ and $\frac{df(\sigma_i|G)}{d\sigma_i} = 0$. Review of Proposition 4 (ii) and Proposition 3 yields the following equivalent but more parsimonious statement for the case of monotone likelihood ratios.

Corollary 1. *If conditional signal distributions are continuous and satisfy the MLRP then a necessary and sufficient condition for a CJT2 under unanimity rule* $(r = 1)$ *is that* $\lim_{\sigma_i \downarrow 0} \frac{f(\sigma_i|I)}{f(\sigma_i|G)} = \infty$.

Proof. Sufficiency follows from the facts that step 1 of the proof of Proposition 3 only requires the weaker condition $\lim_{\sigma_i \downarrow 0} \frac{f(\sigma_i|I)}{f(\sigma_i|G)} = \infty$ and no other part of the proof uses the boundary properties.

Necessity. By way of contradiction assume the converse. But if $\lim_{\sigma_i \downarrow 0} \frac{f(\sigma_i|I)}{f(\sigma_i|G)} < \infty$ then (6) and Lemma 1 imply that (8) holds and the same contradiction as in the proof of Proposition 4 attains. ■

The combined logic of Propositions 3 and 4 yield the following conclusion. If conditional densities satisfy MDP and are continuous, then the possibility of perfectly informative signals, is necessary and sufficient for a CJT2 under unanimity rule. If conditional densities are continuous and satisfy MLRP then the fact that the likelihood ratio is unbounded at 0 is necessary and sufficient for a CJT2 under unanimity rule. While the result for MLRP may seem stronger in the sense that $\lim_{\sigma_i \downarrow 0} \frac{f(\sigma_i|I)}{f(\sigma_i|G)} = \infty$ is weaker than $f(0|G) = 0$ it turns out that with MDP and continuous conditional distributions the two are equivalent, as densities are always bounded and thus for continuous conditional densities that satisfy MDP, $\lim_{\sigma_i \downarrow 0} \frac{f(\sigma_i|I)}{f(\sigma_i|G)} = \infty$ iff $f(0|G) = 0$. In the case of MLRP the fact that $\lim_{\sigma_i \downarrow 0} \frac{f(\sigma_i|I)}{f(\sigma_i|G)} = \infty$ is weaker than $f(0|G) = 0$ and $f(0|I) > 0$ is exactly why the necessity result is not true in Proposition 4 (ii) without the additional conditions (I) and (II). Reconsidering the conditions assumed in D&M will complete the picture. They assume that the likelihood ratio is bounded as $\sigma_i \to 0$ and therefore exclude the cases when the necessary condition for a CJT2 under unanimity rule attains.

3.4 Finite population informational efficiency

To consider the validity of a CJT1 we define the probability that a single agent receiving a single signal will make the correct decision. It is well known that the agent's optimal strategy is:

$$v_i(\sigma_i) = C \quad \text{iff} \left(\frac{\pi f(\sigma_i|G)}{\pi f(\sigma_i|G) + (1 - \pi) f(\sigma_i|I)} \right) \geq \frac{1}{2}. \tag{9}$$

We denote the policy chosen by individual i using this rule as $x_i^1(\sigma_i)$. We denote the probability that this rule makes the correct decision as $p^1(\theta)$. Given Propositions 2 and 3 it is obvious that for all rules and for all $\theta \in \Theta^W$ there exists a finite n for which $p^*(\hat{\sigma}(r,n,\theta)) > p^1(\theta)$.

Proposition 5. *For all $\theta \in \Theta^W$ and $r \in (0.5, 1] \cap \mathbb{Q}$ a CJT1 holds eventually, that is $\exists n'$ s.t for all $n > n'$ we have $p^*(\hat{\sigma}(r,n,\theta)) > p^1(\theta)$.*

Proof. Fix $\theta \in \Theta^W$ and $r \in (0.5, 1] \cap \mathbb{Q}$. Since $\pi \in (0,1)$, $p(s = G) \in (0,1)$. This and the fact that the rule in (9) is non deterministic implies that $p^1(\theta) < 1$. But by Proposition 2, $\lim_{n\to\infty} p^*(\hat{\sigma}(r,n,\theta)) = 1$, so that $\forall \varepsilon > 0$, $\exists n'$ s.t for all $n > n'$ we have $p^*(\hat{\sigma}(r,n,\theta)) > 1 - \varepsilon$. But for ε sufficiently small we also have $1 - \varepsilon > p^1(\theta)$. Thus the result follows. ■

Admittedly, Proposition 5 is a weak CJT1 as it does not rule out the possibility of a dictator outperforming the collective when there is a small number of jurors. The proposition is as strong a statement as we can make without restricting r or θ further. This point is illustrated by the following simple example.

Example 1. A CJT1 not holding for small population.

Consider the following parameterization of $\theta \in \Theta^P$:

$$\pi = 0.7, \quad f(\sigma_i|s) = \begin{cases} 2\sigma_i & \text{if } s = G \\ 2 - 2\sigma_i & \text{if } s = I \end{cases}.$$

Integrating yields

$$F(\sigma_i|s) = \begin{cases} \sigma_i^2 & \text{if } s = G \\ 2\sigma_i - \sigma_i^2 & \text{if } s = I \end{cases}.$$

Let $r = \frac{2}{3}$ and $n = 3$. Applying Proposition 1 yields the equation

$$\frac{0.7(2\sigma_i(\sigma_i^2)(1-\sigma_i^2))}{0.7(2\sigma_i(\sigma_i^2)(1-\sigma_i^2)) + 0.3(2-2\sigma_i)(2\sigma_i - \sigma_i^2)(1 - 2\sigma_i + \sigma_i^2)} = \frac{1}{2}. \tag{10}$$

The solution is $\hat{\sigma}(r,n,\theta) = 0.41$. Calculating Eq. (4) yields

$$\begin{aligned} p^*(\hat{\sigma}(r,n,\theta)) = {} & 0.7[(1-(0.339)^2)^3 + (1-(0.339)^2)^2(0.339)^2] \\ & + 0.3[(2(0.339) - (0.339)^2)^3 \\ & + (1-(2(0.339)-(0.339)^2))(2(0.339)-(0.339)^2)^2], \end{aligned} \tag{11}$$

which yields $p^*(\hat{\sigma}(r,n,\theta)) = 0.61$.

However inspection of (9) illustrates that it is equivalent to the rule $v_i(\sigma_i) = G$ iff $\sigma_i \geq 0.3$. Given this,

$$p^1(\theta) = 0.7(1 - 0.3^2) + 0.3(2(0.3) - 0.3^2) = 0.79. \tag{12}$$

A consequence of this is that a CJT1 does not hold for (θ, r, n).

The key to understanding why the above example exists is to acknowledge that not all rules are equally efficient (in the sense of maximizing $p^*(\hat{\sigma}(r, n, \theta))$ for a fixed population and environment). As a consequence it is possible that some rules will outperform other rules, even with a few less decisionmakers. In the example unanimity rule in a one person society dominates two-thirds rule in a population of three.

4 Conclusion

We consider a more realistic model of common interests decision making with a binary state space. When the signal space is the unit interval, under reasonable assumptions on the distributions generating private signals, the unique symmetric Bayesian Nash equilibrium in responsive strategies is for agents to vote informatively. This result differs significantly from A-S&B's result on the near impossibility of informative equilibrium because the signal space is much smaller in A-S&B. We establish CJT2's for all q-rules (including unanimity). This result is consistent with D&M and F&P save the result regarding unanimity. We are alone in finding that unanimity rule results in efficient pure strategy equilibria to the game-form (modulo signal spaces) originally proposed by A-S&B. The asymptotic efficiency of unanimity rule is shown to hinge on the existence of perfectly informative signals in the case of monotone conditional densities and unbounded likelihood ratios in the case of monotone likelihood ratios.

The observation of A-S&B that rationality may result in the inefficient use of information for all but a few rules may hold here, but in a much diminished sense. When the boundary conditions do not hold asymptotic criterion like the CJT2 are able to discern between unanimity rule and other q-rules, finding that the latter are better than the former. However, when the boundary conditions hold asymptotic analysis does not distinguish between unanimity and non unanimity rules. The reader will note, however, that the efficiency of unanimity rule seems slightly perverse as in the limit most voters are becoming less responsive to their own signals (but not completely unresponsive). The probability of someone being sufficiently well informed increases as population grows. This phenomena is different from that of Feddersen and Pesendorfer (1997) where the percentage of informative voters vanishes. Here, all voters remain informative in the limit, but the probability that any one voter receives a sufficiently low signal so as to induce a vote to acquit vanishes. The rate of this convergence is sufficiently fast (slow) so that with probability one when the state is guilty (innocent) in the limit no-one (at least one person) votes to acquit.

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