Frequency Domain Analysis of In-Line Forces on Circular Cylinders in Random Oscillatory Flow

Experiments were conducted to study random hydrodynamic loading of smooth, rigidly mounted circular cylinders exposed to random oscillatory flow conditions. The experiments were conducted in a water tunnel for statistical Keulegan-Carpenter numbers ranging from $K_c = 6.8$ to 11.6 and statistical Reynolds’s numbers from $Re = 8211$ to 20169 under broad and narrow-band flow conditions. A two-input/single-output frequency domain model for the in-line force is used to identify frequency-dependent inertia and drag coefficients. This model was also used to show the relative strength of the inertia and drag components. Estimates of the in-line force power spectral density under random flow conditions are determined using the two-input/single-output model and compared to the method for estimating wave forces on a cylindrical structure by Borgman (1967, 1972).

1 Introduction

Experimental studies continue to provide knowledge for designers and researchers interested in hydrodynamic loading of slender structures. This is primarily due to the use of empirical models for estimating the in-line and transverse forces induced by oscillatory flows. In the present study, the in-line force induced on a circular cylinder is studied in the frequency domain using laboratory data obtained from experiments in a water tunnel in which the flow is oscillating randomly with a zero mean. In the tradition of the classical planar oscillatory flow studies by Keulegan and Carpenter (1958) and Sarpkaya (1977), use of planar oscillatory flow permits isolation of the fundamental mechanism between the in-line force and the input fluid flow and provides an excellent environment in which to test new force modeling techniques.

Significant efforts have been made in the past to generate frequency domain models for the in-line force, primarily based on the Morison equation. Estimation of the horizontal force is a random sea environment using frequency domain techniques was first presented by Borgman (1965, 1967, 1972). Borgman used models on the Morison equation with constant $C_d$ and $C_m$ and assumed zero-mean Gaussian wave-induced flow kinematics. The primary aim of Borgman’s work was to estimate the power spectral density of the force on a cylindrical structure. The methods of Borgman have been used by many investigators to identify frequency-dependent $C_d$ and $C_m$ coefficients, including analysis of Ocean Test Structure (OTS) data by Bostrom and Ovrevik (1986) and Boström (1987). More recent developments for frequency domain models have been made by Bendat and Piersol (1986). The model presented by Bendat and Piersol uses time-dependent coefficients for decomposed linear and nonlinear components of the in-line force. The model also assumes that the input flow velocity is Gaussian. Vugts and Bouquet (1985) report on use of the Bendat and Piersol model to analyze random forces from wave basin data. Detailed development of methods for identification of frequency domain coefficients for the Bendat and Piersol model from random data and estimation of the uncorrelated linear and nonlinear components of the decomposed in-line force are presented in Bendat and Piersol (1986). More general techniques for nonlinear system identification are presented by Bendat (1990).

Vugts and Bouquet (1985) have concluded that the nonlinear modeling technique of Bendat and Piersol is dependent on the flow velocity conditions. Consequently, the use of the nonlinear modeling techniques in actual offshore design analysis remains contingent upon the ability of experimentalists to acquire a significant data base for characterizing the dependence of $C_d(f)$ and $C_m(f)$ on statistical flow parameters. Such an endeavor can be considerably expensive in prototype ocean studies. In addition, ocean-based studies do not offer the stationary random conditions necessary for statistically accurate measurements nor allow assessment of the effect of flow parameters on the identified quantities. It follows that additional experimentation should be conducted in water tunnels or laboratory wave basins. This study reports on random planar oscillatory flow experiments and analysis of data collected using a two-input/single-output model. The results are interpreted with respect to governing statistical analogs of the Keulegan-Carpenter number, $K_c$, Reynolds’s number, $Re$, and flow velocity spectral width, $q$. 

Contributed by the OMAE Division for publication in the Journal of Offshore Mechanics and Arctic Engineering. Manuscript received by the OMAE Division, August 31, 1991; revised manuscript received August 6, 1992. Technical Editor: S. K. Chakrabarti.
2 Random Force Analysis Methods

2.1 Borgman Method for In-Line Force Estimation. The analysis of random hydrodynamic forces on an object subjected to a Gaussian, zero-mean fluid flow was first developed by Borgman (1965a). Borgman used the Morison equation as a model of the wave force on a cylindrical pile at a particular point in space and time, using constant drag and inertia coefficients, $C_d$ and $C_m$, and with the appropriate values of velocity and acceleration obtained from linear wave theory. The Morison equation estimates the in-line force by superposing an inertia ($f_i(t)$) and drag ($f_d(t)$) component and is commonly written as

$$f(t) = f_i(t) + f_d(t) = pC_d A_d \cdot u(t) + 0.5pC_m d u(t) \cdot u(t)$$

(1)

where $f(t)$ is the per unit length in-line force, $p$ is the fluid density, $A_d$ is the cross-sectional area of the cylinder, $u(t)$ the flow velocity and $d(t)$ the flow acceleration. Borgman obtained a least-square error first approximation to the force power-spectral density function, $G_{ff}(f)$, by linearizing the drag term yielding

$$G_{ff}(f) = \frac{8}{\pi} K_d^2 \sigma_d^2 G_{uu}(f) + \sigma_i^2 G_{ww}(f)$$

(2)

where $G_{uu}(f)$ and $G_{ww}(f)$ are the one-sided power spectral density functions of the flow velocity and acceleration, respectively, $\sigma_d$ is the root-mean-square value of the flow velocity, $K_d = 0.5pC_d D$ and $K_i = 0pC_m A_d$. Borgman (1965b) related the power-spectral density function of the in-line force to the ocean wave height spectrum using linear wave theory to replace $G_{uu}(f)$ and $G_{ww}(f)$. Also, Eq. (2) requires constant drag and inertia coefficients appropriately selected for a given sea state. Values of $C_d$ and $C_m$ can be obtained from experimental data using a method of moments technique described by Borgman (1972). In addition, note that the linearization of the drag term in Eq. (2) assumes that the flow kinematics are Gaussian. Finally, Borgman and others have demonstrated that higher order terms can provide improved estimation of the power spectral density. The reader is referred to Sarpkaya and Isaacson (1981) for a more thorough discussion or random force analysis methods, in general.

2.2 A Two-Input/Single-Output In-Line Force Analysis. In this section, a general two-input/single-output model is described for modeling hydrodynamic in-line forces induced on cylinders and for identifying $C_m(f)$ and $C_d(f)$ from data collected in planar random oscillatory flow experiments. This model makes no assumption on the Gaussianity of the flow velocity. The in-line forces induced on a circular cylinder by a random oscillatory flow can be modeled as a single-output system. It is assumed that the in-line and transverse forces induced on the cylinder are independent of each other, although both are inherently dependent on the time-dependent reversing flow field established about the cylinder. The transverse loading will not be discussed in this paper. The in-line force, $f(t)$, will be decomposed into inertia and drag components as traditionally done using the Morison equation (or "formulae"). Following Bendat and Pierson (1986a), the inertia force will be expressed as a function of the velocity

$$f(t) = f_i(t) + f_d(t) + n(t) = h_i(t) \cdot u(t) + h_d(t) \cdot x_d(t) + n(t)$$

(3)

where $u(t)$ = flow velocity, $x_d(t)$ = $u(t)$ $u(t)$, and $h_i(t)$ and $h_d(t)$ are functions of time-dependent inertia and drag coefficients, respectively. Note that $h_i(t)$ includes a differentiation operator so that Eq. (3) is reconciled with the traditional Morison formulation. In Eq. (3), $*$ denotes time-domain convolution and the noise term, $n(t)$, accounts for unmodeled quantities. Figure 1 shows the block diagram of this model which has been Fourier transformed to yield

$$F(f) = F_i(f) + F_d(f) + N(f) = H_i(f) U(f) + H_d(f) X_d(f) + N(f)$$

(4)

In Eq. (4) and in Fig. 1, $U(f)$, $X_d(f)$ and $N(f)$ are the Fourier transforms of $u(t)$, $x_d(t)$ and $n(t)$, respectively. Note that in this model the force components, $F_i(f)$ and $F_d(f)$, are correlated. The power spectral density (PSD) computed from this formulation will result in a cross-term, which is difficult to interpret physically, although it can be said to incorporate the correlation between the inertia and drag components. To overcome this difficulty, the method used here (see Bendat, 1990) decomposes the force into uncorrelated linear and nonlinear components. After identifying linear transfer functions in a revised model, the original model transfer functions are derived to allow determination of $C_m(f)$ and $C_d(f)$.

The revised model is shown in Fig. 2 where $H_d(f)$ represents the optimum linear system between the in-line force, $f_i(t)$, and the input flow velocity, $u(t)$

$$H_d(f) = \frac{G_{yd}(f)}{G_{uu}(f)}$$

(5)

$G_{yd}(f)$ is the one-sided cross-power spectral density function between the in-line force and flow velocity and $G_{uu}(f)$ is the auto-power spectral density function of the flow velocity. Note that $F_i(f)$ + $F_d(f) = F_0(f) + F_d(f)$. The output of $H_d(f)$, $F_d(f)$, is the modeled linear portion of the measured in-line force.

In the Morison equation, the drag component is a function of the abscare of the flow velocity, $u(t)$ $u(t)$. The effectiveness of the overall Morison-type model depends on the ability of this assumed nonlinear function to model the drag portion of an in-line force induced in reversing flow. The revised force model shows that the abscare nonlinearity output has a linear portion which is incorporated into $F_d(f)$ and is subtracted from $X_d(f)$ to yield $W(f)$; that is

$$W(f) = X_d(f) - \frac{G_{yd}(f)}{G_{uu}(f)} X_d(f)$$

(6)

In this way, the model can be represented with the linear and nonlinear uncorrelated force outputs, $F_0(f)$ and $F_d(f)$, as shown in Fig. 2. This revised model permits direct measurement of the contribution of a modeled nonlinearity to the measured in-line force. A nonlinear coherence function can then be included to assess how well the frequency domain model estimates the in-line force.

It can be shown (e.g., see Bendat, 1990) that the $H_d(f)$ transfer function can be determined from

$$H_d(f) = \frac{G_{wd}(f)}{G_{uw}(f)}$$

(7)

where $G_{uw}(f)$ is given by
3 Results

3.1 Summary of Experiments and Procedure. The experiments were conducted using the laboratory setup shown in Fig. 3. The length of the test section is 0.762 m and the square cross section is 0.219 m on the side. Smooth circular cylinders of 18 and 25 mm in diameter were used giving blockage ratios \((H/D)\) of 12.2 and 8.8, respectively. The cylinders were mounted rigidly between two X-Y force transducers which measure the flow-induced in-line and transverse forces. A differential \((\Delta p)\) pressure transducer is used to provide a measure of the instantaneous flow acceleration which is integrated to obtain the flow velocity. The follower side shaft has a displacement transducer which provides an independent measurement of the flow displacement. Random oscillation of the fluid contained in the water tunnel is accomplished by forcing the driven side piston with a hydraulic actuator controlled by a random vibration controller (see Fig. 3).

In the present experiments the controller was programmed to generate a flow acceleration power spectral density in the water tunnel with a shape derived from the analytical form of the dimensionless Pierson-Moskowitz spectrum. This was done to derive a general shape typical of what might be encountered in random sea conditions. The frequency at the peak value in the control spectrum for flow acceleration was chosen as \(f_p = 3.4\) Hz, approximately equal to the damped natural frequency.
of the water tunnel. The magnitude of the acceleration power-spectral density was increased or lowered to achieve desired test conditions in the water tunnel. The control spectrum for acceleration generated using the shape of the P-M spectrum is shown in Fig. 4(a) along with the measured acceleration spectra for two of the experiments. Additional details concerning the experimental setup can be found in Longoria (1989) and/or Longoria et al., (1991).

Random flow test conditions are parameterized by defining statistical analogs of the Keulegan-Carpenter number and the Reynolds' number, or

$$K_C = \sqrt{\frac{2\sigma_v T_p}{D}} \quad \text{Re}_q = \sqrt{\frac{2qD}{v}}$$

where $q = \int_0^\infty \omega^3 g_m(\omega) d\omega$ for $n = 0, 1, 2, \ldots$, and $\omega = 2\pi f$.

Data collected from five experiments were examined in this study and each experiment is parameterized by the values for $K_C$, $\text{Re}_q$, and $q$ shown in Table 1. For Cases 1 to 4, 180 realizations were collected and for Case 5, 208 realizations. Each realization contained 1024 points digitized at a rate of 2.5 ms, resulting in a Nyquist frequency of 400 Hz. The frequency range for observing physical phenomena of interest was below about 20 Hz. For each set of data, values of $C_d$ and $C_m$ determined using the method of moments described by Borgman (1972) and for use in the Borgman method estimation of the in-line force are shown in Table 2.

Measured velocity power spectral densities (PSDs) for Cases 1, 2 and 5 are shown plotted in Fig. 4(b) to illustrate the distribution of flow velocity energy across the frequency domain and to quantify the difference in the bandwidth properties of Case 5 relative to the first four experiments. The flow velocity PSD had the same spectral width, $q$, of 0.27 to 0.28 for Cases 1 to 4, and for Case 5 the control PSD bandwidth was narrowed to yield a value of $q = 0.16$. Note that the peak frequency in the velocity auto-power spectrum for all five cases was $f_p = 3.125$ Hz.

The random flow velocity measured from experiments in the water tunnel appeared nearly Gaussian when qualitatively compared to the estimated probability density functions. However, a chi-square goodness-of-fit test (using binned data with equal intervals) and a Kolmogorov-Smirnov test (using binned data) showed that the acceleration and velocity data was non-Gaussian. Consequently, it was advantageous that the two-input/single-output model described in the foregoing was used in the analysis of data since no assumption is made on the Gaussianity of the input (see Bendat, 1990).

### Table 1 Summary of random oscillatory flow experiments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_C$</td>
<td>6.8</td>
<td>11.6</td>
<td>6.8</td>
<td>8.6</td>
<td>8.6</td>
</tr>
<tr>
<td>$\text{Re}_q$</td>
<td>8211</td>
<td>14395</td>
<td>15936</td>
<td>20169</td>
<td>10227</td>
</tr>
<tr>
<td>$\beta_p$</td>
<td>1204</td>
<td>1241</td>
<td>2347</td>
<td>2348</td>
<td>1192</td>
</tr>
<tr>
<td>$q$</td>
<td>0.27</td>
<td>0.28</td>
<td>0.27</td>
<td>0.27</td>
<td>0.16</td>
</tr>
</tbody>
</table>

### Table 2 Table of drag and inertia coefficients for random oscillatory flow determined by the method of moments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_C$</td>
<td>6.8</td>
<td>11.6</td>
<td>6.8</td>
<td>8.6</td>
<td>8.6</td>
</tr>
<tr>
<td>$\text{Re}_q$</td>
<td>8211</td>
<td>14395</td>
<td>15936</td>
<td>20169</td>
<td>10227</td>
</tr>
<tr>
<td>$q$</td>
<td>0.27</td>
<td>0.28</td>
<td>0.27</td>
<td>0.27</td>
<td>0.16</td>
</tr>
<tr>
<td>$C_m$</td>
<td>1.57</td>
<td>1.38</td>
<td>1.58</td>
<td>1.52</td>
<td>1.68</td>
</tr>
<tr>
<td>$C_d$</td>
<td>1.01</td>
<td>1.09</td>
<td>0.88</td>
<td>0.91</td>
<td>1.19</td>
</tr>
</tbody>
</table>

3.2 Drag and Inertia Coefficient Identification Results. Frequency-dependent drag and inertia coefficients for Cases 1 through 5 are shown plotted in Figs. 5(a) to (e), respectively. Each graph plots the magnitude and phase of $C_d(f)$ and $C_m(f)$ versus dimensionless frequency, and a legend appears in Fig. 5 applicable to all five graphs. The changes in $C_d(f)$ and $C_m(f)$ due to changes in $K_C$, $\text{Re}_q$ and $q$ demonstrate that these functions are adjusting to compensate for limitations in the assumed Morison equation model of the in-line force. The change in $C_d(f)$ and $C_m(f)$ is not unlike the dependence of constant $C_d$ and $C_m$ values on the oscillatory flow parameters as observed in sinusoidal flow experiments (e.g., Sarpkaya, 1977). As traditionally done with constant $C_d$ and $C_m$ values obtained from sinusoidal flow experiments, comparing measured frequency-dependent coefficients under different random flow conditions provides insight into the mechanisms which govern the induced in-line forces.

For the range of statistical Reynold's number, $\text{Re}_q$, observed, there is no observable difference between the $C_m(f)$ and $C_d(f)$ functions for increases in $\text{Re}_q$. This observation is based on results from Case 1 and Case 3 which had the same $K_C$, and spectral width, $q$, but $\text{Re}_q$ was approximately doubled from $\text{Re}_q = 8211$ to $\text{Re}_q = 15936$. More significant changes are associated with increases in $K_C$, as evidenced by comparing Cases 2 and 3. For Case 2, $K_C$ is greater for approximately the same $\text{Re}_q$, but the drag coefficient magnitude decreases slightly at low frequencies ($f/f_p < 2$) and increases at high frequencies ($f/f_p > 2$); appearing to flatten. In addition, the inertia coefficient magnitude for Case 2 decreases across all frequencies when compared to Case 3. Increases in the Keulegan-Carpenter number have pronounced changes in the separated flow phenomena (i.e., vortex shedding and pairing) around the cylinder and hence it should be expected that the drag force component of the in-line force increases. The inertia and drag components of the in-line force are inherently correlated, both in the Morison model and, more importantly, by virtue of their shared dependence on the flow separation and vortex shedding dynamics. Since the frequency domain model decomposes the inertia and drag forces, it should be expected that the observed increases in the drag coefficient magnitude have an associated decrease in the inertia coefficient magnitude.

The identification results for Case 5, which had a smaller flow velocity spectral width, $q$, than the previous four cases, appear difficult to interpret due to a band from about $f/f_p = 1.5$ to $f/f_p = 2.5$, where low-energy content in the velocity and in-line force spectra result in low coherence. For regions centered about $f/f_p = 1$ and $f/f_p = 3$ where spectral energy levels lead to high coherence levels, the results are reliable. For Cases 1 to 4, it can be seen that the in-line forces are, on average, inertia-dominated, although individual cycles of oscillation can result in large drag-dominated forces. Case 5 is the only experiment which resulted in a region of $f/f_p$ where the magnitude of $C_m(f)$ is smaller than $C_d(f)$. This region is at $f/f_p = 3$ to 4 and is certainly due to the fact that the drag component of the in-line force dominates at these frequencies. The relatively narrow-banded nature of the flow oscillations result in extended cycles where the induced forces have larger drag components (this is concluded from observed time domain signals, particularly during actual experimentaion).

3.3 Effect of Flow Parameters on PSD Estimation. The previous section examined the effect of statistical flow parameters on measured drag and inertia coefficients. In this section,
the interpretation is extended to describe how the total in-line force PSD estimated using the two-input/single-output model compares to the estimate using the linear Borgman model. In addition, the decomposed uncorrelated components are compared in order to interpret how the drag and inertia components are dependent on the flow parameters.

Figure 6 contains graphs of the results from decomposition of the in-line force into inertia and drag components using the two-input/single-output model for Cases 1 to 4. Figure 7 contains measured in-line force PSDs for Cases 1 and 2 compared to those estimated using the linear model of Borgman and the two-input/single-output model. Figure 7 also contains plots of the coherence functions for this model for each respective case. Note that estimation of the in-line force PSD using the \( C_d(f) \) and \( C_m(f) \) determined from the two-input/single-output model (and computed from Eq. (4)) results in a cross-term which represents the correlation between the modeled drag and inertia components. This component is complex and typically has a negative real part, so in Figs. 6 and 8 the absolute value of this cross-term is plotted to indicate its magnitude relative to the inertia and drag terms. Unfortunately this term is difficult to interpret physically. While Fig. 6 contains plots of the correlated drag and inertia components, Fig. 7 illustrates the uncorrelated linear and nonlinear components (computed for Cases 1 and 2 only. The revised model (Fig. 2) eliminates any cross-terms since the force components (linear and
nonlinear) are uncorrelated. With respect to interpreting the measured force PSDs decomposed into inertia and drag components, results from the revised model are difficult to interpret by themselves because the "linear" component contains contributions from both inertia and drag loading. Therefore, both forms are used in the following discussions.

The results for Cases 1 and 3 (which differ only by increased Re) show that the in-line force is dominated by the inertia component (see Figs. 6(a) and (c)). In particular, Fig. 7 shows that the linear component predicts the total measured in-line force across most of the frequency range of interest. The predicted drag at $f/f_p = 1$ is about 50 percent of the inertia force.
component in both cases, however, the cross-term being of equal magnitude cancels most of its contribution at these frequencies. Consequently, the inertia component plotted in Figs. 6(a) and (c) effectively models the total in-line force. Comparison of the total in-line force PSDs for Cases 1 and 3 show that there is no significant change in the frequency content (i.e., shape), which might be expected since there is no significant change in the measured $C_m(f)$ and $C_d(f)$ coefficients. The magnitudes of the inertia and drag components naturally increase overall with increased $Re$ from Cases 1 to 3.

In the frequency domain, nonlinear functional behavior is commonly associated with generation of output energy in a frequency range other than at the excitation frequencies. The square law ($u'^2$) used to model drag forces in uni-directional fluid flow is adapted as an absquare ($u' u'$) in the Morison equation to model the drag force in reversing flow. Because the absquare is often approximated by a cubic ($u'^3$), a subjective measure of the nonlinear behavior is to check for energy in the in-line force PSD at three times the excitation frequencies. Following this course, it can be seen that the drag components in Fig. 6(a)-(d) contain energy in the vicinity of $f/f_p = 3$.

While the overall drag forces increase, in the range of $Re$ observed the in-line forces were primarily inertia-dominated. The general shape of the in-line force PSD in Cases 1, 3 and 4 does not change. On the other hand, an increase in $KC$, from Case 3 conditions to Case 2 conditions, where $KC$ increases from 6.8 to 11.6 for a similar $Re$ value, results in a significant increase in the drag component, which affects the total in-line force PSD. Additionally, the nonlinear drag is well predicted by the two-input/single-output model. Note in Fig. 6(b) that the drag component is beginning to approach the inertia component in magnitude and the cross-term's effect is decreasing, especially in the vicinity of $f/f_p = 1$ and $f/f_p = 3$. Also, the nonlinear coherence for Case 2 (Fig. 7) in the region about $f/f_p = 3$ increases significantly compared to Case 1 and indicates that the nonlinear portion of the drag component in this region can account for over 55 percent of the energy in the in-line force PSD. Such a significant portion of the energy in the in-line force due to the nonlinear drag forces quantifies the statement that the $KC$ value more fully characterizes the nonlinear behavior of the in-line force.

Figure 8 summarizes results for Case 5. The velocity PSD had a smaller spectral parameter, $q$, than the previous four cases, although the controller was not able to eliminate secondary flow excitations as shown in Fig. 8(a). Firstly, note that the effect of decreasing $q$ is to significantly increase the drag component, although it is still highly correlated with the inertia component (see cross-term in 8(b)). The most interesting aspect of this case, however, is that the coherence functions in Fig. 8(d) show a significant amount of energy in the in-line force from $f/f_p = 2.5$ to 3.5 attributable to a linear component. Further, as shown in Fig. 8(b), decomposition of the in-line force PSD into inertia and drag components shows that the energy in this region is largely due to drag forces. We should expect that the secondary flow excitations in this frequency band would excite primarily linear inertia forces, which actually appear to be less than the drag. Drag forces in this region of frequency for peaked spectra are typically attributed to nonlinear drag forces. These observations lead to a conclusion that the energy in this region of the in-line force PSD must be due to high frequency linear drag forces. This result corresponds with the earlier observation that the $C_d(f)$ magnitude in this region was larger than the magnitude of $C_m(f)$ (see Fig. 5, Case 5, 3 < $f/f_p < 4$).

The linear Borgman model estimates the in-line force PSD well for inertia-dominated cases such as Case 1 (see Fig. 7), 3 and 4. However, for Case 2, where the drag term becomes
more significant, the Borgman model is not as accurate because of linearization (see Fig. 7). Linearization of the drag term can also be shown to result in underprediction of the peak in-line forces which are drag dominated. Based on the root-mean-square (rms) level of the in-line force computed from the estimated PSDs using the Borgman model compared to the two-input/single-output model, there is not a significant difference in the root-mean-square level of the in-line force, $\sigma_f$, which would be predicted by the two PSD models. For example, for Case 2 (KC = 11.6, Re = 14395, $\varphi$ = 0.28), the Borgman model yields $\sigma_f = 6.96$ N/m, and the two-input/single-output model yields $\sigma_f = 7.07$ N/m compared to a measured rms level of $\sigma_f = 7.14$ N/m.

4 Discussion

This study has examined the behavior of in-line forces induced on a circular cylinder in random oscillatory flow by using frequency domain modeling based on the Morison equation. The traditional Borgman technique was compared to a two-input/single-output nonlinear frequency model. As far as reported, the Borgman model appears sufficient to predict the rms level of the force (or total energy across the frequency range of interest). It can also be shown that the estimation of the magnitude of the force spectrum at $f/f_p = 1$ by the linear Borgman model is quite effective considering that constant $C_D$ and $C_m$ values are used. In the frequency domain, the Borgman model leads to overprediction of higher frequency components for Case 2 and underprediction for Cases 1, 3 and 4. Additionally, the Borgman model does not accurately predict the low-frequency region of the in-line force spectrum (see Fig. 7, Case 1), whereas the two-input/single-output model predicts these components by accounting for them in the drag force. For prediction of stationary random response of a structure in the frequency domain, the decision to use a more sophisticated in-line force PSD model is up to the designer. Such decisions must consider that estimation of rms levels is not sufficient for inherently non-Gaussian hydrodynamic loading, particularly because it leads to underprediction of extreme force probabilities.

In general, it appears that the absquare nonlinearity assumed in the Morison equation is quite effective in predicting the PSD of the in-line force. In the two-input/single-output model, the nonlinear coherence function quantifies the relative importance of the drag portion of the in-line force, which is modeled by the absquare nonlinearity. However, the physics not incorporated in the model are inherent in the measured $C_m(f)$ and $C_D(f)$ functions. The fact that these coefficients do not remain invariant with changes in the flow parameters is an indication of the lack of robustness of the Morison equation; that is, the physics must be incorporated into the coefficients as is typically accomplished by constant drag and inertia coefficients generated in oscillatory flow experiments (Sarpkaya, 1977; Longoria, et al., 1991).

5 Conclusions

If improved power spectral density (PSD) modeling of the in-line force is to be achieved through the use of nonlinear modeling techniques in the frequency domain, experimental studies which characterize the frequency domain coefficients will be required for the necessary range of the flow parameters KC, Re, and $\varphi$. This study has shown that by using a two-input/single-output nonlinear model of the in-line force which incorporates an absquare nonlinearity of the flow velocity (i.e., as in the Morison equation, $u(t)$ vs $u(t)$), the accuracy of the PSD of the in-line force over all frequencies is significantly improved over traditional methods using a linear Borgman model. Frequency-dependent drag, $C_D(f)$, and inertia, $C_m(f)$, coefficients were determined for five random oscillatory flow experiments, and the effects of the statistical flow parameters, KC, Re, and $\varphi$, were described. It was found that the statistical Keulegan-Carpenter number, KC, is most critical in gauging the changes in $C_D(f)$ and $C_m(f)$ and respective changes in the drag and inertia components of the in-line force PSD. One random flow experiment which had a narrower flow velocity bandwidth (Case 5, $\varphi = 0.16$) was examined to illustrate force prediction at higher frequencies due to multi-peaked flow excitation. For all experiments, the two-input/single-output model was effective in illustrating the contribution of both the inertia and drag components versus frequency. In particular, linear and nonlinear coherence functions were used to assess the quality of the assumed modeling technique.

Acknowledgments

This material represents results of research supported in part by the National Science Foundation, Grant No. MSM-8352272, Texas Advanced Technology Program, Grant No. ATP 4604, Office of Naval Research Grant No. N00014-88-K-0638 and the National Science Foundation sponsored Offshore Technology Research Center Grant No. CDR-8721512. Special thanks to Richard Swanson at Shell's Westhollow Research (Houston) for providing support in the form of fabrication of the X-Y force transducers.

References