Identification for Sucker-Rod Pumping System’s Damping Coefficients Based on Chain Code Method of Pattern Recognition

In this paper, a method for identifying the damping coefficients of a directional well sucker-rod pumping system is put forward by means of the chain code method of pattern recognition. The 24-directional chain code is provided to encode the dynamometer card curve. The parametric equation of the dynamometer card curve is transformed into Fourier series whose coefficients can be computed according to the curve’s chain codes. By means of these coefficients, shape characteristics of the curve are extracted. The Euclidean distance is introduced as the measurement of similar degree between the shape characteristics of measured dynamometer card and that of simulated dynamometer card. Changing the value of viscous damping coefficient and Coulomb friction coefficient in the simulation program, different simulated dynamometer cards are obtained. Substituting their shape characteristics to the Euclidean distance, respectively, a series of distances are acquired. When the distance is less than the given error, the corresponding values of the damping coefficients in the simulation program are regarded as real damping coefficients of the sucker-rod pumping system of directional well. In the end, an example is provided to show the correctness and effectiveness of the presented method. [DOI: 10.1115/1.2748464]

1 Introduction

Mainly due to its long history, sucker-rod pumping is a very popular means of artificial lift all over the world; roughly two-thirds of the producing oil wells are on this type of lift. According to statistic data, it is known that the mean efficiency of current sucker-rod pumping systems does not reach 30%. This means that most sucker-rod pumping systems run in low efficiency and seriously waste energy. Thus, how to improve the efficiency of the sucker-rod pumping system is an important problem needed to solve imminently. Analyzing the ingredients of energy dissipated in this system, it can be seen that most dissipation is caused by damping. In a sucker-rod pumping system, there are two forms of damping. One is the viscous damping between the oil and sucker rod, the other is the Coulomb damping due to the friction between sucker rod and oil pipe. For the vertical well, a system’s damping only involves the former form, but for the directional well whose center locus is a three-dimensional curve, both forms of damping affect the system at the same time. Thus, it is necessary to obtain these two kinds of damping coefficients for the simulation and efficiency optimization of a sucker-rod pumping system. Characterization of damping forces in a vibrating structure has long been an active area of research in structural dynamics. For the identification of viscous damping of a linear system, there are already some ready methods [1–3]. However, for the identification of viscous damping and Coulomb friction of a real mechanical system, it is still a challenging problem. The involved sucker-rod pumping is a complicated system; it is nearly impossible to obtain its damping coefficients by means of the conventional dynamic parameter identification methods, which need system’s actuation and response at the same time, because the actuation to the sucker-rod pumping system is difficult to measure.

Now, there are two kinds of methods to obtain the sucker-rod pumping system’s damping coefficient. One is the computation method, and the other is identification method. In the computation method, the damping coefficient is computed according to the equivalent theory and empirical formula. Without considering the Coulomb friction coefficient, Gibbs deduced an expression of equivalent viscous damping coefficient for the sucker-rod pumping system via the relationship between the energy dissipation caused by the viscous damping and real energy dissipation of sucker-rod system [1]. Everit and Jennings provided a computational formula for the damping coefficient of a sucker-rod pumping system whose rod string is constituted with different materials and obtained the damping coefficient by means of iterative computation of the water power and pump power [2].

In the above-mentioned methods, measurement data of the system are not necessary. Thus, they are easy to use, but the damping coefficient obtained using these methods is rough. In order to obtain precise damping coefficients, the identification research computing the damping coefficient from the measurement data of a sucker-rod pumping system was carried out. Yu et al. deduced the relationship between the measured dynamometer card—a closed curve of pumping system loads against the displacement of sucker rod—and viscous damping coefficient according to the Gibbs equation for a vertical well [3]. However, their method only involves the viscous damping coefficient; thus, it is only applicable to computing the damping coefficient of the vertical well. For the directional well, how to obtain the viscous damping coefficient and Coulomb damping coefficient is still an open problem.

The purpose of this paper is to give a novel approach to the damping coefficients identification of a sucker-rod pumping system of a direction well. The main contribution is that the image process technique is introduced for damping identification of sucker-rod pumping system [4]. The well’s dynamometer card is treated as a kind of image information, and the system’s damping coefficients are identified based on the chain code method of pattern recognition. Its main idea is to digitize the dynamometer card curve by means of the chain code method, extract the curve’s...
characteristics via the Fourier descriptor to condense the dynamometer card information from high-dimensional measurement space to low-dimensional characteristics space, and then identify the damping coefficients of a sucker-rod pumping system through the pattern recognition method.

Usually, image’s shape characteristics, such as perimeter, area, circular degree, slightness degree, and shape factor, are extracted through the methods of chain code [5–9] and Fourier descriptor [10,11]. The chain code method is among the most widely used techniques for boundary shape description. In [5], the boundary curve is approximated via a sequence of connected straight line segments of preselected direction and length. Every line segment is coded with a special coding number depending on its direction. In the current eight-directional chain code method, especially, there are eight possible directions of the line segment that connects boundary pixel \((x_i, y_i)\) with the next one \((x_{i+1}, y_{i+1})\), sweeping the boundary in the clockwise sense. These directions are respectively coded with coding number 0, 1, 2, 3, 4, 5, 6, and 7. For even-coded direction, the length of the corresponding straight line segment is the sampling step length expressed as \(d\), and for the odd-coded directions, it is \(\sqrt{2}d\) (from Pythagoras’ theorem). Obviously, for the eight-directional chain code, the angle between two neighboring directions is \(\pi/4\). There are two disadvantages in the eight-directional chain code: one is the angle between two adjacent line segment is larger, the other is grid sampling, i.e., the sampling step length is equal along the \(x\) and \(y\) directions. For the curve with simple and regular shape, the error caused by the eight-directional chain code is little; however, for the curve with complex shape, some important information of boundary curve will be omitted.

In this paper, a new chain code method, 24-directional chain code method, is put forward to identify damping coefficients of a sucker-rod pumping system of directional well. First, by means of the simulation program of the sucker-rod pumping system, a dynamometer card is obtained under a pair of initial values of viscous damping coefficient and Coulomb damping coefficient. At the same time, the real dynamometer card is obtained through experiment. Next, the 24-directional chain code is used to encode the curve of simulated and measured dynamometer cards. Because the parametric equation of the dynamometer card curve is a periodic function, it can be transformed into the Fourier series and the coefficients of the series can be computed via the curve’s chain code. Then, the shape characteristics of the curve are extracted by the Fourier coefficients. Finally, the Euclidean distance is taken as the measurement of similar degree between the shape characteristics of measured dynamometer card and that of simulated dynamometer card. With the viscous damping coefficient and Coulomb damping coefficient changing from their initial values in the program, a different simulated dynamometer card is obtained. Substituting the measured and simulated dynamometer cards’ shape characteristics into the expression of Euclidean distance, if the distance is less than the given error, the viscous damping coefficient and Coulomb damping coefficient in the simulation program are regarded as the actual damping coefficients of the sucker-rod pumping system of directional well. Otherwise, continue the simulation with a new pair of viscous and Coulomb damping coefficients and obtain a new Euclidean distance, until the distance is less than the given error. In the end, examples are provided to show the correctness of the presented method.

### 2 24-Directional Chain Code

Refer to Fig. 1, in the 24-directional chain code method, it is defined that the angle between two neighboring directions of the line segment that connects boundary pixel \((x_i, y_i)\) with the next one \((x_{i+1}, y_{i+1})\) is \(\pi/12\). Thus, there are 24 possible directions, and each direction is coded with a corresponding number shown in Fig. 1, where \(d\) is the sampling step length in the horizontal direction. The length of the corresponding straight line segment to the coding number is shown in Table 1. It should be mentioned here that line length \(d\), corresponding to coding number 6 and 18, is prescribed because, for the \(\theta = \pi/2\) direction, it is impossible to express this line length using the sampling step length \(d\).

Besides, for line segment \(AB\) (or \(CD\)) whose angle with \(x\)-axis \(a\) (or \(B\)) is close to \(\pi/2\) (or \(3\pi/2\), shown in Fig. 2, the 24-directional chain code defined above cannot be directly applied because cosine of the angle \(a\) (or \(B\)) is close to zero and little change of the angle will lead to sharp variation of the reciprocal of cosine. In order to encode this line segment, the following modification is provided. First, refer to Fig. 2, line \(AB\) (or \(CD\)) is resolved into two segments; one is \(B’\) (or \(D’\)), and the other is \(AB’\) (or \(CD’\)). Second, dividing the length of segment \(AB’\) (or \(CD’\)) by the length of the corresponding line of the coding number 6 (or 18) and rounding this operation’s result, an integer result for the segment \(AB’\) is obtained. Thus, the length of segment \(AB’\) (or \(CD’\)) can be approximately expressed with this integer and the corresponding line length of coding number 6 (or 18). Therefore, the resulting chain code of the segment \(AB’\) (or \(CD’\)) is a sequence of coding number 6 (or 18). Similarly, segment \(B’\) (or \(D’\)), whose length equates \(d\), is also expressed using the coding number 0 (or 12). Adding the coding number of segment \(B’\) (or \(D’\)) to the end of the chain code of the segment \(AB’\) (or \(CD’\)), we can obtain the chain code of the line \(AB\) (or \(CD\)). For example, when the length of line \(AB\) is 8\(d\), the length of segment \(AB’\) is 7.937\(d\) from the Pythagoras’ theorem, dividing the length of \(AB’\) by \(d\), which is the corresponding line length of chain code 6, and rounding the operation’s result, the integer number 8 is obtained. Therefore, the chain code of the segment \(AB’\) is expressed as

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**Table 1 Coding numbers and their corresponding line lengths**

<table>
<thead>
<tr>
<th>Coding number (c)</th>
<th>Corresponding line length (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 6, 12, 18</td>
<td>(d)</td>
</tr>
<tr>
<td>1, 11, 13, 23</td>
<td>1.0353(d)</td>
</tr>
<tr>
<td>2, 10, 14, 22</td>
<td>1.1547(d)</td>
</tr>
<tr>
<td>3, 9, 15, 21</td>
<td>1.4142(d)</td>
</tr>
<tr>
<td>4, 8, 16, 20</td>
<td>2(d)</td>
</tr>
<tr>
<td>5, 7, 17, 19</td>
<td>3.8657(d)</td>
</tr>
</tbody>
</table>
where \( \alpha \) describe the shape of a curve and can be computed by means of curve’s chain codes. In the following, the relationship between the Fourier coefficients and chain code is deduced. Let

\[
t_m = \frac{2\pi s_m}{S} \quad m = 0, 1, 2, \ldots, M
\]

where \( S \) is the perimeter of the closed boundary and \( s_m \) is the arc length variable corresponding to the angle variable \( t_m \).

According to Eq. (3), when \( n \neq 0 \)

\[
p_n = -\frac{1}{2n\pi i} \int_0^{2\pi} e^{-in\theta} dU(t) + \frac{1}{2n\pi} \int_0^{2\pi} e^{-in\theta} dU(t)
\]

\[
= \frac{1}{2n\pi} \int_0^{2\pi} e^{-in\theta} dU(t) \equiv \frac{1}{2n\pi} \sum_{m=1}^{M} e^{-in\theta} [U(t_m) - U(t_{m-1})]
\]

when \( n = 0 \)

\[
p_0 = \frac{1}{2\pi} \int_0^{2\pi} U(t) dt \equiv \frac{1}{2\pi} \sum_{m=1}^{M} U(t_m)(t_m - t_{m-1}) = \frac{1}{2\pi} U(t_M) t_M
\]

\[
+ \frac{1}{2\pi} \sum_{m=1}^{M-1} U(t_m) t_m - \frac{1}{2\pi} \sum_{m=2}^{M} U(t_m) t_{m-1} = U_0 - \frac{1}{2\pi} \sum_{m=2}^{M} [U(t_m) - U(t_{m-1})] t_{m-1}
\]

where \( U_0 \) is the coordinate vector of the initial point.

On the other hand, the approximate expression for the boundary perimeter can be expressed as follows:

\[
S = \sum_{k=1}^{M} a_k
\]

where \( M \) is the number of total curve’s chain code, \( a_k \), shown in Table 1, is the corresponding line length of the coding number \( c_k \).

Substituting Eq. (7) into Eq. (4), one obtains

\[
t_m = \frac{2\pi s_m}{S} \equiv \frac{2\pi}{M} \sum_{k=1}^{M} a_k
\]

(8)

Examining the term \( U(t_m) - U(t_{m-1}) \), it can be seen that this term represents the boundary line segment corresponding to coding number \( c_m \). Thus, the following expression can be obtained according to the definition of the 24-directional chain code

\[
U(t_m) - U(t_{m-1}) \equiv a_m e^{i\pi/12} \quad m = 1, 2, \ldots, M
\]

(9)

Substituting Eqs. (8) and (9) into Eqs. (5) and (6), respectively, we obtain

\[
p_n = \frac{1}{2n\pi i} \sum_{m=1}^{M} a_m e^{i\pi/12} e^{-in\theta_m} n = \pm 1, \pm 2, \ldots
\]

(10)

\[
p_0 \equiv U_0 = \sum_{m=2}^{M} a_m e^{i\pi/12} e^{-i\phi_m}
\]

(11)

From Eqs. (10) and (11), it can be seen that the Fourier coefficients are only related to the coding number \( c_m \) and its corresponding line length \( a_m \). Therefore, once the curve is encoded, its Fourier coefficients can be obtained using Eqs. (10) and (11).

4 Reconstruction of the Closed Curve

In order to test the validity of above methods, reconstruction of corresponding curve is completed. Substituting Fourier coeffi-
coefficients obtained from Eqs. (10) and (11) into Eq. (2), the curve’s parametric equation is obtained. By means of this parametric equation, the curve can be reconstructed. Figure 3 is a reconstruction example of real dynamometer card curve measured from the sucker-rod pumping system.

From Fig. 3, it can be seen that the reconstructed curve by 24-directional chain code matches the original curves well and the result by 24-directional chain code is much better than that of an eight-directional chain code, which validates the effectiveness of the 24-directional chain code method. It also means that the Fourier coefficients obtained through the 24-directional chain code are correct.

5 Shape Characteristics and Euclidean Distance

Usually, following five shape characteristics, such as perimeter, area, circular degree, slightness degree, and shape factor, are used to describe the curve’s shape. Their expressions by Fourier coefficients obtained in Sec. 3 are as follows, respectively:

- **Perimeter**
  \[ S = \sum_{k=1}^{M} a_k \]

- **Area**
  \[ A = \pi \sum_{n=1}^{\infty} n(|p_n|^2 - |p_{n-1}|^2) \]

- **Circular degree**
  \[ F_1 = \frac{|p_1|}{\sum_{n=1}^{\infty} (|p_n| + |p_{n-1}|)} \]

- **Slightness degree**
  \[ F_2 = 1 - \frac{|p_1| - |p_{-1}|}{|p_1| + |p_{-1}|} \]

- **Shape factor**
  \[ F_3 = \frac{S^2}{4\pi A} \]

The above five shape characteristics for a closed curve can be written in the characteristics vector form

\[ \mathbf{F} = [S,A,F_1,F_2,F_3]^T \]  

In order to measure the similar degree between two curves, the Euclidean distance is introduced, whose expression is as follows:

\[ D = \sqrt{(\mathbf{F}_a - \mathbf{F}_b)^T(\mathbf{F}_a - \mathbf{F}_b)} \]

where \( \mathbf{F}_a \) and \( \mathbf{F}_b \) are the characteristics vector of the curves \( a \) and \( b \), respectively. When the distance is less than a given error, it can be considered that the shapes of the two curves are close.

6 Identification of the Damping Coefficients of Sucker-Rod Pumping System

In this section, the above characteristics recognition method based on a chain code is applied for identifying the damping coefficients of sucker-rod pumping system. In a directional well sucker-rod pumping system, the sucker-rod, connected with the ground linkage mechanism’s hanging point with displacement \( s_0 \), velocity \( v_0 \), and acceleration \( a_0 \), can be regarded as a multistage rod string reciprocating in the tubing, as shown in Fig. 4. In order to study the dynamic characteristic of the rod string, an infinitesimal element having distance \( s \) from hanging point \( 0 \) is taken for analysis, forces acting on the element are shown in Fig. 5.
In Fig. 5, $P(s,t)$ and $P(s,t) + \Delta P$ represent the axial forces of $s$ and $s+ds$ section, respectively, $\Delta W$ is the element’s gravitation force under oil liquid, $F_g$ is the element’s inertia force, $F_r$ is the viscous force from the oil liquid, $F_f$ is the Coulomb friction force from the tubing, $N(s)$ (written as $N$ for short in the following), is the reaction force from tubing of unit length. According to the d’Alembert’s principle, the force equilibrium equation along the axis can be written as

$$F_s = [P(s,t) + \Delta P] + \Delta W \cos \alpha - [P(s,t) + F_f + F_r]$$ (14)

where $\alpha$ is the well’s inclined angle. After some operations, Eq. (14) can be expressed as

$$\rho_s A_s \frac{\partial^2 u}{\partial t^2} = E_s A_s \frac{\partial^2 u}{\partial s^2} - \nu_s \frac{\partial u}{\partial t} - \text{sgn} \left( \frac{\partial u}{\partial t} \right) fN + \rho_s' A_s g \cos \alpha$$ (15)

where $u$ is the elastic displacement of the rod’s arbitrary section, $E_s$ is the elasticity modulus of the rod material, $A_s$ is the rod section area, $\rho_s'$ is the relative density of the rod under oil liquid, $\rho_s = \rho_r - \rho_r - \rho_o$ are density of the rod material and oil liquid, respectively, $g$ represents gravitation acceleration, $\nu_s$ is the viscous damping coefficient of the unit length rod, $\text{sgn}$ ( ) is the sign function, and $f$ is Coulomb friction coefficient of the rod against tubing.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Shape characteristics of measured dynamometer card curve and final simulated dynamometer card curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well number</td>
<td>Measured value</td>
</tr>
<tr>
<td>Perimeter $S$</td>
<td>96.2</td>
</tr>
<tr>
<td>Area $A$</td>
<td>28.25</td>
</tr>
<tr>
<td>Circular degree $F_1$</td>
<td>0.2788</td>
</tr>
<tr>
<td>Slightness degree $F_2$</td>
<td>0.8833</td>
</tr>
<tr>
<td>Shape factor $F_3$</td>
<td>26.07</td>
</tr>
<tr>
<td>Maximum load $Q_{\text{max}}$</td>
<td>53.59</td>
</tr>
<tr>
<td>Minimum load $Q_{\text{min}}$</td>
<td>27.05</td>
</tr>
<tr>
<td>Damping coefficients</td>
<td>Viscous damping $\nu_s$</td>
</tr>
<tr>
<td>Coulomb damping $f$</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Combining the initial and boundary conditions, solving Eq. (15), we can obtain a closed curve of the rod’s hanging point load relative to the displacement of stroke, i.e., the simulated dynamometer card using a simulation program for a sucker-rod pumping system. For a given well, the density and elasticity modulus of the rod material, rod section area are invariables; thus, the main factors influencing the rod’s hanging point load are the viscous damping coefficient and Coulomb friction coefficient. Figure 6 shows the simulated dynamometer cards under different viscous and Coulomb coefficients.

It can be seen from Fig. 6 that the shape of simulated dynamometer cards changes with the variation of inputted viscous and Coulomb coefficients. Thus, the similar degree between the measured and simulated dynamometer cards can be evaluated by means of the cards’ characteristics vectors extracted using the chain code and Fourier descriptor methods, when the Euclidean distance is less than the given error. In the above identification, according to experience and a handbook on sucker-rod pumping system [14], 0.4 and 0.05 are adopted as the initial values of the two coefficients, and their step lengths of change are 0.05 and 0.01, respectively.

Table 2 shows the shape characteristics of measured dynamometer cards and that of the final simulated dynamometer cards, and the identified viscous and Coulomb damping coefficients of the sucker-rod pumping system of the four wells, respectively.

The measured dynamometer cards and corresponding final simulated dynamometer cards of the sucker-rod pumping system of well I and II are shown in Figs. 8 and 9, respectively.

7 Conclusions

The identification of viscous damping and Coulomb friction coefficients of the sucker-rod pumping system of a directional well is researched. The main contribution is that a new chain code method, 24-directional chain code method, is put forward to identify damping coefficients of the sucker-rod pumping system. Otherwise, continue the simulation with a new pair of viscous and Coulomb damping coefficients and obtain a new Euclidean distance, until the distance is less than the given error. In the above identification, according to experience and a handbook on sucker-rod pumping system [14], 0.4 and 0.05 are adopted as the initial values of the two coefficients, and their step lengths of change are 0.05 and 0.01, respectively.

The measured dynamometer cards and corresponding final simulated dynamometer cards of the sucker-rod pumping system of well I and II are shown in Figs. 8 and 9, respectively.

In this section, four measured dynamometer cards marked with numbers I-IV, are provided to identify the viscous and Coulomb damping coefficients of the corresponding sucker-rod pumping systems. On the other hand, different simulated dynamometer cards for these four wells can be obtained by inputting different viscous damping and Coulomb friction coefficients in simulation.

The identification is completed by following process. First, 1% of the stroke of rod is taken as the step length to sample the dynamometer card curve in horizontal direction. Second, the discrete dynamometer card curve is encoded using the present 24-directional chain code method. Third, the curve’s Fourier coefficients are expressed by means of chain codes. In this step, selecting the number of terms of the Fourier series properly is a problem. Obviously, the greater the number of terms included, the more precise the computation result on shape characteristics, but the computation task will be heavy. Through a lot of computations and comparisons, it is obtained that 32 is the suitable number of terms. Fourth, the shape characteristics of the measured and the simulated dynamometer card curves are respectively extracted by means of the corresponding Fourier coefficients. Substituting the two groups of characteristics into the expression of Euclidean distance, if the Euclidean distance is less than the given error, the viscous damping coefficient and Coulomb damping coefficient in the simulation program are regarded as the actual damping coefficients of the sucker-rod pumping system. Otherwise, continue the simulation with a new pair of viscous and Coulomb damping coefficients and obtain a new Euclidean distance, until the distance is less than the given error. In the above identification, according to experience and a handbook on sucker-rod pumping system [14], 0.4 and 0.05 are adopted as the initial values of the two coefficients, and their step lengths of change are 0.05 and 0.01, respectively.

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