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FINDING PRIME PAIRS WITH PARTICULAR GAPS

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ABSTRACT. By a prime gap of size g, we mean that there are primes p and $p+g$ such that the $g-1$ numbers between p and $p+g$ are all composite. It is widely believed that infinitely many prime gaps of size g exist for all even integers g. However, it had not previously been known whether a prime gap of size 1000 existed. The objective of this article was to be the first to find a prime gap of size 1000, by using a systematic method that would also apply to finding prime gaps of any size. By this method, we find prime gaps for all even integers from 746 to 1000, and some beyond. What we find are not necessarily the first occurrences of these gaps, but, being examples, they give an upper bound on the first such occurrences. The prime gaps of size 1000 listed in this article were first announced on the Number Theory Listing to the World Wide Web on Tuesday, April 8, 1997. Since then, others, including Sol Weintraub and A.O.L. Atkin, have found prime gaps of size 1000 with smaller integers, using more ad hoc methods. At the end of the article, related computations to find prime triples of the form $6m+1$, $12m-1$, $12m+1$ and their application to divisibility of binomial coefficients by a square will also be discussed.

0. INTRODUCTION

In 1928, D. H. Lehmer realized that if $p-1=FR$ where $F>p^{1/2}$ is factored, then there is a quick, practical way to show if p is prime. In 1975, this result was extended by Lehmer, Brillhart and Selfridge [1], so that one only needs to have the factored part $F>p^{1/3}$ to get a quick test. The test is based on the following theorem.

Theorem 1. *Let $N-1=FR$, where F is factored, $F>N^{1/3}$ and $(F+1)^2\neq N$. Assume that, for each prime p_k dividing F, there exists an a_k such that*

(i) $a_k^{N-1}\equiv 1 \pmod N$, *and*

(ii) $\gcd(a_k^{\frac{N-1}{p_k}}-1,N)=1$.

Write $R=2Fs+r$ where $1\le r<2F$. Then N is prime if and only if $s=0$ or r^2-8s is not the square of an integer.

To apply this to the prime gap problem, we need to have partial factorizations of $p-1$ and $p+g-1$. To do this, we proceed as follows: For a given even integer g, we select a and b to be the largest integers with $2^a,3^b<e^{g/4}$. We select p_0 to be the

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least positive integer $\pmod{2^a 3^b}$ as follows, by the Chinese Remainder Theorem:

$$\begin{aligned} &\text{if} \quad g \equiv 2 \pmod 3, \text{ then } p_0 \equiv 1 \pmod{2^a} \text{ and } p_0 + g \equiv 1 \pmod{3^b}, \\ (1) \quad & \\ &\text{if} \quad g \not\equiv 2 \pmod 3, \text{ then } p_0 \equiv 1 \pmod{3^b} \text{ and } p_0 + g \equiv 1 \pmod{2^a}. \end{aligned}$$

We need two cases, for, if $g \equiv 2 \pmod 3$ and we select $p_0 \pmod{2^a 3^b}$ such that $p_0 \equiv 1 \pmod{3^b}$ and $p_0 + g \equiv 1 \pmod{2^a}$, then we would have that 3 divides $p_0 + g$. If we were to only select $p_0 \pmod{2^a 3^b}$ such that $p_0 \equiv 1 \pmod{2^a}$ and $p_0 + g \equiv 1 \pmod{3^b}$, then we would have that 3 divides p_0 when $g \equiv 1 \pmod 3$. Selecting $p \equiv p_0 \pmod{2^a 3^b}$ with $p < \min(2^{3a-1}, 3^{3b-1})$, we get $p-1$ and $p+g-1$ divisible by 2^a and 3^b (the order depending on $g \pmod 3$). Notice that 2^a and 3^b are factors $> (p+g)^{1/3} > p^{1/3}$. Thus we can use Theorem 1 above to determine whether p and $p+g$ are prime.

In addition to the prime gap problem, Sections 4 and 5 will show how Theorem 1 can be applied to find prime triples of the form $6m+1$, $12m-1$, $12m+1$. Prime triples of this form will be used to complete the proof of the following theorem:

Theorem 2. *$\binom{2n}{n}$ is divisible by the square of a prime $p \geq \sqrt{n/5}$ for all $n \geq 2082$.*

1. The algorithm

The algorithm used to find consecutive primes with a given gap g is as follows.

Step 1: Find p_0, 2^a and 3^b as stated above.

Step 2: The numbers p that we will be testing are of the form $p = p_0 + i2^a 3^b$, as i runs from 1 to 10^6. We do not actually want to put all of these numbers through the primality test, as some of them may be divisible by small primes. We will sieve these numbers out as follows: For each prime q up to 10^4, if i is in either of the residue classes

$$\frac{-p_0}{2^a 3^b} \pmod q \text{ or } \frac{-p_0 - g}{2^a 3^b} \pmod q,$$

then either p or $p+g$, respectively, will be divisible by q. So we will discard this value of i. The remaining i's we store in a table, which we call $T_1(g)$.

Step 3: For each value of i in $T_1(g)$, we use Theorem 1 to test if $p = p_0 + i2^a 3^b$ is a prime. To do this, we start with $a_k = 2$. If, along with the other hypotheses, conditions (i) and (ii) of Theorem 1 are met, we have a prime. If they are not satisfied, we let a_k run through primes, up to the limit 255. If we reach this limit with no success in finding an a_k, or proving p composite, then we give up trying to determine primality of this value p and move on to the next i in $T_1(g)$. There are three ways we will learn that p is composite from our algorithm:

- ${a_k}^{p-1} \not\equiv 1 \pmod p$
- $1 < \gcd({a_k}^{(p-1)/b} - 1, p) < p$ where b is 2 or 3, depending on p
- In the theorem, we have $s > 0$ and $r^2 - 8s$ is the square of an integer

If we find an a_k which satisfies conditions (i) and (ii) of Theorem 1, we then perform the same test with $p+g$. If we have primality of both p and $p+g$, we store the values p, $p+g$ and the respective a_k's in a table, call it $T_2(g)$.

Step 4: For every prime pair p, $p+g$ in $T_2(g)$, we need to determine whether the numbers $p+j$, as j runs from 1 to $g-1$, are composite. We do this in two steps.

(a) We first check if the numbers $p+j$ are divisible by primes up to 10^4. For each prime $q < 10^4$, whenever $j \equiv -p \pmod{q}$, we have $p+j$ divisible by q. Thus $p+j$ is composite. We store the remaining j's (those where $p+j$ is not divisible by a prime $< 10^4$) in a table $T_{3,p}(g)$.

(b) For each j in $T_{3,p}(g)$, we run at least one pseudoprime test on $n = p+j$. If n is not a pseudoprime to either base 2 or 3, then we know it is composite and we go on to the next j in $T_{3,p}(g)$. If n should happen to pass pseudoprime tests to both the bases 2 and 3, we say n is probably prime and we disregard this prime pair p, $p+g$. We then move to the next prime pair p, $p+g$ in $T_2(g)$. If all of the numbers $p+j$ where $j = 1, .., g-1$, are not pseudoprimes to either base 2 or 3, then they are all composite, and we have found a consecutive pair of primes with gap g, as desired.

2. Expected values

Let $\pi_g(x)$ be the number of primes n up to x such that $n+g$ is also prime. Then we have the following conjecture, due to Hardy and Littlewood [3].

Conjecture 1. *If g is even, then*

$$\pi_g(x) \sim \alpha \prod_{\substack{p \mid g \\ p>2}} \frac{p-1}{p-2} \cdot \frac{x}{\log^2 x}$$

where

$$\alpha = 2 \prod_{p>2} \frac{\left(1 - \frac{2}{p}\right)}{\left(1 - \frac{1}{p}\right)^2}.$$

If we first just assume that the primality of the two numbers n, $n+g$ near x, randomly chosen, are independent events, then we would have

$$\pi_g(x) \sim \frac{x}{\log^2 x}.$$

But we know these events are not independent, so we correct this argument by multiplying by the factor

$$\prod_p \frac{\left(1 - \frac{\omega(p)}{p}\right)}{\left(1 - \frac{1}{p}\right)^2},$$

since the probablity that two random integers are not divisible by p is $\left(1 - \frac{1}{p}\right)^2$, and the probablity that our particular numbers are not divisible by p is $\left(1 - \frac{\omega(p)}{p}\right)$, where $\omega(p)$ is the number of solutions to $n(n+g) \equiv 0 \pmod{p}$. Noting that $\omega(p) = 2$ unless p divides g, whence $\omega(p) = 1$, we obtain the stated conjecture (with some rearrangement).

Recall that, in Step 2 of our algorithm, we did not want to put all of our numbers p through the primality test. So we sieved out the values of p which were divisible by primes up to 10^4. We can thus make a further adjustment to our expected values to account for this, and we arrive at the following:

Heuristic 1. *Knowing that the numbers n, $n+g$ are not divisible by primes less than 10^4, we have, assuming $g < 10^4$, that the probability that n and $n+g$ near x are both prime is $\approx \beta \frac{1}{\log^2 x}$ where $\beta = \prod_{p<10^4} \frac{1}{\left(1-\frac{1}{p}\right)^2} \approx 269.76$.*

We justify the heuristic as follows:

We previously had that the probability that n and $n+g$ are both prime was

$$\begin{aligned} &\sim \prod_{\substack{p \\ \omega(p)\neq 2}} \frac{\left(1-\frac{1}{p}\right)}{\left(1-\frac{1}{p}\right)^2} \cdot \prod_{\substack{p \\ \omega(p)=2}} \frac{\left(1-\frac{2}{p}\right)}{\left(1-\frac{1}{p}\right)^2} \cdot \frac{1}{\log^2 x} \\ &= \prod_{p|g} \frac{\left(1-\frac{1}{p}\right)}{\left(1-\frac{1}{p}\right)^2} \cdot \prod_{p\nmid g} \frac{\left(1-\frac{2}{p}\right)}{\left(1-\frac{1}{p}\right)^2} \cdot \frac{1}{\log^2 x}. \end{aligned}$$

Since we know n and $n+g$ are not divisible by primes up to 10^4, we can make the following adjustment: The probability that n and $n+g$ are both prime is now

$$\sim \prod_{\substack{p|g \\ p>10^4}} \frac{\left(1-\frac{1}{p}\right)}{\left(1-\frac{1}{p}\right)^2} \cdot \prod_{\substack{p\nmid g \\ p>10^4}} \frac{\left(1-\frac{2}{p}\right)}{\left(1-\frac{1}{p}\right)^2} \cdot \prod_{p<10^4} \frac{1}{\left(1-\frac{1}{p}\right)^2} \cdot \frac{1}{\log^2 x}.$$

Computing this last product, we get our heuristic, provided the contribution of the first two products here is negligible. Now, if $p > 10^4$, then $p > g$, so $p \nmid g$. Thus the first product equals 1; and it remains to show that $\prod_{p>10^4}(1-\frac{2}{p})/(1-\frac{1}{p})^2$ is close to 1. Now

$$\begin{aligned} 0 &< -\sum_{\substack{p\text{prime} \\ p>10^4}} \log\left(\frac{1-\frac{2}{p}}{\left(1-\frac{1}{p}\right)^2}\right) = -\sum_{\substack{p\text{prime} \\ p>10^4}} \log\left(1-\frac{1}{(p-1)^2}\right) \\ &\leq c\sum_{\substack{p\text{prime} \\ p>10^4}} \frac{1}{(p-1)^2} \leq c\sum_{n>10^4} \frac{1}{(n-1)^2} \\ &\leq c\int_{10^4}^{\infty} \frac{1}{(x-1)^2}\,dx = c\left[\frac{-1}{x-1}\right]_{10^4}^{\infty} = \frac{c}{10^4-1} \end{aligned}$$

where, since $p \geq 10^4+1$, we get $c < 1 + \frac{1}{10^8-1} \approx 1.00000001$.

Thus, taking exponentials of the above,

$$1 \geq \prod_{p>10^4} \frac{\left(1-\frac{2}{p}\right)}{\left(1-\frac{1}{p}\right)^2} \geq e^{-c/(10^4-1)} \approx 1 - .0001,$$

so is close to 1, justifying the heuristic.

Next, we would like to determine how many of these prime pairs $n, n+g$ we would expect to be consecutive. If we have a random number m near x, the probability that m is composite is $1 - \frac{1}{\log x}$. Assuming independence of compositeness of the $g-1$ numbers between n and $n+g$, the probablity that these numbers are all composite is $\left(1-\frac{1}{\log x}\right)^{g-1}$. Now, for our gaps g, the values of x are around $e^{g/2}$.

So we would have

$$\left(1-\frac{1}{\log x}\right)^{g-1} \sim \left(1-\frac{2}{g}\right)^{g-1} \approx e^{-2}.$$

We thus expect that roughly 1 out of every $e^2 \approx 7.39$ prime pairs $p, p+g$ in $T_2(g)$ should be consecutive primes (i.e., have no primes between p and $p+g$). Upon looking at some of the data below, this crude estimate seems to be reasonable.

3. Data

For each gap g, Table 1 (on the next page) contains the exponents a and b of 2 and 3, respectively, and the first value of i, which give consecutive primes p, $p+g$, where $p = p_0 + i2^a3^b$ (recall we have i running from 1 to 10^6), and p_0 is constructed as described by equation (1.1). Also listed is the number of values of i for which consecutive primes $p, p+g$ were obtained. There may be more values, since the algorithm ignores a pair $p, p+g$ if some number $p+j$, $1 \le j \le g-1$, passes 2 pseudoprime tests; however, it is highly unlikely that there is such a problem with the data. Table 1 is arranged with the original gap of size 1000 listed first, followed by several gaps of size larger than 1000 that were found. It then proceeds numerically, starting with gap 746. Gaps g listed with an asterisk are gaps that were already known to exist at the time these computations were performed ([4], [7], [8]). These gaps were found again just to make the list complete.

4. Other calculations

In addition to the prime gap problem, the primality testing ideas of Brillhart, Lehmer and Selfridge can be applied to find prime triples of the form $6m+1$, $12m-1$, $12m+1$. Prime triples of this form will be used to complete the proof of the following theorem:

Theorem 2. *$\binom{2n}{n}$ is divisible by the square of a prime $p \ge \sqrt{n/5}$ for all $n \ge 2082$.*

Granville and Ramaré have verified this theorem for $2082 \le n \le 10^{10}$ by using a direct consequence of Kummer's theorem, and for $n \ge 2^{1617}$ by using bounds on exponential sums [2]. By using the following proposition of Granville and Ramaré, it becomes a practical computational problem to establish this theorem for $10^{10} \le n \le 2^{1617}$.

Proposition 1. *If m is a positive integer for which $p = 6m+1$, $q = 12m-1$, and $r = 12m+1$ are all prime, then at least one of p^2, q^2, r^2 divides $\binom{2n}{n}$, for each integer n in the interval $[96m^2 - 2m, 108m^2 + 3m - 2]$, with the one exception, namely $m = 1, n = 104$.*

The biggest difficulty in applying this proposition is in primality proving. In general, when the integers involved are large, it is difficult to prove that p, q and r are all prime in a reasonable amount of time. However, since we have Theorem 1, we can construct the integers p, q and r in a specific manner, to make primality testing easier.

To apply Theorem 1 in finding prime triples p, q, r as in the proposition, we need to have the factorizations of part of m and $6m-1$. To do this, we proceed as follows: For a given odd integer b, choose a to be the smallest integer such that $2^a > 5^b$. By the Chinese Remainder Theorem, select m_0 to be the least positive integer satisfying the congruences $m_0 \equiv 0 \pmod{2^a}$ and $6m_0 - 1 \equiv 0 \pmod{5^b}$.

TABLE 1.

gap g	a	b	# of values of i	first value of i	gap g	a	b	# of values of i	first value of i	gap g	a	b	# of values of i	first value of i
1000	360	227	2	360878	824	297	187	8	71371	912	328	207	4	381747
1002	360	227	3	548723	826	297	187	13	208058	914	328	207	7	1929
1100	396	250	2	30276	828	298	188	5	178067	916	330	208	1	56173
1500	541	341	4	218942	830	298	188	12	101315	918	330	208	4	45015
2000	721	455	1	588652	832	300	189	8	53722	920	331	209	7	451458
*746	268	169	13	14308	834	300	189	4	266194	922	331	209	8	31794
*748	269	170	9	57235	836	301	190	3	140350	924	333	210	10	85698
*750	269	170	6	364122	838	301	190	5	116952	926	333	210	6	488010
*752	271	171	14	44909	840	302	191	6	105445	928	334	211	9	83306
*754	271	171	8	65890	842	302	191	9	74978	930	334	211	4	380958
*756	272	172	7	41673	844	304	192	5	510836	932	336	212	3	386782
*758	272	172	6	183807	846	304	192	10	194215	934	336	212	6	34178
*760	274	172	12	236393	848	305	192	2	356988	936	337	212	4	243622
*762	274	172	3	85	850	305	192	9	6907	938	337	212	6	178835
*764	275	173	3	253279	852	307	193	8	562	940	339	213	6	243000
*766	275	173	7	215388	854	307	193	9	38684	942	339	213	5	71245
768	276	174	7	289399	856	308	194	7	119756	944	340	214	8	39019
*770	276	174	8	27243	858	308	194	6	25518	946	340	214	5	8276
*772	278	175	7	125253	860	310	195	4	32742	948	341	215	3	507289
774	278	175	7	40978	862	310	195	7	266748	950	341	215	5	59634
*776	279	176	8	74202	*864	311	196	6	284541	952	343	216	12	25613
*778	279	176	8	91165	866	311	196	2	428141	954	343	216	5	19058
*780	281	177	8	86527	868	313	197	13	173495	956	344	217	2	210615
782	281	177	9	21943	870	313	197	4	489215	958	344	217	1	481114
*784	282	178	12	228620	872	314	198	12	147292	960	346	218	9	67752
786	282	178	8	52671	874	314	198	5	199077	962	346	218	6	420982
*788	284	179	5	225376	876	315	199	5	57648	964	347	219	8	45026
*790	284	179	13	53100	878	315	199	6	206082	966	347	219	9	19068
*792	285	180	7	147987	*880	317	200	12	40815	968	349	220	5	233833
794	285	180	6	58703	882	317	200	5	7559	970	349	220	5	75279
796	287	181	7	121321	884	318	201	6	214418	972	350	221	3	141435
*798	287	191	7	41947	886	318	201	5	549797	974	350	221	8	78969
800	288	182	12	18465	888	320	202	3	390981	976	352	222	3	236366
*802	288	182	9	85131	890	320	202	6	100213	978	352	222	6	106959
*804	289	182	11	24329	892	321	202	3	197853	980	353	223	7	44678
*806	289	182	6	82037	894	321	202	7	43284	982	353	223	8	17732
808	291	183	8	324033	896	323	203	4	108269	984	354	223	5	497244
810	291	183	7	34971	898	323	203	6	121508	984	354	223	5	497244
812	292	184	6	124459	900	324	204	2	68389	986	354	223	6	86991
*814	292	184	6	106578	902	324	204	7	10429	988	356	224	9	36947
*816	294	185	7	303334	904	326	205	5	495510	990	356	224	4	210425
818	294	185	3	76118	*906	326	205	3	683868	992	357	225	5	75636
820	295	186	6	2834	908	327	206	2	894181	994	357	225	7	87748
822	295	186	5	324077	910	327	206	5	45052	996	359	226	1	898299
										998	359	226	3	364937

Then for any $m \equiv m_0 \pmod{2^a 5^b}$, we have $2^a \mid m$ and $5^b \mid (6m-1)$. Notice that $p-1 = 6m$, $q-1 = 12m-2 = 2(6m-1)$, and $r-1 = 12m$, so we have $2^a \mid (p-1)$, $5^b \mid (q-1)$, and $2^a \mid (r-1)$. Thus we have factorizations of part of $p-1$, $q-1$, and $r-1$. Now, if $m \leq 5^{3b-1}$, then the factored parts of $p-1$, $q-1$, and $r-1$ are greater than $p^{1/3}$, $q^{1/3}$ and $r^{1/3}$, respectively; so we can use Theorem 1 to determine primality.

5. The algorithm

The following algorithm was used to find prime triples of the form $6m+1$, $12m-1$, $12m+1$, so that the intervals $[96m^2-2m, 108m^2+3m-2]$ overlap each other and cover the entire interval $[10^{10}, 2^{1617}]$.

Step 1: Find 2^a, 5^b and m_0 as stated above.

Step 2: The numbers p, q and r that we will be testing are of the form $p = 6(m_0 + i2^a5^b) + 1$, $q = 12(m_0 + i2^a5^b) - 1$, $r = 12(m_o + i2^a5^b) + 1$, as i varies. As with the prime pairs, we do not want to put all of these numbers through the primality test, since some of them may be divisible by small primes. We will sieve these numbers out as follows: For each prime l up to 10^4, if i is in any of the residue classes

$$\frac{-(6m_0+1)}{6\cdot 2^a5^b}, \frac{-(12m_0-1)}{12\cdot 2^a5^b} \text{ or } \frac{-(12m_0+1)}{12\cdot 2^a5^b} \pmod{l},$$

then either p, q, or r, respectively, will be divisible by l. So we will discard this value of i. The remaining i's we store in a table which we call T_1.

Step 3: For each value of i in T_1, we use Theorem 1 to test whether $p = 6(m_0 + i2^a5^b) + 1$ is a prime. This is done in the same manner as was done for prime pairs (see Step 3 in Section 1). If we find that p is prime, we perform the test with q. If q is also prime, we then test r. If we have primality of p, q and r, we proceed to Step 4.

Step 4: Once we have a prime triple $6m+1$, $12m-1$, $12m+1$, we want to find a new value, say $\tilde{m}$, so that the integers $6\tilde{m}+1$, $12\tilde{m}-1$, and $12\tilde{m}+1$ are all prime and the intervals $[96m^2-2m, 108m^2+3m-2]$ and $[96\tilde{m}^2-2\tilde{m}, 108\tilde{m}^2+3\tilde{m}-2]$ overlap. We find $\tilde{m}$ in the following manner: We want $96\tilde{m}^2 < 108m^2$, so we need $\tilde{m} \le \sqrt{108/96}m = \sqrt{9/8}m \approx 1.06066m$. So we will look for $\tilde{m}$ in the range $1.03m \le \tilde{m} \le 1.06m$ by first choosing values for a and b such that 2^a, $5^b \approx m^{2/5}$. Then we let $i = \lfloor \frac{1.03m}{2^a5^b} \rfloor$, and select m_0 as in Step 1. Take $\tilde{m} = m_0 + i2^a5^b$, as i varies. Return to Step 2 and repeat.

6. Data

To find prime triples of the form $6m+1$, $12m-1$, $12m+1$ so that the intervals $[96m^2-2m, 108m^2+3m-2]$ overlap each other and cover the entire range $[10^{10}, 2^{1617}]$, we need m to vary from about 10206 to 10^{243}. The first prime triple found was constructed with

$$m = 8896.$$

To find this triple, the starting values for the exponents a and b of 2 and 5 were 6 and 1, respectively. This value of m is less than 10206. The largest prime triple found was constructed with

$$\begin{aligned} m \;=\; & 865811619055227296688381075294124473916879909874 0 \\ & 1254690645211635431634072427787417224738578003507 \\ & 9334280517714221366437396824089500530872622682674 \\ & 4987801486383661769150183229722484031893648332404 \\ & 2942946151731516598223000376795729001363118489 6. \end{aligned}$$

This value of m is around 10^{243}. To find this triple, the starting values of a and b were 323 and 139, respectively. The algorithm described above produced over 6000 triples to cover the range $[10^{10}, 2^{1617}]$.

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