# SHEAR CENTRE OF THIN-WALLED SECTIONS

**C. GRANT** *Department of Mechanical, Materials and Manufacturing Engineering, University of Newcastle upon Tyne, UK. MIMechE*

Shear centre is an important geometric property of thin-walled sections that can be difficult to determine in practice. A computer based solution is developed for sections comprising an arbitrary number of limbs attached to each other at end nodes. Linear equations are identified that are sufficient to determine the shear flow in each limb and the shear centre is derived directly from their solution in a compact, closed form. The method is applied to sections with straight uniform limbs, and a specific example is evaluated.

## 1 INTRODUCTION

Thin-walled sections as produced by fabrication and by extrusion have become increasingly important due to their economy of material use and manufacture. Although shear stress levels in these structures are often negligibly small, the position of the shear centre in the section can be of crucial significance to the designer where twisting of the section is undesirable.

The solution to the general problem of combined bending and torsion is well known and can in principle be solved (**1**)† for any arbitrary section. The location of the shear centre is of particular importance in thin-walled sections where, under certain assumptions, a greatly simplified theory is appropriate (see Timoshenko (**2**))

Using the simplified theory, open sections such as the channel, *T* and *I* section can be handled analytically without much difficulty. Closed sections, which are torsionally very much stiffer than open sections and, therefore, more useful structurally, are more tedious to compute. Even with geometries of relatively modest complexity, the simplified theory can become intractable in practical terms. Alternative computer based methods such as the Finite Element Method are difficult to apply economically to this problem without special coding which is seldom available.

The purpose of this work is to formulate a computer based solution for the shear centre based on the simplified theory that is applicable to a wide range of thin-walled sections. The established method (**3**) treats a section as a group of 'cells' and analyses the shear flow around each cell. Although appropriate to a manual solution, this representation is difficult to incorporate into a general solution algorithm. In this work, the section is assumed to comprise an arbitrary number of limbs attached to each other at end nodes. With this representation, the section can be treated as a directed graph, enabling the use of established algorithms (**4**) to determine the section's connectivity, and to identify the cells etc.–questions that are vital in a computer based solution but do not arise in a manual solution.

*The MS. of this paper was received at the Institution on 28 August 1991 and accepted for publication on 4 March 1992*

† *References are given in the Appendix.*

Elementary matrix methods are employed to provide a concise solution suitable for implementation on a computer. A particular solution is developed for sections comprising straight limbs of uniform thickness.

### 1.1 Notation

| | |
|---|---|
| $\{\ \}$ | A 2 × 1 column matrix |
| $\{\ \}^T$ | A 1 × 2 row matrix |
| $[\ ]$ | A 2 × 2 square matrix |
| $\alpha$ | Local orientation of a limb |
| $\sigma_{sz}$ | Shear stress on the section |
| $\sigma_{zz}$ | Normal stress on the section |
| $\theta$ | Twist per unit length of the section |
| $\{\mathbf{a}\}$ | Local orientation matrix (see (**9**)) |
| $A$ | Section area |
| $\{c\}$ | Position of the shear centre |
| $e_{sz}$ | Shear strain on the section |
| $e_{zz}$ | Normal strain on the section |
| $E$ | Young's modulus of elasticity |
| $\{F\}$ | Shear force on the section |
| $G$ | Elastic modulus of rigidity |
| $[\mathbf{I}]$ | Unit matrix |
| $[\mathbf{J}]$ | Second moment of area matrix (see (**5**)) |
| $\{k\}$ | Curvature due to bending |
| $L$ | Length of a limb |
| $m$ | Number of limbs comprising the section |
| $\{M\}$ | Bending moment on the section |
| $n$ | Number of nodes; in-plane coordinate normal to $s$ |
| $p$ | Number of independent closed paths in the section |
| $q$ | Shear flow $\sigma_{sz} t$ |
| $q_0$ | Initial shear flow in a limb |
| $\delta q$ | Change in shear flow along a limb |
| $\{Q\}$ | Shear flow per unit shear force |
| $\{Q_0\}$ | Initial shear flow per unit shear force |
| $\{\delta Q\}$ | Change in shear flow per unit shear force |
| $[\mathbf{R}]$ | Orthogonal transformation matrix (see (**6**)) |
| $s$ | In-plane coordinate along a limb |
| $t$ | Local thickness of a limb |
| $T$ | Torque on the section |
| $\{u\}$ | Displacement of the section centroid |
| $\{x\}$ | In-plane Cartesian coordinates with origin at the section centroid |



0309–3247/92/0700–0151 $03.00 + 0.05

$\{\bar{x}\}$ Local centroid of a limb
$z$ Axial coordinate, normal to the section

## 2 BASIC EQUATIONS

The notation employed for bending on which the simplified theory is based is illustrated in Fig. 1. Using this notation, the basic equations of the simplified theory can be expressed in an appropriate form.

The simple theory of bending rests on the assumption that plane sections of a prismatic beam remain plane and normal to its centre line after bending by a uniform moment. With no axial load, the normal strain on the section is a linear function of the in-plane coordinates $x_1$, $x_2$ with origin at the section centroid.

$$e_{zz} = -(k_1 x_1 + k_2 x_2) = -\{x\}^{\mathrm{T}}\{k\} \tag{1}$$

The curvature matrix $\{k\}$ is defined as the second derivative of the centroid's displacement $\{u\}$

$$\{k\} = \frac{\mathrm{d}^2}{\mathrm{d}z^2}\{u\} \tag{2}$$

The normal stress on the section is

$$\sigma_{zz} = e_{zz} E = -\{x\}^{\mathrm{T}}\{k\}E \tag{3}$$

Moments on the section are determined by integrating the contributions due to normal stress $\sigma_{zz}$ over the section. They may be conveniently expressed in matrix form using the transformation matrix $[\mathbf{R}]$ defined in (6).

$$\{M\} = -\int_A [\mathbf{R}]\{x\}\sigma_{zz}\,\mathrm{d}A = \int_A [\mathbf{R}]\{x\}\{x\}^{\mathrm{T}}\{k\}E\,\mathrm{d}A$$
$$= [\mathbf{R}][\mathbf{J}]\{k\}E \tag{4}$$

where

$$[\mathbf{J}] = \int_A \{x\}\{x\}^{\mathrm{T}}\,\mathrm{d}A \tag{5}$$

is the symmetrical second moment of area matrix.

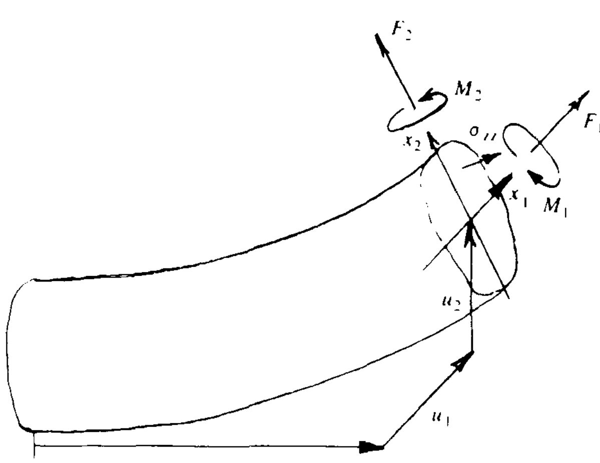


Fig. 1. Bending notation

The orthogonal transformation matrix $[\mathbf{R}]$ is defined as

$$[\mathbf{R}] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{6}$$

and has the properties $[\mathbf{R}]^{\mathrm{T}} = [\mathbf{R}]^{-1} = -[\mathbf{R}]$.

Solving (4) for the curvature $\{k\}$ gives

$$\{k\} = [\mathbf{J}]^{-1}[\mathbf{R}]^{\mathrm{T}}\{M\}E^{-1} \tag{7}$$

For a section comprising thin-walled limbs, the in-plane shear stress acts entirely along the limb. The component of shear stress acting normal to the limb, $\sigma_{nz}$, is zero. Equilibrium requires

$$\frac{\mathrm{d}}{\mathrm{d}z}(\sigma_{zz}t) + \frac{\mathrm{d}}{\mathrm{d}s}(\sigma_{sz}t) = 0$$

The local thickness $t$ may in general vary with the in-plane coordinate $s$, but is independent of the normal coordinate $z$.

$$\frac{\mathrm{d}}{\mathrm{d}s}(\sigma_{sz}t) = -t\frac{\mathrm{d}}{\mathrm{d}z}(\sigma_{zz}) = t\frac{\mathrm{d}}{\mathrm{d}z}(\{x\}^{\mathrm{T}}\{k\}E)$$
$$= t\{x\}^{\mathrm{T}}[\mathbf{J}]^{-1}[\mathbf{R}]^{\mathrm{T}}\frac{\mathrm{d}}{\mathrm{d}z}\{M\}$$

With a uniform bending moment, the shear force and the shear stress are everywhere zero. In the general case with non-zero shear force equilibrium requires that $\{F\} = [\mathbf{R}]^{\mathrm{T}}\,\mathrm{d}\{M\}/\mathrm{d}z$. Bending stress is assumed not to vary significantly from that given by the simple theory, since distortion of the plane section due to shear stresses is negligible. The shear stress $\sigma_{sz}$ varies along the limb according to (8), expressed in terms of the shear flow $q(s) = \sigma_{sz}t$.

$$\frac{\mathrm{d}q}{\mathrm{d}s} = t\{x\}^{\mathrm{T}}[\mathbf{J}]^{-1}\{F\} \tag{8}$$

The contribution of the shear flow in an individual limb to the total torque on the section is the moment of the shear flow components, Fig. 2, integrated over the limb

$$\delta T = \int_s (x_1 \sin\alpha - x_2\cos\alpha)q\,\mathrm{d}s$$

The torque can be expressed in matrix form using a local orientation matrix $\{\mathbf{a}\}$ defined in (9) below.

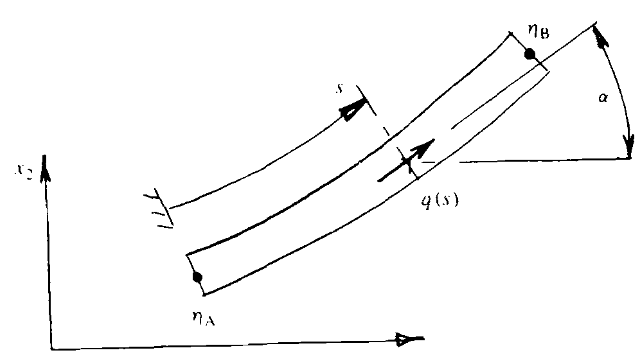


Fig. 2. Shear flow in a limb

$$\delta T = \int_s \{x\}^{\mathrm{T}}[\mathbf{R}]^{\mathrm{T}}\{\mathbf{a}\}q \cdot \mathrm{d}s$$

The local orientation matrix $\{\mathbf{a}\}$ defines the slope of the limb

$$\{\mathbf{a}\}^{\mathrm{T}} = (\cos\alpha \ \sin\alpha) \tag{9}$$

The total torque on the whole section is the sum of the contributions from each limb

$$T = \sum_1^m \int_s \{x\}^{\mathrm{T}}[\mathbf{R}]^{\mathrm{T}}\{\mathbf{a}\}q \cdot \mathrm{d}s \tag{10}$$

## 3 DISTRIBUTION OF SHEAR FLOW

The change in shear flow $\delta q$ along a typical limb is obtained by integrating (8)

$$\delta q(s) = \int_s t\{x\}^{\mathrm{T}}[\mathbf{J}]^{-1}\{F\}\,\mathrm{d}s$$
$$= A(s)\{\bar{x}(s)\}^{\mathrm{T}}[\mathbf{J}]^{-1}\{F\} \tag{11}$$

where $A\{\bar{x}\}$ is the first moment of area of the limb. The integration is taken over an arbitrary length $s$, not necessarily the full length $L$ of the limb. The total shear flow at any point in a typical limb, Fig. 2 is, therefore, the sum of an initial shear flow $q_0$ and the change given by (11)

$$q(s) = q_0 + \delta q(s) \tag{12}$$

## 4 INITIAL SHEAR FLOWS

The initial shear flow $q_0$ must be determined for each limb according to the constraints imposed by equilibrium and compatibility between limbs.

### 4.1 Open sections

In an open section comprising $m$ thin walled limbs, let each limb have two nodes, $n_A$ at the start ($s = 0$) and $n_B$ at the end ($s = L$) respectively as shown in Fig. 2. The total number of nodes $n$ in the whole section, assuming it to be connected, is related to the number of limbs $m$ by

$$m = n - 1 \tag{13}$$

Equilibrium at each node requires that the net shear flow out of the node is zero. This condition provides $n$ nodal equations (**14**) to define the $m$ initial shear flows $q_0$ in terms of the changes in shear flow $\delta q(L)$, the latter calculated over the whole length $L$ of each limb according to (**11**).

$$\sum q(s) = \sum \pm(q_0 + \delta q(s)) = 0 \tag{14}$$

The summations are taken over all limbs connected to a given node. $s$ takes the value 0 for a start node $n_A$ when $\delta q(0) = 0$ and the flow in accordance with Fig. 2 is $+q_0$, or $L$ for an end node $n_B$ when the flow is $-(q_0 + \delta q(L))$, negative as it is into the node. Figure 3 shows a typical node with limbs $a$ and $b$ connected at their start nodes and limbs $c$ and $d$ connected at their end nodes. The equilibrium equation in this case would be

$$(q_0)_a + (q_0)_b - (q_0 + \delta q(L))_c - (q_0 + \delta q(L))_d = 0$$

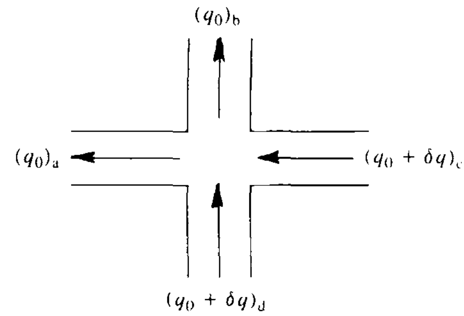


Fig. 3. Shear flow at a typical node

The number $n$ of nodal equations (14) exceeds the number of unknown initial flows $q_0$ by 1, so the set of equations (14) are redundant. For a given limb, $q_0$ will appear in exactly two nodal equations, namely those associated with its two nodes. At the start node $n_A$ the coefficient will be $+1$ as $q_0$ flows out of the node. At the end node $n_B$, the shear flow will be $-(q_0 + \delta q(L))$ *out* of the node, giving a coefficient of $-1$. Adding together all of the $n$ nodal equations gives a total contribution of zero for each of the $m$ initial flows $q_0$.

By the same reasoning, the change in shear flow $\delta q(L)$ for a given limb calculated from (11) will appear in only one nodal equation, namely that associated with its end node, $n_B$. Adding all of the $n$ nodal equations will, therefore, give

$$\sum_1^m \delta q(L) = 0 \tag{15}$$

Equation (15) is correct providing that the origin of the coordinates $\{x\}$ coincides with the centroid of the whole section. $\delta q(L)$ for each limb is proportional to the first moment of area of the limb in accordance with (11). The sum over all limbs is, therefore, proportional to the first moment of area for the whole section, and this is zero about the centroid by definition.

The $n$ nodal equations are, therefore, redundant but consistent. Any one equation may be discarded, and those remaining solved for the $m = n - 1$ initial flows $q_0$. The redundant equation would determine the position of the centroid if an arbitrary coordinate system were employed.

### 4.2 Closed sections

If an open section is constructed by adding new limbs such that each is attached to the existing section at only one node, each new limb added will also add one new node. The total number of limbs will increase with the number of nodes, maintaining the open section criterion (13). If, however, a new limb is attached to two existing nodes, no new node is created although the number of limbs is increased by one. This creates a closed circuit in the section and an excess of unknown initial shear flows $q_0$. The $n - 1$ independent nodal equations are no longer sufficient to solve for the $n$ unknown initial shear flows $q_0$. The required additional equation is derived from the continuity of shear strain around the circuit.

$$e_{sz} = \tfrac{1}{2}(\mathrm{d}u_z/\mathrm{d}s + \mathrm{d}u_s/\mathrm{d}z) = \frac{\sigma_{sz}}{2G} = \frac{q}{2}\,Gt$$

If the section suffers no in-plane distortions, the displacement $u_s$ parallel to the limb depends on the twist $\theta$ and the normal distance $n$ to the origin.

$$\frac{du_s}{dz} = n\theta$$

As the normal displacement $u_z$ must be continuous around the circuit, integrating the shear strain around the circuit gives

$$\oint \frac{q}{Gt}\, ds = \theta \oint n\, ds = 2A\theta$$

If the shear forces $\{F\}$ act through the shear centre then $\theta = 0$ by definition, and with uniform elastic properties

$$\oint \frac{q}{t}\, ds = 0 \tag{16}$$

Equation (16) is the additional equation required to supplement the nodal equations when the section contains a closed path.

For a section with multiple circuits, equation (16) must be applied to each independent circuit. The number of independent closed paths $p$ is the excess of limbs over nodes minus one.

$$p = m - (n - 1) \tag{17}$$

In a section containing multiple circuits, there are usually more than $p$ possible closed paths, but only $p$ of these are independent, in that they create independent versions of equation (16). Otherwise, the choice of which paths to use is arbitrary. Minimum length paths should be preferred on the grounds of computational efficiency.

## 5 SHEAR CENTRE

When equations (14) and (16) are solved for the $m$ initial shear flows $q_0$, the shear flow, and hence shear stress is essentially known throughout the section. The total torque on the section can then be calculated from (10).

If the shear forces $\{F\}$ are applied at the shear centre $\{c\}$ then by definition they do not cause twisting of the section. The torque about the centroid may be expressed as

$$T = F_2 c_1 - F_1 c_2 = \{c\}^{\mathrm{T}}[\mathbf{R}]^{\mathrm{T}}\{F\} \tag{18}$$

Local shear flow $q(s)$ in (10), may be expressed with the shear force $\{F\}$ as a factor, since changes in shear flow $\delta q(L)$ along each limb, from which $q_0$ are derived, also have $\{F\}$ as a factor. Let the specific shear flow per unit shear force $\{Q\}$ be defined by equation (19).

$$\begin{aligned} q(s) &= \{Q(s)\}^{\mathrm{T}}\{F\} \\ &= (\{Q_0\} + \{\delta Q(s)\})^{\mathrm{T}}\{F\} \end{aligned} \tag{19}$$

Equation (19) also defines the specific initial shear flow $\{Q_0\}$ and specific change in shear flow $\{\delta Q(s)\}$ analogous to (12). The corresponding expression for $\{\delta Q(s)\}$ based on (11) is

$$\{\delta Q(s)\}^{\mathrm{T}} = A(s)\{\bar{x}(s)\}^{\mathrm{T}}[\mathbf{J}]^{-1} \tag{20}$$

Rewriting (10) in terms of specific shear flow gives

$$T = \sum_1^m \int_s \{x\}^{\mathrm{T}}[\mathbf{R}]^{\mathrm{T}}\{\mathbf{a}\}\{Q\}^{\mathrm{T}}\{F\}\, ds \tag{21}$$

Equating the two expressions for torque (18) and (21) allows the shear centre $\{c\}$ to be expressed independently of the applied shear force $\{F\}$.

$$\{c\} = \sum_1^m \int_s [\mathbf{R}]^{\mathrm{T}}\{Q\}\{a\}^{\mathrm{T}}[\mathbf{R}]\{x\}\, ds \tag{22}$$

## 6 APPLICATION TO SECTIONS WITH STRAIGHT, UNIFORM LIMBS

The shear centre (22) can be applied to general thin walled sections with limbs of arbitrary shape and varying thickness. Before the integration in (22) can be performed explicitly and before the section properties can be determined, the actual shape and thickness must be specified for each limb.

For the particular case of a straight uniform limb the shear flow varies quadratically along the limb. Equation (20) becomes

$$\begin{aligned} \{\delta Q(s)\} &= [\mathbf{J}]^{-1}\{\bar{x}(s)\}A(s) \\ &= [\mathbf{J}]^{-1}(\{x_{\mathrm{A}}\} + \{a\}s/2)st \end{aligned}$$

where

$\{x_{\mathrm{A}}\}$ is the coordinate of the start node $n_{\mathrm{A}}$
$\{a\}$ is a constant limb orientation (see (9))

The total shear flow within a limb can then be expressed as

$$\{Q\} = \{Q_0\} + \{Q_1\}s + \{Q_2\}s^2 \tag{23}$$

where

$$\begin{aligned} \{Q_1\} &= [\mathbf{J}]^{-1}\{x_{\mathrm{A}}\}t \\ \{Q_2\} &= [\mathbf{J}]^{-1}\{a\}t/2 \end{aligned}$$

Equation (23) defines the distribution of shear flow in a uniform straight limb. It can be used to determine explicitly the $m$ initial shear flows $\{Q_0\}$ using $n - 1$ nodal equations (14) and $p$ compatibility equations (16), and the shear centre (22).

For sections that contain closed paths, (16) can be expressed in terms of the specific shear flow $\{Q\}$ and integrated around the circuit

$$\oint \frac{\{Q\}}{t}\, ds = \sum \pm \int_s \frac{\{Q\}}{t}\, ds = 0$$

$$\sum \pm(\{Q_0\}L + \{Q_1\}L^2/2 + \{Q_2\}L^3/3)/t = 0 \tag{24}$$

The summation is taken over all limbs in the closed path when the sign $\pm$ depends on the direction taken around the path relative to the assumed flow direction $n_{\mathrm{A}}$ to $n_{\mathrm{B}}$.

Since the centroid of the section would not in general be known initially, nodal coordinates would be prescribed in a global coordinate system with an arbitrary origin. The area and its first and second moments $A$, $\{H\}$ and $[\mathbf{J}_0]$ relative to the global origin are accumulated first for each limb. $\delta A$, $\{\delta H\}$, and $[\delta \mathbf{J}_0]$ define the contributions from each limb.

For each limb

$$\begin{aligned} \{\mathrm{d}\} &= \{x_{\mathrm{B}}\} - \{x_{\mathrm{A}}\} \\ L &= \sqrt{(\{\mathrm{d}\}^{\mathrm{T}}\{\mathrm{d}\})} \end{aligned}$$

$$\begin{aligned} \delta A &= Lt \\ \{\bar{x}\} &= \{x_{\mathrm{A}}\} + \{\mathrm{d}\}/2 \\ \{\delta H\} &= \{\bar{x}\}\delta A \\ [\delta \mathbf{J}_0] &= \{\bar{x}\}\delta A\{\bar{x}\}^{\mathrm{T}} + \{\mathrm{d}\}\delta A/12\{\mathrm{d}\}^{\mathrm{T}} \end{aligned}$$

The section centroid $\{\bar{x}\}$ and second moment of area $[\mathbf{J}]$ relative to the centroid can then be determined

$$\{\bar{x}\} = \{H\}/A$$
$$[\mathbf{J}] = [\mathbf{J}_0] - \{\bar{x}\}A\{\bar{x}\}^{\mathrm{T}}$$

The only other non-constant term in (22) is $\{x\} = \{x_A\} + \{a\}s$ with $\{x_A\}$ relative to the section centroid. Using these results and the identity $\{a\}^{\mathrm{T}}[\mathbf{R}]\{a\} = 0$, integration of (22) results in

$$\{c\} = \sum_{1}^{m} [\mathbf{R}]^{\mathrm{T}}\{\tilde{Q}\}\{\mathrm{d}\}^{\mathrm{T}}[\mathbf{R}]\{x_A\} \quad (25)$$

where

$$\{\tilde{Q}\} = \{Q_0\} + \{Q_1\}L/2 + \{Q_2\}L^2/3$$

The total shear force $\{F\}$ on the section can be determined by integrating the shear flow over the section, Fig. 2

$$\{F\} = \sum_{1}^{m} \int_s \{\mathbf{a}\}q \, \mathrm{d}s = \sum_{1}^{m} \{\mathrm{d}\}\{\tilde{Q}\}^{\mathrm{T}}\{F\}$$

Therefore, $\sum \{\tilde{Q}\}\{\mathrm{d}\}^{\mathrm{T}} = [\mathbf{I}]$, the identity matrix. This result can be used to define an error matrix to be used as a check on the numerical errors present in a complex calculation

$$(\text{error}) = \sum_{1}^{m} (\{\tilde{Q}\}\{\mathrm{d}\}^{\mathrm{T}} - [\mathbf{I}]) \quad (26)$$

A section is described by a node table in which each node consists of global position coordinates $x_1$ and $x_2$, and a limb table with each limb consisting of a thickness $t$, and a pair of end nodes $n_A$, $n_B$. Data for the channel section illustrated in Fig. 4 are given in Tables 1 and 2.

Results for this case were obtained using Gaussian elimination procedures to solve the nodal equations, as described in Grant (**5**) and are given in Table 3 with the shear centre $\{c\}$ transformed back into global coordinates. The shear flow distribution vectors $\{Q_0\}$, $\{Q_1\}$, and $\{Q_2\}$ in Table 3 should be pre-multiplied by the second moment of area matrix $[\mathbf{J}]^{-1}$ to conform with the definitions in (23). This calculation was deferred on the grounds of computational efficiency and performed only once on the final calculation for the shear centre (25) rather than twice for every limb as implied by (23). The results are presented in this modified form.

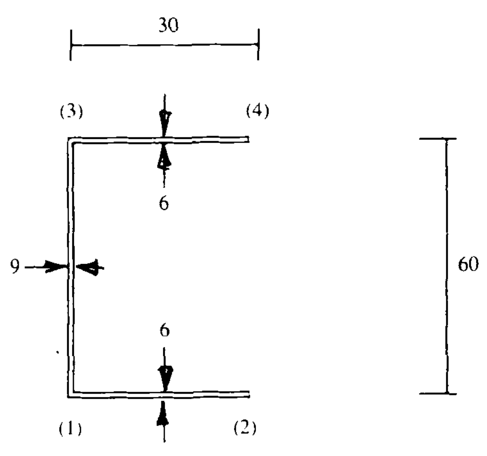

Fig. 4. Channel section

Table 1. Node table for the channel section

| Node | x1 | x2 |
|---|---|---|
| 1 | 0.0 | 0.0 |
| 2 | 30.0 | 0.0 |
| 3 | 0.0 | 60.0 |
| 4 | 30.0 | 60.0 |

Table 2. Limb table for the channel section

| Limb | $t$ | $n_A$ | $n_B$ |
|---|---|---|---|
| 1 | 6.0 | 1 | 2 |
| 2 | 9.0 | 1 | 3 |
| 3 | 6.0 | 3 | 4 |

Table 3. Results for the channel section

| Nodes 4 | | Limbs 3 | Loops 0 |
|---|---|---|---|
| | Area | 900.0 | |
| | Centroid | 6.0 | 30.0 |
| Second Moment | | 75 600.0 | 0.0 |
| | | 0.0 | 486 000.0 |
| Limb 1 | Q0 | 1620.0 | −5400.0 |
| | Q1 | 36.0 | 180.0 |
| | Q2 | −3.0 | 0.0 |
| | dQ | −1620.0 | 5400.0 |
| Limb 2 | Q0 | −1620.0 | 5400.0 |
| | Q1 | 54.0 | 270.0 |
| | Q2 | 0.0 | −4.5 |
| | dQ | 3240.0 | 0.0 |
| Limb 3 | Q0 | 1620.0 | 5400.0 |
| | Q1 | 36.0 | −180.0 |
| | Q2 | −3.0 | 0.0 |
| | dQ | −1620.0 | −5400.0 |
| | Error | −9.09E-13 | 0.00E + 00 |
| | | 0.00E + 00 | 0.00E + 00 |
| Shear centre | | −1.0 | 30.0 |

## 7 CONCLUSIONS

A method has been developed for finding the shear centre of a thin walled section automatically on a digital computer. The method can be applied to any section that can be described as a number of connected limbs, and has been implemented for straight uniform limbs in particular. In addition to finding the shear centre in two dimensions the method also enables the detailed distribution of shear flow in each limb to be determined as well as the second moments of area and centroid of the section.

## APPENDIX

### REFERENCES

(**1**) TIMOSHENKO, S. P., *History of strength of materials*, 1953, McGraw-Hill.
(**2**) TIMOSHENKO, S., *Strength of materials*, Part 1, 1955, Kreiger.
(**3**) GOODIER, J. N., 'A theorem on the shearing stress in beams with applications to multicellular sections', J. Aeronaut. Sci., 1944, **11**, 272–280.
(**4**) GIBBONS, A., *Algorithmic graph theory*, 1985, Cambridge University Press.
(**5**) GRANT, C., *Computing for Engineers*, 1989, Prentice-Hall.