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MATHEMATICAL MODELLING OF ELECTROMAGNETIC SCATTERING FROM A THIN PENETRABLE TARGET

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Abstract—Three mathematical models based on approximate surface integral equations for electromagnetic analysis of scalar wave scattering from thin extended target are considered. Such models include different systems of the second kind singular integral equations determined on the target median. The effective algorithm for direct (without preliminary regularization) numerical solution of the systems is based on the special quadrature formulae for singular integrals. Verification of the mathematical models and their comparison is performed in the case of penetrable cylindrical shell in homogeneous non-magnetic medium.

1. INTRODUCTION

Electromagnetic scattering from thin sheets of materials with different dielectric and magnetic properties is of relevance in the context of different practical applications: radome, antenna, and frequency-selective surfaces design, polarizing attenuators and converters production, non-destructive testing etc. The computational complexity of total diffracted field calculation (the reflected, transmitted and absorbed radiation including) becomes significantly less if the target can be modeled as infinitesimally thin surface with special boundary conditions [1–5].

In this case the internal field of the target is excluded, and its thickness supposed to be zero. Electromagnetic properties of material are modeled using some effective parameters included to appropriate boundary conditions. Generally speaking, such parameters depend on the frequency of illumination and allow for the target thickness as well as for electromagnetic properties (permittivity, permeability and conductivity) of its material and environment. Several types of the mentioned boundary conditions have been developed up to now. The corresponding survey one can find, for instance, in [6–8]. In fact, such boundary conditions have been widely analyzed and the corresponding boundary (surface) integral equations extensively treated during analysis of various electromagnetic structures in a number of recently published articles (e.g., see [9–16]).

In this paper we concentrate on comparison of some numerical results obtained with different surface integral equations for a thin penetrable target. Such obstacle fits adequately (regarding electrodynamics) of pore, crack-type defect etc. and is of interest of material non-destructive examination. We realize all the mentioned mathematical models using the same singular integral equations (SIE) technique. However, our main purpose is not to study their computational effectiveness, but rather to determine the application range of the considered mathematical models as to the non-destructive testing needs.

For simplicity we consider below the scalar diffraction problem. In this case we use the term "*thin target (obstacle)*" to mean the cylindrical shell when its largest thickness is much less than the length along the guide. The last one is assumed to be comparable with the excitation wavelength.

Let the material of target is homogeneous and isotropic. The primary electromagnetic wave is monochromatic (time dependence $\exp(-i\omega t)$, ω is the circular frequency), arbitrarily polarized, and does not depend on longitudinal Cartesian coordinate Z. The cross-section Σ of the considered thin obstacle is shown in Fig. 1. In this figure $2h = const$ is the target thickness; the numbers ε, μ and σ are permittivity, permeability and conductivity of material in domain Σ; the positive normal $\vec{n}$ points leftward to the median L (we have in mind the directed arc with positive trend from a to b) and makes the angle ψ with X-axis. It is evident that the initial diffraction problem includes two (TM- and TE-) cases. We denote the longitudinal component of a total field by W. It is the electrical or magnetic component in TM- or TE-cases correspondingly. Then the transverse field components can be easily determined through the function $W(x, y)$ by differentiation.

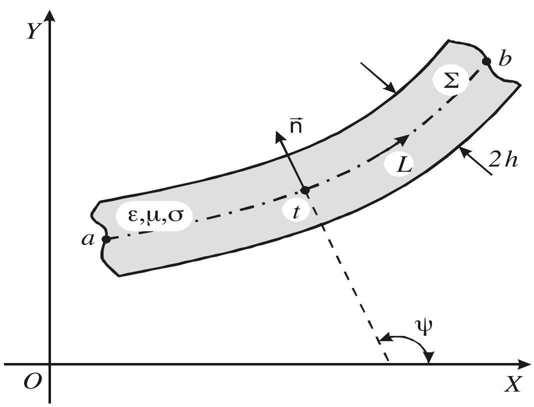

Figure 1. The target cross-section.

In this paper we consider three mathematical models of the mentioned diffraction problem. One of them is based, in fact, on the volume integral equations approach (e.g., see [17–19]). The other two models use different boundary conditions on the curve L: the higher-order ones (the kind of [20]) and the zero-order sheet-boundary conditions in the form of "Ohm's law" [6]. These three models we designate by I, II, and III correspondingly.

2. SURFACE INTEGRAL EQUATIONS BASED ON THE POLARISATION CURRENTS CONCEPT (MODEL I)

Well known approach to derivation of the volume (determined in domain Σ in the scalar case) integral equations includes introduction of the electric and magnetic polarization currents concept. For the considered diffraction problem it was applied for the first time probably in [21, 22]. The obtained in such a way integral equations are rigorous and equivalent to the initial diffraction problem. In the case of small thickness of the target they can be transformed to the surface integral equations determined on the curve L. The mentioned procedure for the first time was performed most likely in [23]. In the case of E-polarization the corresponding surface integral equation looks like

$$E_z(s_0) - 2K \int_L E_z(s) H_0^{(1)}(\chi_0 r) ds = E_z^0(s_0), \quad K = ih(\chi^2 - \chi_0^2)/4. \quad (1)$$

Here $E_z(s)(\equiv W)$ is electric component of a total electromagnetic field (superscript "0" regards to the excitation), $H_0^{(1)}(z)$ is Hankel function; $\chi = \omega(\mu(\varepsilon + i\sigma/\omega))^{1/2}$ and $\chi_0 = \omega(\mu_0(\varepsilon_0 + i\sigma_0/\omega))^{1/2}$ mean wave numbers inside and outside domain Σ (for simplicity below we consider the case of non-magnetic media when μ and μ_0 coincide with permeability of vacuum); $r = |t - t_0|$ denotes the distance between two complex points of curve L with affixes $t = x + iy$ and $t_0 = x_0 + iy_0$; arc abscissa s (natural parameter of the contour L) corresponds to t.

In case of H-polarization we obtain the next system of coupled surface integral equations regarding to the transverse electric components $E_{x,y}$ (see also [24]):

$$\begin{aligned}
&A^+(s_0)E_x(s_0) + B(s_0)E_y(s_0) \\
&- [I_1(E_x, s_0) + I_2(E_x, s_0) + i(I_1(E_y, s_0) + I_2(E_y, s_0))] \\
&- \int_L [E_x(s)C^+(s, s_0) - E_y(s)D(s, s_0)]\, ds = E_x^0(s_0), \qquad (2) \\
&B(s_0)E_x(s_0) + A^-(s_0)E_y(s_0) \\
&- [i(I_1(E_x, s_0) - I_2(E_x, s_0)) - I_1(E_y, s_0) - I_2(E_y, s_0)] \\
&- \int_L [-E_x(s, s_0)D(s, s_0) + E_y(s, s_0)C^-(s, s_0)]\, ds = E_y^0(s_0).
\end{aligned}$$

The following designators are used in the integral Equations (2):

$$A^{\pm}(s) = \frac{1}{2}\left[\frac{\chi^2}{\chi_0^2} + 1 \pm \left(\frac{\chi^2}{\chi_0^2} - 1\right)\cos(2\psi)\right], B(s) = \frac{1}{2}\left(\frac{\chi^2}{\chi_0^2} - 1\right)\sin(2\psi); \tag{3}$$

$$\begin{aligned}
C^{\pm}(s, s_0) &= K\left[H_0^{(1)}(\chi_0 r) \pm H_2(\chi_0 r)\mathrm{Re}\left(\frac{\bar{t} - \bar{t}_0}{t - t_0}\right)\right],\\
D(s, s_0) &= K H_2(\chi_0 r)\mathrm{Im}\left(\frac{\bar{t} - \bar{t}_0}{t - t_0}\right);\\
I_1(f, s_0) &= \frac{h}{2\pi}\left(\frac{\chi^2}{\chi_0^2} - 1\right)\int_L \frac{(f(s)d\bar{t}/ds)'_s}{t - t_0}ds,\\
I_2(f, s_0) &= \frac{h}{2\pi}\left(\frac{\chi^2}{\chi_0^2} - 1\right)\int_L \frac{(f(s)dt/ds)'_s}{\bar{t} - \bar{t}_0}ds;\\
H_2(z) &= H_2^{(1)}(z) + 4i/\pi z^2, \quad t = t(s), \quad t_0 = t(s_0),
\end{aligned}$$

Re and Im mean the real and imaginary parts, hyphen over symbol stands for complex conjugation.

Equations (1) and (2) are derived under condition $2h|dE_{x,y,z}/dn| \ll |E_{x,y,z}|$. This requirement leads, in fact, to some limitation of the illumination frequency range and physical parameters of materials: waves inside target should be essentially lengthy than its thickness.

Taking into account formulae (3) and definition of Hankel functions $H_{0,2}^{(1)}(z)$ it is clear that (1) is Fredholm type integral equation of the second kind with logarithmic kernel while (2) is the system of singular integro-differential equations with Cauchy type singularities.

3. APPROXIMATE BOUNDARY CONDITIONS ON THE CURVE L

Let domain Σ separates two different media. We index by "1" and "2" all the variables and physical parameters for exterior material situated to the left and to the right of curve L correspondingly. Then some kind of general boundary conditions which function $W(x, y)$ should satisfy on contour L is proposed in [25]. They look like

$$F_1\left(e^{-kh}W^{(1)} + e^{kh}W^{(2)}\right) - F_2\left(W^{(1)} + W^{(2)}\right) - kh\left(W^{(1)} - W^{(2)}\right)$$
$$+2h\xi\left(\frac{1}{\xi_1}\frac{\partial W^{(1)}}{\partial n} - \frac{1}{\xi_2}\frac{\partial W^{(2)}}{\partial n}\right) = 0; \tag{4}$$

$$F_1\left(e^{-kh}W^{(1)} - e^{kh}W^{(2)}\right) + F_2\left(W^{(1)} - W^{(2)}\right) + kh\left(W^{(1)} + W^{(2)}\right)$$
$$-2h\xi\left(\frac{1}{\xi_1}\frac{\partial W^{(1)}}{\partial n} + \frac{1}{\xi_2}\frac{\partial W^{(2)}}{\partial n}\right) = 0$$

and include two operators — the bounded operator F_1 and unbounded one F_2:

$$\begin{aligned} F_1 &= 2hQ/sh(2hQ); \qquad F_2 = 2hQcth(2hQ); \\ 2hQ &= \left((kh)^2 - (2hp)^2\right)^{1/2}, \quad p^2 = \chi^2 + \partial^2/\partial s^2. \end{aligned} \tag{5}$$

Besides, the following designations are used in formulae (4): $\xi = \mu$ or $\xi = \varepsilon + i\sigma/\omega$ in the case of TM- or TE-polarization (similarly for ξ_1 and ξ_2); $k(s) = \partial\psi/\partial s$ is the curvature of contour L; $\partial/\partial n$ means normal differentiation.

The idea of conditions (4) derivation is adopted from [26] (see also [23]) and presumed an introduction of the following thickness-averaged integral characteristics of function $W(s, n)$:

$$W^*(s) = \frac{1}{2h}\int_{-h}^{h} W(s, n)dn; \qquad W^{**}(s) = \frac{3}{2h^2}\int_{-h}^{h} nW(s, n)dn. \tag{6}$$

We can obtain formulae (4) after the corresponding thickness averaging of Helmholtz equation and consequent excluding of these characteristics as well as the internal (in domain Σ) boundary values of function $W(x, y)$ using the general solution of wave equation as well as continuity property of the tangential field components.

Some application of conditions (4) for diffraction problems solving one can find in [27]. Of course, the effectiveness of these boundary conditions depends aloud on the kind of approximation of operators $F_{1,2}$. One of the simplest approximation is

$$F_1 \approx 1 - (2hQ)^2/6, \quad F_2 \approx 1 + (2hQ)^2/3, \tag{7}$$

obtained by Taylor expansion of functions $f_1 = 2hz/sh(2hz)$; $f_2 = 2hzcth(2hz)$. As it is well known such an expansion is the best near the origin and worse out the point $z = 0$. So the obtained in this way approximation of operators $F_{1,2}$ are suitable for the region of their small eigenvalues.

From the mathematical point of view formulae (7) mean that we replace bounded operator F_1 by the unbounded one. Also we change a linear growth at infinity of the operator F_2 to its quadratic growth.

Note, that (7) are valid for both cases of open ($a \neq b$) and close ($a = b$) curve L. From the physical standpoint the approximation (7) hold true only under conditions: $2h/l \ll 1$; $|\sqrt{((hk)^2 - (2h\chi)^2)}| \ll 1$ which obviously restrict the geometry as well as electrical, and magnetic properties of a target.

4. SURFACE INTEGRAL EQUATIONS BASED ON HIGH-ORDER BOUNDARY CONDITIONS (MODEL II)

To demonstrate the main idea of application the approximate boundary conditions to solving the diffraction problems for thin target we regard the simple cases of operators $F_{1,2}$ fitting in conditions (4). Again for simplicity we consider an arbitrary cylindrical dielectric shell (L is smooth curve, and $a = b$) in homogeneous non-magnetic environment. Then we mark by suffix "0" all the physical parameters out of shell, put $\mu/\mu_0 = 1$, and rewrite formulae (4) as

$$\begin{aligned} & F_1\left(e^{-kh}W^{(+)}+e^{kh}W^{(-)}\right)-F_2\left(W^{(+)}+W^{(-)}\right)-kh\left(W^{(+)}-W^{(1)}\right) \\ & +2h\Delta\left(\frac{\partial W^{(+)}}{\partial n}-\frac{\partial W^{(-)}}{\partial n}\right)=0; \qquad (8) \\ & F_1\left(e^{-kh}W^{(+)}-e^{kh}W^{(-)}\right)+F_2\left(W^{(+)}-W^{(-)}\right)+kh\left(W^{(+)}+W^{(-)}\right) \\ & -2h\Delta\left(\frac{\partial W^{(+)}}{\partial n}+\frac{\partial W^{(-)}}{\partial n}\right)=0. \end{aligned}$$

Here the superscripts mark boundary values to the right (+) and to the left ($-$) of contour L; $\Delta = 1$ or $\Delta = (\chi/\chi_0)^2$ in the case of TM- or TE-polarization respectively.

If approximation (7) of operators $F_{1,2}$ occurs the boundary conditions (8) result in

$$\begin{aligned} & [\exp(-2g)R-S^-]\,W^+ + [\exp(2g)R-S^+]\,W^- - T^+ + U^+ + \Delta V^- = 0; \\ & [\exp(-2g)R+S^+]\,W^+ - [\exp(2g)R+S^-]\,W^- - T^- - U^- - \Delta V^+ = 0, \end{aligned} \qquad (9)$$

where

$$\begin{aligned} R &= 1+\frac{h^2}{6}\left[4\left(\chi^2+\left(h\frac{\partial k}{\partial s}\right)^2-h\frac{\partial^2 k}{\partial s^2}\right)-k^2\right];\ g=kh/2; \qquad (10) \\ S^{\pm} &= 1+\left(\frac{h^2}{3}\right)\left(4\chi^2-k^2\right)\pm 2g;\, U^{\pm}=\frac{4}{3}h^2\left(\frac{\partial^2 W^+}{\partial s^2}\pm\frac{\partial^2 W^-}{\partial s^2}\right); \end{aligned}$$

$$\begin{aligned} V^{\pm} &= 2h\left(\frac{\partial W^{+}}{\partial n} \pm \frac{\partial W^{-}}{\partial n}\right); \\ T^{\pm} &= -\frac{2}{3}h^{2}\left[\exp(-2g)\frac{\partial^{2}W^{+}}{\partial s^{2}} \pm \exp(2g)\frac{\partial^{2}W^{-}}{\partial s^{2}}\right. \\ &\quad \left. -2h\frac{\partial k}{\partial s}\left(\exp(-2g)\frac{\partial W^{+}}{\partial s} \mp \exp(2g)\frac{\partial W^{-}}{\partial s}\right)\right]. \end{aligned}$$

To derive the required surface integral equations we represent $W(x,y) \equiv W(z,\bar{z})$ as superposition of generalized single- and double-layer potentials (e.g., see [23]):

$$\begin{aligned} W(z,\bar{z}) &= W^{0}(z,\bar{z}) + W^{s}(z,\bar{z}); \quad z = x+iy, \quad \bar{z} = x-iy; \qquad (11) \\ W^{s}(z,\bar{z}) &= \frac{\pi i}{2}\int_{L}\left[j(s)H_{0}^{(1)}(\chi_{0}r) - \chi_{0}m(s)H_{1}^{(1)}(\chi_{0}r)\mathrm{Re}\left(\frac{t-z}{r}e^{-i\psi}\right)\right]ds, \end{aligned}$$

where W^0 and W^s are longitudinal components of excited and scattered field; functions $j(s)$ and $m(s)$ are unknown potential densities; $H_{0,1}^{(1)}(z)$ mean the Hankel functions; $r = |t-z|$ denotes the distance between observation and current points with affixes z and t accordingly.

Function $W(z,\bar{z})$ satisfies all the necessary conditions of the scalar diffraction problem except the boundary ones (9) on the curve L. As it is well known the term W^s is discontinuous function in the complex plane $(x,y) \equiv (z,\bar{z})$. Both this term and some its derivatives jump when z moves across L. In particular, we have

$$\begin{aligned} W^{\pm} &= \pm\pi m(s_0) + W^{0} + W^{s}; \qquad (12) \\ \frac{\partial W^{\pm}}{\partial n_0} &= \mp\pi j(s_0) + \frac{\partial W^{0}}{\partial n_0} + \frac{\partial W^{s}}{\partial n_0}; \\ \frac{\partial^{2} W^{\pm}}{\partial s_0^{2}} &= \pm\pi m''(s_0) + \frac{\partial^{2} W^{0}}{\partial s_0^{2}} + \frac{\partial^{2} W^{s}}{\partial s_0^{2}}. \end{aligned}$$

Here the arc abscissa s_0 corresponds to the point $z \to t_0 \in L$; the superscript $\pm$ is absent in the direct value of corresponding function on L.

Applying formulae (11) and (12) to conditions (9) we derive, after some calculations, the following system of surface integral equations:

$$\begin{aligned} h^{2}\frac{\partial^{2}W^{s}}{\partial s_0^{2}} + A_1 W^{s} + \pi B_1 h j(s_0) + \pi C_1 m(s_0) + \pi D_1 h^{2} m''(s_0) &= E_1; \qquad (13) \\ h\frac{\partial W^{s}}{\partial n_0} + A_2 W^{s} + \pi B_2 h j(s_0) + \pi C_2 m(s_0) + \pi D_2 h^{2} m''(s_0) &= E_2. \end{aligned}$$

Here

$$\begin{aligned}
E_1 &= E_1(s_0) = -\left[h^2\frac{\partial^2 W^0}{\partial s_0^2} + A_1 W^0\right]; \\
E_2 &= E_2(s_0) = -\left[h\frac{\partial W^0}{\partial n_0} + A_2 W^0\right]; \\
A_1 &= A_1(s_0) = T - \frac{th(g)}{2P};\ A_2 = A_2(s_0) = -\frac{1}{\Delta}\left[\frac{1}{2P} + g\right]; \\
B_1 &= B_1(s_0) = \frac{\Delta H}{2P};\ D_1 = D_1(s_0) = \frac{1}{3P}; \\
B_2 &= B_2(s_0) = -D_1;\ D_2 = D_2(s) = -\frac{1}{6\Delta}\frac{H}{P}; \\
C_1 &= C_1(s_0) = \frac{1}{2P}\left[1 + gH\right] + D_1 T; \\
C_2 &= C_2(s_0) = \frac{1}{\Delta}\left[B_2 g + \frac{cth(g)}{2P}\right] + D_2 T; \\
P &= P(s_0) = (th(g)/3 - cth(g))/2; \\
H &= H(s_0) = cth(g) - th(g);\ T = h^2(\chi^2 - k^2/4).
\end{aligned} \tag{14}$$

Let us introduce the following regular functions

$$\begin{aligned}
H_1(z) &= H_1^{(1)}(z) - \frac{2}{\pi i z},\ H_1(0) = 0; \\
H_2(z) &= H_2^{(2)}(z) - \frac{4}{\pi i z^2},\ H_2(0) = \frac{1}{\pi i}.
\end{aligned} \tag{15}$$

Then we can write the normal derivative appearing in Equations (13) as

$$\begin{aligned}
\frac{\partial W^s}{\partial n_0} = & \int_L m'(s)\mathrm{Re}\left[\frac{\exp(i\psi_0)}{i(t-t_0)}\right] ds \\
& + \frac{\pi i}{2}\chi_0 \int_L \left\{ j(s) H_1^{(1)}(\chi_0 r)\mathrm{Re}\left[\frac{t-t_0}{r}\exp(-i\psi_0)\right] \right. \\
& + m(s)\chi_0\left[H_0^{(1)}(\chi_0 r)\cos\left[i(\psi - \psi_0)\right]\right. \\
& \left.\left. - H_2(\chi_0 r)\mathrm{Re}\left[\frac{\overline{t-t_0}}{t-t_0}\exp\left(i(\psi+\psi_0)\right)\right]\right]\right\} ds.
\end{aligned} \tag{16}$$

If we consider the differential operator

$$G = -\left[\exp(2i\psi_0)\frac{\partial^2}{\partial t_0^2} + \exp(-2i\psi_0)\frac{\partial^2}{\partial \bar{t}_0^2}\right], \tag{17}$$

and take into account the equalities [23]

$$\begin{aligned}\nabla^2 &= \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=4\frac{\partial^2}{\partial z\partial\bar{z}}=\frac{\partial^2}{\partial s^2}+\frac{\partial^2}{\partial n^2}-k(s)\frac{\partial}{\partial n}; \qquad (18)\\ \frac{\partial^2}{\partial s^2} &= k(s)\frac{\partial}{\partial n}+2\frac{\partial^2}{\partial t\partial\bar{t}}+G;\ \frac{\partial^2}{\partial n^2}=2\frac{\partial^2}{\partial t\partial\bar{t}}-G,\end{aligned}$$

as well as the fact that function W^s satisfies Helmholtz equation outside L, then, after differentiating and keeping in mind well known properties of cylindrical functions, we are led to formulae

$$\frac{\partial^2 W^s}{\partial s_0^2}=\left(G+k(s_0)\frac{\partial}{\partial n_0}-\frac{\chi_0^2}{2}\right)W^s; \qquad (19)$$

$$\begin{aligned}GW^s = &\int_L\left\{j'(s)\mathrm{Im}\left(\frac{e^{i(2\psi_0-\psi)}}{t-t_0}\right)-j(s)k(s)\mathrm{Re}\left(\frac{e^{i(2\psi_0-\psi)}}{t-t_0}\right)\right.\\ &\left.-m'(s)\mathrm{Im}\left(\frac{e^{2i\psi_0}}{(t-t_0)^2}\right)\right\}ds-\pi i\left(\frac{\chi_0}{2}\right)^2\\ &\times\int_L\left\{j(s)H_2(\chi_0 r)\mathrm{Re}\left(\frac{\overline{t-t_0}}{t-t_0}e^{2i\psi_0}\right)\right.\\ &+\chi_0 m(s)\left[\left(H_1^{(1)}(\chi_0 r)-2\frac{H_2(\chi_0 r)}{\chi_0 r}\right)\mathrm{Re}\left(\frac{\overline{t-t_0}}{r}e^{i\psi}\frac{\overline{t-t_0}}{t-t_0}e^{2i\psi_0}\right)\right.\\ &\left.\left.+H_1^{(1)}(\chi_0 r)\mathrm{Im}\left(\frac{\overline{t-t_0}}{r}e^{i\psi}\right)\mathrm{Im}\left(\frac{\overline{t-t_0}}{t-t_0}e^{2i\psi_0}\right)\right]\right\}ds.\end{aligned}$$

Substituting (11), (16) and (19) into (13), we obtain the system of singular integro-differential equations with extracted Cauchy type singularities. Also the obtained in such a way surface integral equations have the weak (logarithmic) singularities due to presence of function $H_0^{(1)}(z)$ in their kernels.

5. SURFACE INTEGRAL EQUATIONS BASED ON ZERO-ORDER BOUNDARY CONDITIONS (MODEL III)

If we neglect derivative $\partial^2/\partial s^2$ in formulae (5) then we have $2hQ\equiv 2ihq$, $q\equiv q(s)=(\chi^2-(k(s)/2)^2)^{1/2}$. Thus operators $F_{1,2}$ degenerate to functions: $F_1=f(s)\equiv 2hq/\sin(2hq)$; $F_2=f(s)\cos(2hq)$, and (8) transforms to the familiar zero-order boundary conditions [6].

We can perform all the mentioned above calculation to derive two connected surface integral equations which correspond to the zero-order boundary conditions. Of course, such the system becomes simpler and looks like

$$\begin{aligned} W^s(s_0) - \pi h A_1 j(s_0) - \pi B_1 m(s_0) &= -W^0(s_0), \\ h\frac{\partial W^s(s_0)}{\partial n_0} - \pi h A_2 j(s_0) - \pi B_2 m(s_0) &= -h\frac{\partial W^0(s_0)}{\partial n_0}. \end{aligned} \tag{20}$$

here

$$\begin{aligned} A_1 &= \frac{2\Delta}{P}; \qquad B_1 = \frac{kh + fsh(kh)}{P}; \\ A_2 &= \frac{kh - fsh(kh)}{P}; \qquad B_2 = \frac{2(h\chi)^2}{\Delta P}; \end{aligned} \tag{21}$$

$P \equiv P(s_0) = f\,[ch(kh) - \cos(2hq)]$; $f \equiv f(s_0) = 2hq/\sin(2hq)$; $2hq \equiv 2hq(s_0) = \sqrt{(2h\chi)^2 - (kh)^2}$. Note that both function $W^s(s_0)$ and its normal derivative are assigned by formulae (11) and (16).

If, in addition, we neglect the curvature $k = k(s)$ of contour L in expressions (21) then we obtain

$$A_1 = \Delta ctg(\chi h)/\chi h;\ A_2 = B_1 = 0;\ B_2 = \chi h ctg(\chi h)/\Delta. \tag{22}$$

In this case the surface integral Equations (20) are outcome of application the formulae (11) to the presented in paper [6] boundary conditions for penetrable (dielectric) sheet.

6. NUMERICAL REALIZATION OF THE MODELS

At first we note that all the surface integral equations appeared in models I–III are singular integro-differential ones. They have the Cauchy type kernels as well as logarithmic singularities. Such singularities should be taken into account while constructing numerical algorithm for solution of SIE. Here we give only the numerical solution of system (20). Other two cases are similar (their regular kernels make some difference). All the details for the systems (2) and (13) are presented in [23, 27].

Let the parametric equation of curve L is given by complex-valued function $t = t(\tau),\ 0 \le \tau \le 2\pi$. We introduce the following unknown functions:

$$\varphi_1(\tau) = j\,[s(\tau)]\,t'(\tau)|;\ \varphi_2(\tau) = m\,[s(\tau)] \tag{23}$$

and use N order trigonometric polynomial

$$\varphi_{1,2}(\tau) \approx \frac{1}{N}\sum_{i=1}^{N}\varphi_{1,2}(\tau_i)\sin\left[n(\tau-\tau_i)\right]\cos\left(\frac{\tau-\tau_i}{2}\right) \tag{24}$$

to fit the continuous 2π-periodic functions $\varphi_{1,2}(\tau)$ in $N = 2n$ nodes $\tau_i = i\pi/n,\ i = 1,\cdots,N$.

Then by passing to the interval $[0,2\pi]$ we rewrite system (20) as the standardized one and apply to the appeared integrals the interpolation quadrature formulae for even number of nodes [23]:

$$\int_0^{2\pi} f(\tau)\ln\left|\sin\frac{\tau-\tau_0}{2}\right| d\tau \approx -\frac{\pi}{n}\sum_{i=1}^{N} f(\tau_i)S_1(\tau_i,\tau_0), \tag{25}$$

$$S_1(\tau_i,\tau_j) = \ln 2 + \frac{(-1)^{i-j}}{N} + \sum_{i=1}^{n-1}\cos(i(\tau_i-\tau_j))/i;$$

$$\int_0^{2\pi} f'(\tau)\cot\left(\frac{\tau-\tau_0}{2}\right) d\tau \approx -\frac{\pi}{n}\sum_{i=1}^{N} f(\tau_i)S_2(\tau_i,\tau_0),$$

$$S_2(\tau_i,\tau_j) = \begin{cases} n^2, & i=j; \\ \left((-1)^{i-j}-1\right)/2\sin^2\left(\dfrac{\tau_i-\tau_j}{2}\right), & i\neq j; \end{cases}$$

$$\int_0^{2\pi} f(\tau)d\tau \approx \frac{\pi}{n}\sum_{i=1}^{N} f(\tau_i).$$

Then assuming $\tau_0 = \tau_j,\ j = 1,\cdots,N$ we arrive to the system of $2N$ linear algebraic equations that determine of $2N$ unknowns $\varphi_{1i} = \varphi_1(\tau_i)$, $\varphi_{2i} = \varphi_2(\tau_i)$:

$$\begin{aligned} &\frac{1}{n}\sum_{i=1}^{N}\{R_{11}(\tau_i,\tau_j)\varphi_{1i}+R_1(\tau_i,\tau_j)\varphi_{2i}\}-\kappa(\tau_j)A_1(\tau_j)\varphi_{1j}-B_1(\tau_j)\varphi_{2j} \\ =\ &\frac{1}{\pi}E_1(\tau_j), \qquad (26) \\ &\frac{1}{n}\sum_{i=1}^{N}\{Q_2(\tau_i,\tau_j)\varphi_{1i}+R_{22}(\tau_i,\tau_j)\varphi_{2i}\}-\kappa(\tau_j)A_2(\tau_j)\varphi_{1j}-B_2(\tau_j)\varphi_{2j} \\ =\ &\frac{1}{\pi}E_2(\tau_j). \end{aligned}$$

Here

$$\begin{aligned}
R_{11}(\tau_i,\tau_j) &= S_1(\tau_i,\tau_j)+Q_1(\tau_i,\tau_j);\quad \kappa(\tau_j)=h/|t'_j|; \qquad (27)\\
R_{22}(\tau_i,\tau_j) &= \kappa(\tau_j)\Big[\Big(\chi_0^2\mathrm{Re}(t'_i,\bar{t}'_j)S_1(\tau_i,\tau_j)-S_2(\tau_i,\tau_j)\Big)/2+R_2(\tau_i,\tau_j)\Big];\\
Q_1(\tau_i,\tau_j) &= \frac{\pi i}{2}H_0^{(1)}(\chi_0 r_{ij})+\ln\left|\sin\frac{\tau_i-\tau_j}{2}\right|;\\
r_{ij} &= |t(\tau_i)-t(\tau_j)|;\quad Q_1(\tau_j,\tau_j)=\frac{\pi i}{2}-C-\ln\left(\chi_0\left|t'_j\right|\right);\\
R_1(\tau_i,\tau_j) &= \chi_0\frac{\pi i}{2}H_1^{(1)}(\chi_0 r_{ij})\mathrm{Im}\left(\frac{\overline{t_i-t_j}}{r_{ij}}t'_i\right);\\
R_1(\tau_j,\tau_j) &= \frac{1}{2}\mathrm{Im}\left(\frac{t''(\tau_j)}{t'(\tau_j)}\right)=\frac{1}{2}k(\tau_j)\left|t'_j\right|;\\
Q_2(\tau_i,\tau_j) &= \kappa(\tau_j)\chi_0\frac{\pi i}{2}H_1^{(1)}(\chi_0 r_{ij})\mathrm{Im}\left(\frac{t_i-t_j}{r_{ij}}\overline{t'_j}\right);\\
Q_2(\tau_j,\tau_j) &= \frac{1}{2}\kappa(\tau_j)\mathrm{Im}\left(\frac{t''(\tau_j)}{t'(\tau_j)}\right);\\
R_2(\tau_i,\tau_j) &= \frac{\chi_0^2}{2}\left[\mathrm{Re}(t'_i\overline{t'_j})\left(\frac{\pi i}{2}H_0^{(1)}(\chi_0 r_{ij})+\ln\left|\sin\frac{\tau_i-\tau_j}{2}\right|\right)\right.\\
&\quad\left.+\frac{\pi i}{2}H_2^{(1)}(\chi_0 r_{ij})\mathrm{Re}\left(\frac{\overline{t_i-t_j}}{t_i-t_j}t'_it'_j\right)\right]-\frac{0.25}{\sin^2((\tau_i-\tau_j)/2)};\\
R_2(\tau_j,\tau_j) &= \mathrm{Re}\left(\frac{t'''_j}{3t'_j}-\frac{1}{2}\left(\frac{t''_j}{t'_j}\right)^2-\frac{1}{6}\right)+\frac{\left(\chi_0\left|t'_j\right|\right)^2}{2}\\
&\quad\times\left(\frac{1+\pi i}{2}-C-\ln\left(\chi_0\left|t'_j\right|\right)\right);\\
E_1(\tau_0) &= -W^0(s(\tau_0));\quad E_2(\tau_0)=-\frac{\partial W^0(s(\tau_0))}{\partial n_0}\\
t_i &= t(\tau_i);\quad t'_i=t'(\tau_i);\quad t''_i=t''(\tau_i);\quad t'''_i=t'''(\tau_i),
\end{aligned}$$

$C = 0.5772156649\cdots$ is the Euler constant, functions $A_{1,2}(\tau)$ and $B_{1,2}(\tau)$ are defined in (21).

The following limiting relations at $\tau\to\tau_0$ are taken into account in formulae (27):

$$\chi_0\frac{\pi i}{2}H_1^{(1)}(\chi_0 r)\mathrm{Im}\left(\frac{\overline{t-t_0}}{r}t'\right)\xrightarrow{\tau\to\tau_0}\frac{1}{2}\mathrm{Im}\left(\frac{t''_0}{t'_0}\right)$$

$$\xleftarrow{\tau_0 \leftarrow \tau} \chi_0 \frac{\pi i}{2} H_1^{(1)}(\chi_0 r) \mathrm{Im}\left(\frac{t - t_0}{r} \overline{t_0'}\right); \tag{28}$$

$$\frac{\pi i}{2} H_0^{(1)}(\chi_0 r) + \ln\left|\sin\frac{\tau - \tau_0}{2}\right| \xrightarrow{\tau \to \tau_0} \frac{\pi i}{2} - C - \ln\left(\chi_0 \left|t_0'\right|\right).$$

Note that we avoid in such a way the non-trivial analytical regularization of singular operators in surface integral Equations (2), (13), and (20). Nevertheless all the existing singularities (the logarithmic one including) are taken into account due to application the special quadrature formulae (25). Thus we can realize all the models mentioned above.

7. FAR FIELD CALCULATION

Taking into account definition (23) we rewrite the second line of representation (11) as the standardized one. Then we have

$$W^s(z, \overline{z}) = i\frac{\pi}{2} \int_0^{2\pi} \left[\varphi_1(\tau) H_0^{(1)}(\chi_0 r) - \chi_0 \varphi_2(\tau) H_1^{(1)}(\chi_0 r) \mathrm{Im}\left(\frac{t-z}{r}\overline{t'}\right)\right] d\tau,$$
$$r = |t - z|. \tag{29}$$

Therefore, we can calculate the longitudinal component of the scattering field at an arbitrary point z of the complex plane (x, y) applying the last (Gaussian) quadrature rule in (25) to (29).

Let us introduce the far field diagram (scattering pattern) $D(\varphi)$, $0 < \varphi < 2\pi$ by means of equality [28]

$$W^s(z, \overline{z}) \xrightarrow{\rho \to \infty} \sqrt{\frac{2}{\pi \rho \chi_0}} \exp\left[i(\chi_0 \rho - \pi/4)\right] D(\varphi), \quad \rho = |z|. \tag{30}$$

Using (29) and (30) and utilizing the last line of (26), we obtain next expression for calculation of the scattering pattern:

$$\begin{aligned} D(\varphi) \quad = \quad & i\frac{\pi^2}{N} \sum_{k=1}^{N} \left[\varphi_{1k} + i\chi_0 \mathrm{Im}\left(t_k' \exp(-i\varphi)\right) \varphi_{2k}\right] \\ & \times \exp\left[-i\chi_0 \mathrm{Re}(t_k \exp(-i\varphi))\right]. \end{aligned} \tag{31}$$

Taking into account the following definition of a total radar cross section $\sigma(\varphi)$

$$2\pi\rho \left|\frac{W^s}{W^0}\right|^2 \xrightarrow{\rho \to \infty} \sigma(\varphi), \tag{32}$$

we can easily determine that

$$\sigma(\varphi)/\lambda = 2|D(\varphi)|^2/\pi, \tag{33}$$

where λ is the length of incident wave outside a target.

The calculation is slightly different in the case of E- and H-polarization when model I is considered. Then instead of (31) we have [23]

$$D_E(\varphi) = \frac{4\pi}{N} K \sum_{k=1}^{N} E_z\left(s(\tau_k)\right) |t'_k| \exp\left[-i\chi_0 \mathrm{Re}(t_k \exp(-i\varphi))\right]; \tag{34}$$

$$\begin{aligned} D_H(\varphi) = -i\frac{4\pi}{N} K \sum_{k=1}^{N} & \left[\varphi_{1k}\sin(\varphi) - \varphi_{2k}\cos(\varphi)\right] \\ & \times \exp\left[-i\chi_0 \mathrm{Re}(t_k \exp(-i\varphi))\right], \end{aligned}$$

where $\varphi_1(\tau) = E_x(s(\tau))\,|t'|$, $\varphi_2(\tau) = E_y(s(\tau))\,|t'|$, K is defined in (1).

This completes the numerical algorithm construction for considered surface integral equations. The consideration of the case when curve L is open ($a \neq b$) can be performed similarly.

8. COMPARISON OF THE RESULTS

This section presents an example to demonstrate the validity of the proposed numerical algorithm as well as to compare the considered electrodynamic models of thin cylindrical shell.

Let the following parametric equation of curve L in the basic coordinate system XOY is given:

$$t(\tau) = c\left[\cos(\tau) + i\varepsilon\sin(\tau)\right]; \quad 0 \leq \tau < 2\pi, \tag{35}$$

where $2c$ is the ellipse axis, ε is the axes ratio.

The material of target is dielectric (conductivity $\sigma = 0$, permeability $\mu = \mu_0$, wave number χ is real). The excited TM- or TE-polarized plane wave is of unit amplitude and makes the angle $\varphi_0 = \pi$ with X-axis. Then particularly in (27) we have

$$\begin{aligned} W^0(s_0) &= \exp\left[-i\chi_0 \mathrm{Re}(t_0 \exp(-i\varphi_0))\right]; \\ \frac{\partial W^0(s_0)}{\partial n_0} &= -i\chi_0 h \mathrm{Im}\left(\frac{t'_0}{|t'_0|}\exp(-i\varphi_0)\right) W^0(s_0). \end{aligned}$$

The normalized backscattering width σ/λ is presented in Tables 1 and 2 for different models and both cases of polarization. Two first columns of these Tables contain geometrical and physical parameters of the problem. We made all the calculations at two numbers of nodal points N (see (26), for example). We putted $N = 31$, $N = 51$ for model I, and $N = 30$, $N = 50$ for models II, and III. Then we observed that four-five significant figures coincide in the results mantissa (we did not use double precision in the computer programs). Thus we have completely stable and robust algorithms for all the considered models.

Table 1.

Wave number and size	ε	Model I		Model II		Model III	
		E-case	*H*-case	*E*-case	*H*-case	*E*-case	*H*-case
$c=\lambda/\pi$	1.00	0.5697(-3)	0.5291(-3)	0.5701(-3)	0.5290(-3)	0.5646(-3)	0.5264(-3)
$h=10^{-3}\lambda/4\pi$	0.75	0.3429(-3)	0.2597(-3)	0.3432(-3)	0.2597(-3)	0.3394(-3)	0.2603(-3)
$\chi=5\chi_0$	0.50	0.1859(-3)	0.8918(-4)	0.1860(-3)	0.8912(-4)	0.1802(-3)	0.8891(-4)
$c=\lambda/\pi$	1.00	0.8918(-5)	0.8377(-5)	0.8925(-5)	0.8376(-5)	0.8400(-5)	0.8175(-5)
$h=10^{-3}\lambda/4\pi$	0.75	0.5276(-5)	0.4283(-5)	0.5282(-5)	0.4283(-5)	0.4819(-5)	0.4469(-5)
$\chi=2\chi_0$	0.50	0.2801(-5)	0.1651(-5)	0.2803(-5)	0.1649(-5)	0.2157(-5)	0.1728(-5)
$c=\lambda/\pi$	1.00	0.8894(-3)	0.8421(-3)	0.8965(-3)	0.8407(-3)	0.8595(-3)	0.8466(-3)
$h=10^{-2}\lambda/4\pi$	0.75	0.5379(-3)	0.4322(-3)	0.5440(-3)	0.4317(-3)	0.4922(-3)	0.4423(-3)
$\chi=2\chi_0$	0.50	0.2934(-3)	0.1663(-3)	0.2958(-3)	0.1647(-3)	0.2381(-3)	0.1760(-3)
$c=\lambda/\pi$	1.00	0.6248(-2)	0.5999(-2)	0.6341(-2)	0.5968(-2)	0.6027(-2)	0.5994(-2)
$h=10^{-2}\lambda/2\pi$	0.75	0.3942(-2)	0.3074(-2)	0.4027(-2)	0.3066(-2)	0.3683(-2)	0.3109(-2)
$\chi=\sqrt{5}\chi_0$	0.50	0.2261(-2)	0.1151(-2)	0.2296(-2)	0.1132(-2)	0.1953(-2)	0.1179(-2)
$c=\lambda/\pi$	1.00	0.9617(-3)	0.9222(-3)	0.9780(-3)	0.9186(-3)	0.8966(-3)	0.9411(-3)
$h=10^{-2}\lambda/2\pi$	0.75	0.5823(-3)	0.4874(-3)	0.5962(-3)	0.4854(-3)	0.4869(-3)	0.5223(-3)
$\chi=1.6\chi_0$	0.50	0.3180(-3)	0.2014(-3)	0.3241(-3)	0.1965(-3)	0.2078(-3)	0.2374(-3)
$c=\lambda/\pi$	1.00	0.9690(-2)	0.9654(-2)	0.1048(-1)	0.9243(-2)	0.7637(-2)	0.9963(-2)
$h=10^{-1}\lambda/4\pi$	0.75	0.6220(-2)	0.5297(-2)	0.6976(-2)	0.5069(-2)	0.4419(-2)	0.6008(-2)
$\chi=\sqrt{2}\chi_0$	0.50	0.3642(-2)	0.2334(-2)	0.4056(-2)	0.2017(-2)	0.1871(-2)	0.3022(-2)
$c=0.275\lambda$	1.00	0.5012(0)	0.5778(0)	0.7067(0)	0.4229(0)	0.1679(0)	0.4101(0)
$h=0.025\lambda$	0.75	0.6143(0)	0.3624(0)	0.6932(0)	0.2735(0)	0.5149(0)	0.2610(0)
$\chi=2\chi_0$	0.50	0.5434(0)	0.1884(0)	0.5187(0)	0.1624(0)	0.4905(0)	0.1604(0)

Parameters $c = \lambda/\pi$; $h = 10^{-2}\lambda/2\pi$; $\chi = 1.6\chi_0$; $\varepsilon = 1$ in Table 1 correspond to the input data of paper [2] (the case of H-polarized plane wave excitation of a circular dielectric shell was considered, and frequency dependencies of normalized backscattering width was plotted there). In this case we observe good agreement of our data with the cited calculation obtained from the moment method. Note that the last one coincides with exact solution of the problem.

Parameters $c = 0.275\lambda$; $h = 0.025\lambda$; $\chi = 2\chi_0$; $\varepsilon = 1$ in Table 1 are the same as in papers [21, 22]. Numerical results in this papers were obtained for TM-case [21] and TE-case [22] with the moment method. Our comparison shows that maximal error of mode I depend on the wave polarization: it is less than 1% in TM-case and run up to 5% in TE-case [23]. As one can see model III is inapplicable in this case. The corresponding calculations are not satisfactory both for E- and H-polarized excitation. Also the results obtained within model II network should be improved, for instance, by minimax theory application instead of approach (7) (e.g., see [27]).

Note that both of the mentioned cases were considered in papers [2, 21, 22] by the volume integral equations technique for arbitrary shaped cylindrical shell and the calculations were performed using the moment method. Of course, the exact solution for the circular geometry ($\varepsilon = 1$) also is available in the literature [29]. So we really have the precise solution of this problem for an arbitrary geometry. It was used as a reference one during the mentioned above comparison of our numerical results.

Table 2.

$c=\lambda/\pi$ $h=10^{-2}\lambda/4\lambda$	ε	Model I		Model II	
		E-case	H-case	E-case	H-case
$\chi = 0.5\chi_0$	1.00	0.5575(-6)	0.7019(-6)	0.5578(-6)	0.7025(-6)
	0.75	0.3288(-6)	0.6346(-6)	0.3291(-6)	0.6345(-6)
	0.50	0.1739(-6)	0.6374(-6)	0.1739(-6)	0.6363(-6)
$\chi = 0.25\chi_0$	1.00	0.8711(-6)	0.2142(-5)	0.8716(-6)	0.2147(-5)
	0.75	0.5137(-6)	0.4223(-5)	0.5142(-6)	0.4219(-5)
	0.50	0.2717(-6)	0.7987(-5)	0.2717(-6)	0.7968(-5)
$\chi = 0.1\chi_0$	1.00	0.9714(-6)	0.8470(-5)	0.9720(-6)	0.8559(-5)
	0.75	0.5729(-6)	0.5635(-4)	0.5734(-6)	0.5621(-4)
	0.50	0.3029(-6)	0.1731(-3)	0.3030(-6)	0.1725(-3)
$\chi = 0.05\chi_0$	1.00	0.9861(-6)	0.2521(-3)	0.9868(-6)	0.2527(-3)
	0.75	0.5816(-6)	0.2025(-3)	0.5821(-6)	0.1829(-3)
	0.50	0.3075(-6)	0.2989(-3)	0.3076(-6)	0.2959(-3)

Table 2 consist some computation of the normalized backscattering width when penetrable shell is placed in optical denser medium. Such a case is especially interesting for the non-destructive testing purpose. However the corresponding accurate calculations which are suitable for testing of our numerical results in this case are not available in the literature. Nevertheless we see very good agreement for both the models I and II realization. Unfortunately the model III is inapplicable in this case. Note that a penetrable target serves as good polarization filter when its wave number is essential less than the medium one (see the last three lines of Table 2).

To understand the cause of the model III inapplicability during some considered cases we recall that surface integral Equations (20) correspond to the zero-order boundary conditions considered in paper [6]. It was settled there that such conditions are valid in the case

Table 3.

Wave number and size	$2h\chi_0$	$(\chi_0/\chi)^2$
$c=\lambda/\pi$, $h=10^{-3}\lambda/4\pi$, $\chi=5\chi_0$	0.1000(-2)	0.4000(-1)
$c=\lambda/\pi$, $h=10^{-2}\lambda/2\pi$, $\chi=\sqrt{5}\chi_0$	0.2000(-1)	0.2000(0)
$c=0.275\lambda$, $h=0.025\lambda$, $\chi=2\chi_0$	0.3142(0)	0.2500(0)

of thin-layer ($2h\chi_0 \ll 1$) and small-refraction angle ($(\chi_0/\chi)^2 \ll 1$) limits. The first limitation regards to target thickness. The second one implies that the wave inside shell medium propagates approximately in the direction normal to curve L. These specific values are shown in Table 3 for some parameters appearing in Table 1. We can see that only the first set of wave number and size satisfies the necessary conditions of model III validity. Every next set of input parameters (from top to down of Table 3) satisfy such conditions worse.

The considered above mathematical models of a thin target give effective approximate solution of the problem in both cases of polarization. Their numerical realization is based mainly on the previously developed special quadrature formulae. In this sense the considered models allow to extend the worked out theory of singular integral equations to a new class of physical objects. All these models are effective if the target size (to be more correct the length of the contour L) is comparable with or less then the wavelength in surrounding medium. It is essential that the models I and II are suitable at arbitrary incidence of the illuminating wave. All these models assume that the target is thin. However the performed calculations show some difference in their application. Model I among them has the widest range of application with regard to the thickness of the target (see Table 1). Unfortunately this model leads to different kind of surface integral equations depending on the polarization. Furthermore its generalization to the system of thin targets is not easy.

Models II and III are attractive due to their universality regarding the polarization of incident wave. Also we can use them to build a variant of diffraction theory for several dielectric objects in a similar way as it is made for perfectly conducting infinitely thin obstacles (screens). Model III based on the zero-order boundary conditions is the simplest one. However its range of application with respect of optical thickness of the target is essentially narrower in comparison with model II that utilize the high-order boundary conditions.

9. CONCLUSION

We apply some boundary conditions for total electromagnetic field to the analysis of wave scattering by a thin penetrable cylindrical shell in homogeneous media. These conditions are determined on the target median L and contain of two differential operators $F_{1,2}$. Two different approximations of these operators in the region of small eigenvalues are considered. One of them leads to the second-order boundary conditions and the other kind of approximation results the zero-order boundary conditions. Both of them coupled with representation of the scattered field as a superposition of single and double-layered potentials lead to the system of two singular integro-differential equations. The type of these equations depends neither on the polarization of excitation nor the physical properties of the obstacle. The obtained SIE system admits an effective numerical solution by mechanical quadrature method which avoids the necessity of analytical regularization of the singular integrals and allows keeping the universality of the approach. The obtained in such a way numerical results are compared with similar calculation based on the surface integral equations derived as an approximation of the rigorous volume integral ones. Some analysis of three mentioned mathematical models is given as to the range of their application.

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