# STRATIFIED ONTOLOGIES: THE CASE OF PHYSICAL OBJECTS

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## 1. INTRODUCTION

When modelling a domain, it is often the case that certain individuals are represented as belonging to multiple categories, generating therefore "tangled" hierarchies:

1. A physical object can be seen as an amount of matter
2. A hole can be seen as a region of space
3. A building can be seen as a place
4. A biological organ can be seen as an aggregate of molecules
5. An association can be seen as a group of people
6. A physical object can be seen as a temporal process

This phenomenon is quite common: the examples above have been taken from existing top-level ontologies used in practice, and they are responsible, in our opinion, of many situations of tangleness, confusion, and lack of semantic rigour. We advocate in the present paper a design principle aimed to clarify such situations and produce more reusable and well-founded ontologies. It can be stated as follows:

> *Ontological Distinction Principle:* Classes corresponding to different identity criteria must be disjoint.

A rigorous definition of the notion of identity criteria is outside the scope of the present paper. It will suffice here to establish that an identity criterion for a class C allows us to recognize two individuals as *being the same instance of C* even if they may differ in some properties. This criterion can be intended as isolating the "core properties" considered to be invariant for an instance of the class.

Identity criteria must be established beforehand in order to draw ontological distinctions. Consider for instance a particular glass: we are able to recognize *the same glass* in another situation by using specific criteria, even if its spatial location has changed or a tiny piece has been lost. On the other hand, when focusing our attention on the matter our glass is made of, we shall use *different criteria* to recognize the existence of that particular amount of matter, which will be the same as long as no piece of it is removed independently of any properties regarding shape, material connection and so on. Even if we are not able to precisely state what the identity criteria are in the two cases (as it would be a hard task, in general), the fact that the two criteria are different allows us to conclude that the class “glass” is disjoint from the class “piece of matter”, making justice of the everyday observation that, when a glass ceases *to exist* after falling on the ground, the matter it is made of is still there.

Applying these considerations to the other examples mentioned above, we can observe that a hole in a piece of cheese may be filled with some material and then disappear, or simply be moved to some other region of space, while the original region of space it was occupying is still there[1]. A similar argument holds for a building, which can be destroyed and reconstructed in a completely different way in the same place, or maybe moved to a different place. Again, a biological organ like a kidney is not identical to a set of molecules; two different associations may involve the same group of persons, and the same physical object can be associated to different processes (think of

[1] For a nice discussion on ontological properties of holes and other "superficial entities", as the authors call them, see [4].

two different possible trajectories of a solid ball)[1].

In all the examples above, the Ontological Distinction Principle tells us that the classes involved are disjoint[2]. In our opinion, such a principle helps us a lot in identifying the relevant classes of a given domain, but of course it doesn't tell us anything about the relevant relations holding among them. To this purpose, it may be useful to identify the *ontological dependence relationships*[3] holding among classes, which give us a sort of *a priori structure* of our domain. We can observe for instance that the concrete existence[4] of a material object necessarily implies the concrete existence of a specific piece of matter, as well as the concrete existence of the latter implies the concrete existence of a region of space: ontological dependence induces therefore a *stratification* on the relevant classes. We adopt the term *stratified ontology* to denote an ontology where classes corresponding to different identity criteria are kept carefully disjoint and represent the roots of separate hierarchies called *strata*, and where the ontological dependencies among strata are made explicit [9].

As well as identity, the issue of dependence belongs to the field of Formal Ontology. We have discussed elsewhere[12] the role played by such a philosophical discipline in the practice of knowledge engineering. Dependence, in particular, has been discussed in detail in [24]. The notion of *ontological stratum* has been discussed in [21][5].

Despite their apparent simplicity, the adoption of the methodological principles introduced above may turn to be rather difficult. In this paper, we limit ourselves to the case of simple physical objects like mechanical components, with a twofold objective. On one hand, we present an ontological theory based on a fundamental distinction between objects and their *substrates*, like chunks of matter and regions of space; its intended purpose is to put the basis of a general ontology of space, matter and physical objects, to be mainly used in the domain of mechanical artifacts. On the other hand, we try to demonstrate and discuss in practice the design principles we have adopted, using the domain of physical object as a test case.

What we present is a particular logical theory which is also an *ontological theory*, in the sense discussed in [13]. It is a *rich* theory in terms of axioms and definitions (in the spirit of [14]), since its main purpose is to convey meaning, in such a way as to characterize the intended models of a logical vocabulary suitable to be used in concrete applications; it is not intended therefore to be directly implemented in a reasoning system.

In the next section we discuss the general assumptions underlying our approach; in Section 3 we introduce our mereological framework, while in Section 4 we present an axiomatic characterization of space which is innovative in many points with respect to theories adopting the topological connection ‘C’ as a primitive [22,1]; in section 5 we introduce matter and objects as separate subdomains, and discuss the various relations holding between objects, matter, and space; finally, in section 6 we show how various useful ontological distinctions among physical objects can be made within our framework.

## 2. General Assumptions About Physical Objects

According to the principles discussed above, our basic assumption is a sharp distinction between physical objects and their substrates. The simplified world we have in mind is characterised by the concrete existence of a certain quantity of matter (all of the same kind), that can assume different configurations within a given tridimensional space. We limit ourselves to the properties of physical objects bound to their *spatio-material configuration*, assuming that an object can be described by the set of its admittable spatio-material configurations. A particular solid cube, for instance, may be described by the class of all (roughly) cube-shaped configurations that involve the same amount of matter; according to the particular identity criteria we are interested to model, this class

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[1] Compare with CYC's ontology [17], where physical objects are assimilated to processes in overt violation of the classical distinction between “continuants” and “occurrents” [24]

[2] Just as a mental exercise, it may be useful to check whether the same conclusion holds for the couples “Student/Person” or “Passenger/Person”. In the former case, it is quite obvious to adopt the same identity criteria for both classes; the latter case is however more subtle, as discussed in [29].

[3] We refer here in particular to *rigid dependence* among classes: we say that a class A is rigidly dependent on B if the concrete existence of an element of A always implies the concrete existence of an element of B

[4] *Concrete existence* is not intended here as related to logical quantification, but rather to ontological status, like existence in a particular world; such a status may correspond to an ad-hoc logical predicate as discussed in [16]

[5] In this work, however, the importance of identity criteria is not so much underlined.

of admitted configurations may also include the case where the cube has “lost” a tiny corner, and still it is considered to be the same cube (Fig. 1).

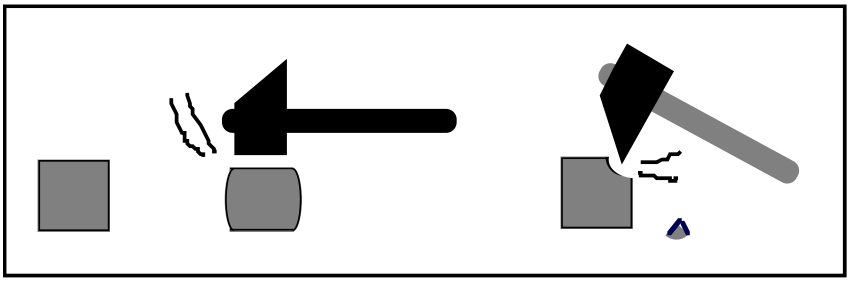

Fig. 1. Still the same cube?

Let us consider these assumptions in greater detail. The first claim we make is that we need to distinguish between physical objects and the space they occupy. An important reason for this distinction is related to the desire to represent *movement*: in order to tell that something has moved across space, we need to recognize two *different* regions of space occupied by *the same* object. Indeed, current AI approaches dealing with the representation of movement, like [23] or [8] postulate an ontological distinction between objects and regions for this purpose.

Our second claim regards a further distinction between objects and chunks of matter. As explained above, the reason of this choice lies in the different identity criteria of the entities involved. Without taking matter into account, no difference can be made between a material body and a hole, or between an imaginary boundary dividing two adjacent parts of a body and a physical boundary marked by matter discontinuity.

Either regions of space and chunks of matter have to be distinguished because only a chunk of matter can move across space or change its shape.

What we propose is to carefully distinguishing among four different subdomains: a set ‘R’ of regions of space, a set ‘M’ of chunks of matter, a set ‘OB’ of physical objects, and a set ‘S’ of system states, intended as particular spatial configurations of the elements of ‘M’ (we assume here a restricted notion of state).

*Regions of space* are intended to be either *self-connected tridimensional regions* of a Euclidean space or mereological sums of such regions. Following [22], we do not distinguish between open and closed regions, focusing on so-called *regular regions*. Moreover, nothing analogous to the mathematical notion of point, line or surface is assumed in the domain.

*Chunks of matter* are either single integral pieces of matter or finite mereological sums of such pieces. Only one kind of (incompressible) material is assumed. The set ‘M’ is also called a *material system.*

*Physical objects* (or simply *objects*) require a preliminary clarification of the notion of *state*, based on the relationship between matter and space. Following [9] (who builds up on [18,19]), we assume two primitive parthood relations for matter and space, generating two separate mereological lattices. A *state* is seen then as a homomorphism, which establishes a relation between the two lattices by taking chunks of matter and returning the region they occupy. Such a homomorphism represents a possible *spatial configuration* of our material system. Notice that we do not make a distinction between two states were only immaterial entities have changed their locations; as a conseguence, we suppose here that every immaterial object (e.g., a crack in a wall) depends on a material entity (the wall). A more general notion of state could be used, but it doesn’t seem very interesting for our task, since our simple notion is enough to express some useful properties of objects we are interested here, such as rigidity and integrity**.**

A crucial aspect of the axiomatization described in section 5 is that states are included in the domain, as current practice in situation calculus [20]. The relationship between matter and space is described by means of the primitive ternary predicate ‘LOC’, that gives the location of a chunk of matter in a particular state. Physical objects are seen as entities related to space and matter by means of *dependence relations*, whose identity criteria are different from space and matter. In fact, the latter are considered as *substrates*, in the sense that they have to exist (at least space, usually matter, too) in order to make possible the existence of a physical object. Moreover, the concrete existence of substrates does not depend on the particular state we consider, while this is the case of physical objects: for example, if *s* is the state corresponding to a (completely) broken glass, it is plausible to assume that the glass doesn’t exist in that state, while its matter does; we say that an object $x$ exists in a state $s$ if there exists a region $r$ where $x$ is localized in the state $s$ ($\exists r\text{LOC}rxs$).

Looking now at the relationship between objects and space, it is quite natural to extend the 'LOC' relation to account for the location of objects in a particular state, and to introduce a similar relation 'MAT' that gives the chunk of matter constituting an object in a particular state. Together, the two relations 'LOC' and 'MAT' completely specify the spatio-material behaviour of a physical object within a material system, by giving all its *admittable* spatial and material extensions. Once these two relations are given, then a physical object can be *recognized*, in the sense that a particular spatial pattern assumed by a particular amount of matter can be ascribed to a particular object. Notice that a given physical object may have no matter associated in a particular state. In fact, we define an *immaterial object* as a physical object which never has matter associated: holes and boundaries, as we shall see, are of this kind.

A further difference between substrates and physical objects regards their mereological properties. An extensional parthood relation is assumed both for regions of space and chunks of matter, such that they are always identical to the sum of their parts independently of the particular state. For physical objects, on the other hand, the very notion of "part" becomes more problematic, since we must assume that they can loose or acquire parts when the state of the system changes. We introduce therefore a notion of *contingent* part of a physical object relativized with respect to a particular state. Two distinct objects may happen to have the same contingent parts in a particular state and therefore *coincide* in that state, without being identical.

In conclusion, we model physical objects as entities depending on a spatio-material substrate, that may or may not maintain their identity when their spatio-material properties change: our goal is to establish a logical theory able to state precisely the behavior of such properties in order to stipulate useful ontological distinctions.

## 3. GENERALITIES AND MEREOLOGICAL FRAMEWORK

We adopt in the following a standard first-order language with identity. Four unary predicates 'R', 'M', 'OB', and 'S' represent the subdomains of *region of space, chunks of matter, physical objects* and *global system's states*, which are assumed as being mutually exclusive and covering the whole universe (the symbol $\underline{\vee}$ stands for exclusive disjunction; free variables are assumed to be universally quantified):

A1. $\text{R}x \,\underline{\vee}\, \text{M}x \,\underline{\vee}\, \text{OB}x \,\underline{\vee}\, \text{S}x$

The two parthood relations for space and matter postulated in our notion of a material system are represented in a rather standard way, by means of a *single* binary predicate 'P' restricted to hold only between *substrates of the same kind*:

A2. $\text{P}xy \rightarrow (\text{M}x \wedge \text{M}y) \vee (\text{R}x \wedge \text{R}y)$

Such a parthood relation is assumed to satisfy the axioms of Closed Extensional Mereology [24,28], which are not reported here because of space limitations. In particular, this means that the so-called "strong supplementation axiom"

A3. $\neg\text{P}xy \rightarrow \exists z(\text{P}zx \wedge \neg\text{O}zy)$, where $\text{O}xy =_{\text{df}} \exists z(\text{P}zx \wedge \text{P}zy)$

is adopted, while the existence of (finite) mereological sums $x$+$y$, mereological differences $x$-$y$ (only defined if $\neg\text{P}xy$), and mereological products $x\cdot y$, (only defined if $\text{O}xy$), is warranted.

Due to A2, the parthood relation is only defined for substrates. In section 6 we shall see how the notion of *contingent parthood* is defined for physical objects.

## 4. SPACE

### 4.1 TOPOLOGICAL LEVEL

Besides the parthood relation, we introduce in the domain of space a further primitive to account for topological properties. We follow with this move the first of the strategies discussed in [28] (the others being adopting topology as a basis for mereology, as in [5,22,1] and adopting mereology as a basis for topology, as in [10]); our approach differs however from those inspired to Clarke's work, since we don't take topological connection ('C') as a primitive: rather, we adopt the notion of simple region (s-region), or region "all in one piece", denoted by the predicate 'SR'. The reasons of this choice come from the desire to make possible a simple and natural interpretation of our primitives, and at the same time to characterize our intended models as best as possible in order to avoid ambiguous interpretations. In particular, in the case of the RCC approach, the exact interpretation of the ‘C’ primitive is not so clear, although they informally state that two regions are connected when they share a point. This is only true in their intended model, however, since it is easy to verify that the axioms of RCC theory don’t exclude to interpret ‘C’ in terms of strong connection or surface connection (s-connection), assuming that two regions are (strongly) connected if they have a surface in common.

The intuition underlying the choice of s-regions as primitives is bound to this notion of surface connection, and it is aimed to the desire to “explain” connection in commonsense terms. In our system, a sum of two arbitrary s-regions counts as a s-region if they are s-connected: in this case, a material object whose location is given by the sum of the two s-regions can be thought of, in the everyday intuition, as a single *thing.* This does not seem the case of line-connected or point-connected regions, since no "drop" of matter would keep the corresponding material object together; the notion of s-connection is therefore bound to that of *physical connection.* We shall see in section 4.2 how the weaker notions of l– and p-connection can be defined in our system by means of a further *morphological* primitive.

After this informal introduction, we briefly present in the following the axioms and definitions characterizing the *topological level* of our theory. A detailed discussion of the advantages and disadvantages of this approach with respect to other spatial logics presented in the literature will be the object of a future paper. Let us first introduce some preliminary definitions:

D6. $\mathrm{PO}xy =_{\mathrm{df}} \mathrm{O}xy \wedge \neg \mathrm{P}xy \wedge \neg \mathrm{P}yx$ *(proper overlap)*
D7. $\mathrm{IP}xy =_{\mathrm{df}} \mathrm{R}x \wedge \mathrm{PP}xy \wedge \forall z\ ((\mathrm{SR}z \wedge \mathrm{PO}zx) \rightarrow \mathrm{O}z(y\text{-}x))$ *(interior part)*
D8. $\mathrm{MCP}xy =_{\mathrm{df}} \mathrm{P}xy \wedge \mathrm{SR}x \wedge \neg \exists z\ (\mathrm{SR}z \wedge \mathrm{PP}xz \wedge \mathrm{P}zy)$ *(maximally connected part)*

Notice that, according to D7, a region being l- or p-connected with a region external to *x* must be considered as an interior part of *x*. So, in our interpretation, ‘IP’ turns to be different from the relation ‘NTPP’ (non-tangential proper part) defined in the RCC theory. The following axioms are assumed:

A4. $\mathrm{SR}x \rightarrow \mathrm{R}x$
A5. $(\mathrm{SR}x \wedge x = y + z) \rightarrow \exists u\ (\mathrm{SR}u \wedge \mathrm{O}uy \wedge \mathrm{O}uz \wedge \mathrm{IP}ux)$
A6. $\mathrm{R}x \rightarrow \exists y\ \mathrm{MCP}yx$
A7. $\mathrm{R}x \rightarrow \exists y\ (\mathrm{SR}y \wedge \mathrm{IP}xy)$

A5 captures the idea of intimate connection between two arbitrary halves of a s-region; we can also prove that it excludes regions which are not manifolds, like the sum of two tangent spheres, to be considered as s-regions. A6 and A7 make some minimal assumptions regarding the structure of space. By means of SR, the relation of *strong connection* between spatial regions can be defined as follows:

D9. $\mathrm{SC}xy =_{\mathrm{df}} \exists uv\ (\mathrm{P}ux \wedge \mathrm{P}vy \wedge \mathrm{SR}(u+v))$

Due to space limitations, we do not discuss here the consequences of the axioms presented above; it will suffice to remark that the notion of s-connection turns out to be quite well characterized. However, we are currently not able to define l-connection and p-connection at the topological level in a satisfactory way, that is, in such a way that some unpleasant non-intended models are excluded. Within the RCC approach, things are a bit easier in this sense due to the expressive power of the ‘C’ primitive [11,6], but we must underline that this is true only in RCC *intended* models: RCC axioms don't exclude the possibility to interpret the ‘C’ relation in the sense of strong connection, while the discussion above shows that the converse (i.e. the interpretation of ‘SC’ in the sense of ‘C’) is not admitted in our system.

### 4.2 MORPHOLOGICAL LEVEL

The expressivity problems bound to the use of mereological and topological primitives alone can be overcome by the introduction of a morphological primitive. We are forced to adopt it in any case, if we want to speak of shapes, holes, edges and various morphological features. A “convex hull” primitive has been used with some success within the RCC theory, but its characterizing axioms are still not clear. A ternary alignement relation has been used in [2], but it commits to the notion of point. We opt here for a different primitive, the most simple we can think of: the *congru-*

*ence relation* between regions, denoted with ‘CG’. In the case of classical geometry based on the notions of points, segments and angles, this relation has been first axiomatized by Hilbert [15], with various semplifications thereafter. In order to take advantage from such work, we need an analogous of points in terms of regions. This analogy has been brilliantly pursued in [27], where a (second order) axiomatic theory taking spheres and parts as primitives has been shown as equivalent to classical geometry. Our strategy to axiomatize ‘CG’ is therefore the following:

1. define a sphere in terms of ‘P’, ‘SR’ and ‘CG’;
2. adopt Tarski’s definitions related to spheres;
3. define a notion of alignment for three spheres;
4. reconstruct standard axioms for congruence between segments and triangles by exploiting the analogy between points and spheres;
5. add further axioms in order to constrain the congruence between spheres and to introduce suitable existential assumptions.

The result will be a first order theory of congruence between regions.

**4.2.1 Defining Spheres.** The crucial step here is the definition of a sphere that makes it possible to link Tarski’s mereo-morphological theory with our mereo-topological theory:

D10. $\mathrm{SPH}x =_{\mathrm{df}} \mathrm{SR}x \wedge \forall y(\mathrm{CG}xy \wedge \mathrm{PO}xy \rightarrow \mathrm{SR}(x\text{-}y))$

It is easy to see that only spherical regions satisfy D10 (Fig. 2), provided that enough regions congruent to the given one exist. Axiom A16 will force this condition.

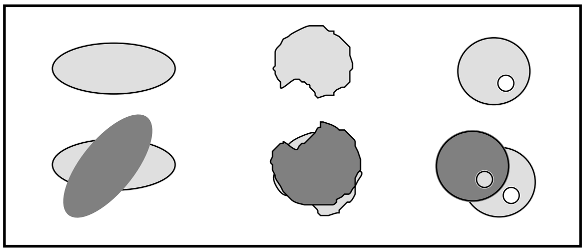

Fig. 2. These regions are not spheres

**4.2.2 Tarski’s definitions.** Let us now introduce Tarski's definitions regarding spheres. For the sake of conciseness, we assume that all variables appearing in D11-D20 below are restricted to the class of spheres.

D11. $\mathrm{ET}xy =_{\mathrm{df}} \neg\mathrm{O}xy \wedge ((\neg\mathrm{O}uy \wedge \neg\mathrm{O}vy \wedge \mathrm{P}xu \wedge \mathrm{P}xv) \rightarrow (\mathrm{P}uv \vee \mathrm{P}vu))$ *(x is externally tangent to y)*
D12. $\mathrm{IT}xy =_{\mathrm{df}} \mathrm{PP}xy \wedge ((\mathrm{P}uy \wedge \mathrm{P}vy \wedge \mathrm{P}xu \wedge \mathrm{P}xv) \rightarrow (\mathrm{P}uv \vee \mathrm{P}vu))$ *(x is internally tangent to y)*
D13. $\mathrm{ED}xyz =_{\mathrm{df}} \mathrm{ET}xz \wedge \mathrm{ET}yz \wedge (\neg\mathrm{O}uz \wedge \neg\mathrm{O}vz \wedge \mathrm{P}xu \wedge \mathrm{P}yv \rightarrow \neg\mathrm{O}uv)$
*(x and y are externally diametrical w.r.t. z)*
D14. $\mathrm{ID}xyz =_{\mathrm{df}} \mathrm{IT}xz \wedge \mathrm{IT}yz \wedge (\neg\mathrm{O}uz \wedge \neg\mathrm{O}vz \wedge \mathrm{ET}xu \wedge \mathrm{ET}yv \rightarrow \neg\mathrm{O}uv)$
*(x and y are internally diametrical w.r.t. z)*
D15. $\mathrm{CNC}xy =_{\mathrm{df}} x = y \vee (\mathrm{PP}xy \wedge (\mathrm{ED}uvx \wedge \mathrm{IT}uy \wedge \mathrm{IT}vy \rightarrow \mathrm{ID}uvy)) \vee (\mathrm{PP}yx \wedge (\mathrm{ED}uvy \wedge \mathrm{IT}ux \wedge \mathrm{IT}vx \rightarrow \mathrm{ID}uvx))$ *(x is concentric with y)*

**4.2.3 Alignment of spheres**. We can now easily exploit Tarski's definition to define the notion of alignement among spheres, and to establish notions analogous to those of segments and triangles in terms of spheres (*s-segments* and *s-triangles*):

D16. $\mathrm{BTW}xyz =_{\mathrm{df}} \exists x'y'z'(\mathrm{CNC}xx' \wedge \mathrm{CNC}yy' \wedge \mathrm{CNC}zz' \wedge \mathrm{ED}y'z'x')$ *(x is between y and z)*
D17. $\mathrm{LIN}xyz =_{\mathrm{df}} \mathrm{BTW}xyz \vee \mathrm{BTW}yxz \vee \mathrm{BTW}zxy$ *(x is aligned w.r.t. y and z)*
D18. $\mathrm{SSD}xyz =_{\mathrm{df}} \mathrm{BTW}xyz \vee \mathrm{BTW}yxz \vee \mathrm{CNC}xy$ *(x and y are on the same side w.r.t. z)*
D19. $\mathrm{SEG}xy =_{\mathrm{df}} \neg\mathrm{CNC}xy$ *(x and y form an s-segment)*
D20. $\mathrm{TRI}xyz =_{\mathrm{df}} \neg\mathrm{CNC}xy \wedge \neg\mathrm{CNC}yz \wedge \neg\mathrm{CNC}xz \wedge \neg\mathrm{LIN}xyz$ *(x, y, and z form an s-triangle)*

**4.2.4 Reconstructing Standard Axioms for Congruence.** We can now introduce the proper axioms of congruence, modifying the formulation presented by Coxeter [7] by exploiting the parallelism between points and spheres. For the sake of conciseness, we require first a variable-arity relation which holds when its arguments are not parts one each other.

D21. PNP$xyz$...$=_{df}$ ($\neg$ P$xy$ $\wedge$ $\neg$P$xz$ $\wedge$...) $\wedge$ ($\neg$P$yx$ $\wedge$ $\neg$P$yz$ $\wedge$...) $\wedge$ ($\neg$P$zx$ $\wedge$ $\neg$P$zy$ $\wedge$...) $\wedge$... *(pairwise not part)*

A8. CG$xy$ $\rightarrow$ R$x$ $\wedge$ R$y$

A9. CG$xy$ $\wedge$ CG$zy$ $\rightarrow$ CG$xz$

A10. PNP$xy$ $\wedge$ SEG$xy$ $\wedge$ CG$xx'$ $\wedge$ SEG$x'w$ $\rightarrow$ $\exists!y'$(CG($x$+$y$)($x'$+$y'$) $\wedge$ SSD$wy'x'$) *(Transportability of s-segments)*

A11. PNP$xyz$ $\wedge$ BTW$yxz$ $\wedge$ BTW$y'x'z'$ $\wedge$ CG$xx'$ $\wedge$ CG($x$+$y$)($x'$+$y'$) $\wedge$ CG($y$+$z$)($y'$+$z'$) $\rightarrow$ CG($x$+$z$)($x'$+$z'$) *(Congruence of s-segments)*

A12. PNP$xyzv$ $\wedge$ TRI$xyz$ $\wedge$ TRI$x'y'z'$ $\wedge$ BTW$yxv$ $\wedge$ BTW$y'x'v'$ $\wedge$ CG$xx'$ $\wedge$ CG$yy'$ $\wedge$ CG$zz'$ $\wedge$ CG$vv'$ $\wedge$ CG($x$+$y$)($x'$+$y'$) $\wedge$ CG($x$+$z$)($x'$+$z'$) $\wedge$ CG($y$+$z$)($y'$+$z'$) $\wedge$ CG($x$+$v$)($x'$+$v'$) $\rightarrow$ CG($z$+$v$)($z'$+$v'$) *(Congruence of s-triangles)*

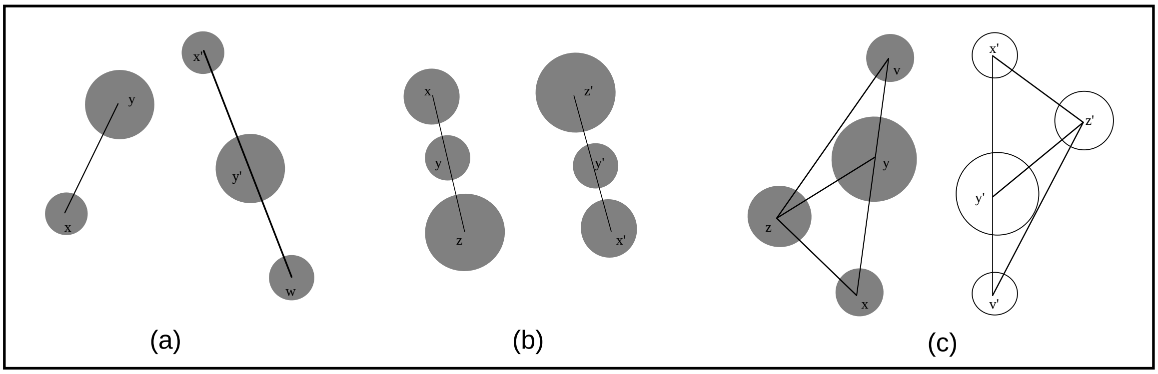


Fig. 3. Revisiting classical axioms for congruence

The axiom A9, together with suitable existential assumptions, implies that 'CG' is an equivalence relation. A10 ensures that, given a s-segment *xy*[1], a sphere *x'* congruent to *x*, and a further sphere *w* individuating a half-line starting from the center of *x'*, there exist a unique s-segment *x'y'* congruent to *xy* such that the center of *y* belongs to this half-line (Fig. 3a). A11 is described by Fig. 3b, and tells us that if *xy* and *yz* are respectively congruent to *x'y'* and *y'z'*, then *xz* is congruent to *x'z'*. A12 is described by Fig. 3c, and allows us to conclude that the s-triangle *xvz* is congruent to *x'v' z'* if *xyz* is congruent to *x'y'z'* and *xv* is congruent to *x'v'*.

**4.2.5 Further Constraints.** Let us first add some axioms which further constrain the congruence between spheres and introduce existential axioms related to congruent regions:

A13. CG$xy$ $\rightarrow$ $\neg$PP$xy$

A14. CG$xy$ $\wedge$ ED$xyz$ $\wedge$ ID$xyw$ $\wedge$ BTW$wxy$ $\wedge$ BTW$zxy$ $\rightarrow$ CNC$zw$

A15. $\exists y$ (SPH$y$ $\wedge$ IP$yx$)

A16. EC$xy$ $\wedge$ EC$xz$ $\rightarrow$ $\exists w$ (CG$xw$ $\wedge$ O$wy$ $\wedge$ O$wz$)

A13 excludes the congruence of two spheres being one internal to the other, while A14 excludes the congruence of two spheres of different size.

Axioms A15-A16 are all *existential axioms*, which are necessary to ensure that enough regions congruent to a given one exist. A15 states that a sphere is contained in any region; it is not independent of topological axioms, and it is stated here in this form just for clarity purposes. A16 excludes a model where congruent regions are only those which can superimpose each other by means of a simple translation. Notice that it excludes a model where spheres were interpreted as moon-shaped regions, and at the same time it guarantees that enough spheres congruent and overlapping a given one exist.

**4.2.6 Using the Congruence Primitive.** Having axiomatized the relation of congruence among

[1] The s-segment *xy* does in fact correspond to the mereological sum *x*+*y*.

regions (and being sure therefore that our definition of spheres (D10) excludes a number of unwanted models), we are now in the position to define l- and p-connection by the help of spheres, and then the usual notion of connection:

D22. $\mathrm{LC}xy =_{\mathrm{df}} \neg\mathrm{SC}xy \wedge \exists z\, (\mathrm{SPH}z \wedge \mathrm{SR}(z\text{-}x) \wedge \mathrm{SR}(z\text{-}y) \wedge \neg\mathrm{SR}(z\text{-}(x+y)))$ *(l-connection)*
D23. $\mathrm{PC}xy =_{\mathrm{df}} \neg\mathrm{SC}xy \wedge \neg\mathrm{LC}xy \wedge \exists z(\mathrm{SPH}z \wedge \forall u(\mathrm{CNC}uz \rightarrow (\mathrm{O}ux \wedge \mathrm{O}uy)))$ *(p-connection)*
D24. $\mathrm{C}xy =_{\mathrm{df}} \mathrm{SC}xy \vee \mathrm{LC}xy \vee \mathrm{PC}xy$ *(connection)*

Now, we can easily define a convex region, used in the RCC approach to make useful distinctions among regions:

D25. $\mathrm{CONV}x =_{\mathrm{df}} (\mathrm{P}(u+v)x \wedge \mathrm{CG}uv \wedge \mathrm{CG}wu \wedge \mathrm{BTW}wuv) \rightarrow \mathrm{P}wx$ *(x is convex)*

## 5. Matter and Physical Objects

Let us now present the axioms for chunks of matter and physical objects. Two primitives are introduced to account for the relationships between our four subdomains: 'LOC' gives the spatial extension (location) of an individual *x* at state *s*, while 'MAT' gives its material extension. The domain of 'LOC' is extended to include also regions, assuming that, for any state, the location of a region coincides with the region itself [3]. The following axioms clarify the domain of these relations:

A17. $\mathrm{LOC}rxs \rightarrow \mathrm{R}r \wedge (\mathrm{OB}x \vee \mathrm{M}x \vee \mathrm{R}x) \wedge \mathrm{S}s$
A18. $\mathrm{MAT}mxs \rightarrow \mathrm{M}m \wedge (\mathrm{OB}x \vee \mathrm{M}x) \wedge \mathrm{S}s$
A19. $\mathrm{R}x \leftrightarrow \mathrm{LOC}xxs$
A20. $\mathrm{M}x \leftrightarrow \mathrm{MAT}xxs$

As discussed in Section 2, we assume that an individual *exists* (in the ontological sense) in a state if it located somewhere in that state (D26). A19 and A21 make sure that regions and pieces of matter always exist; A22 states that an object must exist at least in some state. Moreover, A21 and A22 express, respectively, the ontological dependence between matter and space and between physical objects and space.

D26. $\mathrm{EX}xs =_{\mathrm{df}} \exists r\, \mathrm{LOC}rxs$

A21. $\mathrm{M}x \rightarrow \forall s\, \mathrm{EX}xs$
A22. $\mathrm{OB}x \rightarrow \exists s\, \mathrm{EX}xs$

The following axioms guarantee that i) 'LOC' denotes a function from 'M' to 'R' with respect to the parameter *s* (A23); ii) such a function is injective (A24); and iii) it is an homomorphism between ' M' and 'R' preserving the parthood relation 'P' (A25). A26 shows how the notion of state is bound to such homomorphism.

A23. $\mathrm{LOC}rxs \wedge \mathrm{LOC}r'xs \rightarrow r=r'$
A24. $\mathrm{M}x \wedge \mathrm{M}x' \wedge \mathrm{LOC}rxs \wedge \mathrm{LOC}rx's \rightarrow x=x'$
A25. $\mathrm{M}x \wedge \mathrm{M}x' \wedge \mathrm{LOC}rxs \wedge \mathrm{LOC}r'x's \rightarrow (\mathrm{P}xx' \leftrightarrow \mathrm{P}rr')$
A26. $\forall rm\, (\mathrm{M}m \wedge (\mathrm{LOC}rms \leftrightarrow \mathrm{LOC}rms')) \rightarrow s=s'$

A27 states that, analogously to 'LOC', 'MAT' denotes a function from 'OB' to 'M' with respect to the parameter *s*. We also assume that, in the case of physical objects, their spatial location coincides with the location of the matter they are made of (A28). This last assumption may be removed if we allow for "mixed" objects (i.e., both material and immaterial), such that the region they occupy is larger than the region occupied by their matter; we shall not consider such cases here.

A27. $\mathrm{MAT}mxs \wedge \mathrm{MAT}m'xs \rightarrow m=m'$
A28. $\mathrm{LOC}rxs \wedge \mathrm{MAT}mxs \rightarrow \mathrm{LOC}rms$

## 6. Ontological Distinctions Among Physical Objects

The ontological theory developed so far turns to be quite powerful, allowing to establish – in a rigorous way – useful distinctions within our domain. We give here a preliminary account of some of these distinctions.

### 6.1 GENERAL PROPERTIES

First, it is useful to distinguish between *material* and *immaterial* objects (denoted respectively with 'MO' 'IO') on the basis of the presence or absence of a material substrate in any state where the object exists:

D27. $\mathrm{MO}x =_{\mathrm{df}} \mathrm{OB}x \wedge (\mathrm{EX}xs \rightarrow \exists m\ \mathrm{MAT}mxs)$
D28. $\mathrm{IO}x =_{\mathrm{df}} \mathrm{OB}x \wedge (\mathrm{EX}xs \rightarrow \neg\exists m\ \mathrm{MAT}mxs)$

We define then the notion of *contingent part* for an object in a particular state as follows. Notice that, due to A28, we exclude the case of an immaterial object being part of a material object, and vice-versa. Objects being contingent parts of another object in any state are called *essential parts* of that object.

D29. $\mathrm{CP}xys =_{\mathrm{df}} ((\mathrm{IO}x \wedge \mathrm{IO}y) \vee (\mathrm{MO}x \wedge \mathrm{MO}y)) \wedge \mathrm{LOC}uxs \wedge \mathrm{LOC}vys \wedge \mathrm{P}uv$ *(contingent part)*
D30. $\mathrm{ESP}xy =_{\mathrm{df}} \mathrm{EX}ys \rightarrow \mathrm{CP}xys$ *(essential part)*

We say that two objects *coincide* in a state if they have the same contingent parts in that state. Notice that two objects can be *constantly coincident* in all states without being identical:

D31. $\mathrm{CCD}xys =_{\mathrm{df}} \forall z\ (\mathrm{CP}zxs \leftrightarrow \mathrm{CP}zys)$ *(coincidence)*

Also the notion of *rigidity* for physical objects can be easily defined as follows:

D32. $\mathrm{RIG}x =_{\mathrm{df}} \mathrm{OB}x \wedge (\mathrm{LOC}uxs \wedge \mathrm{LOC}vxs' \rightarrow \mathrm{CG}uv)$ *(rigidity)*

### 6.2 BOUNDARIES AND GRANULARITY

Boundaries are introduced in our framework avoiding to rely on their classical mathematical definition. Rather, we adopt a definition more akin to the commonsense intuition, where surfaces and edges are thought of as concrete entities, and granularity considerations are invoked [26]. We can easily introduce a notion of *granularity* within our system by fixing a particular sphere $g$, and defining a *granule* of granularity $g$ as follows:

D33. $\mathrm{G}xg =_{\mathrm{df}} \mathrm{SPH}g \wedge \mathrm{CG}xg$

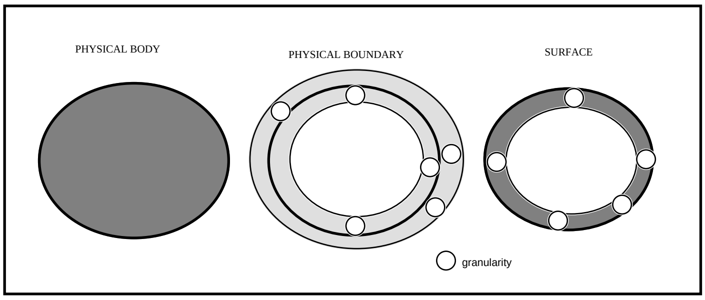


Fig. 3. Boundary and surface of a physical body.

Now we can "approximate" the mathematical notion of the boundary of a region by means of a suitably thin region overlapping the "real" boundary (Fig. 3)

D34. $\mathrm{SB}xyg =_{\mathrm{df}} \mathrm{R}x \wedge \mathrm{R}y \wedge \forall z\ (\mathrm{P}zx \leftrightarrow \forall w\ (\mathrm{P}wz \rightarrow \exists u\ (\mathrm{G}ug \wedge \mathrm{O}uw \wedge \mathrm{PO}uy)))$
*(x is the spatial boundary of y at granularity g)*

In the case of physical objects, boundaries are not intended as regions, but as *immaterial objects* always overlapping the "real boundary" as the state changes. They are called in this case *physical boundaries*.

D35. $\mathrm{PB}xyg =_{\mathrm{df}} \mathrm{IO}x \wedge \mathrm{OB}y \wedge ((\mathrm{LOC}uxs \wedge \mathrm{LOC}vys) \rightarrow \mathrm{SB}uvg)$
*(x is the physical boundary of y at granularity g)*

Now the notion of the *surface* (or "skin") of a physical object can be defined as follows:

D36. $\mathrm{SURF}xyg =_{\mathrm{df}} \mathrm{CP}xys \wedge ((\mathrm{LOC}uxs \wedge \mathrm{LOC}vys \wedge \mathrm{SB}wvg) \rightarrow u=v\times w)$

Many other useful distinctions can be made, which will be discussed in detail in a future work. In particular, we are able to distinguish between so-called "fiat" and "bona-fide" boundaries depending on the presence of matter discontinuity [25], and between contact and material connection among physical bodies on the basis of the morphological properties of the boundary between them, at a given granularity.

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