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On the Natural Selection of Market Choice

AVIV BERGMAN
Center for Computational Genetics and Biological Modeling, Stanford University, 385 Serra Mall (Herrin 428), Stanford, CA 94305-5020

MOSHE TENNENHOLTZ moshet@ie.technion.ac.il
Center for Computational Genetics and Biological Modeling, Stanford University, 385 Serra Mall (Herrin 428), Stanford, CA 94305-5020 and Computer Science Department, Stanford University, Stanford, CA 94305

Abstract. An evolutionary approach to the problem of economic mechanism choice is presented. It demonstrates the power that a single participant has on the choice of a preferred market mechanism. A population of sellers, each with one of two possible economic mechanisms, facing potential buyers, is presented as a test model. It is shown that if two auctions, such as first- and second-price auctions, are to attract an identical number of buyers, although under the model assumptions their expected revenues are identical, sellers using a first-price auction mechanism will be selected for. However, if a second-price auction attracts one additional buyer, then it will be selected for by the evolutionary process. These results are extended to the choice between an arbitrary k- and l-price auctions.

Keywords: market choice, evolutionary dynamics

1. Introduction

Auctions [12, 18, 19] and other economic mechanisms have received much recent attention in Computer Science and Artificial Intelligence [2, 13, 16, 17]. The study of auctions has become a subject of particular importance due to the popularity of auctions over the Internet. In particular, the question of which auction one should choose, and which auctions would survive in the on-line environment is of fundamental importance both for theoreticians and practitioners in the areas of electronic commerce and multi-agent systems.

Most work on auction theory is concerned with equilibrium analysis of auctions. Although learning and adaptation in the context of auctions are fundamental for experimental game-theory and economics (see [9]), analytic results on emergent and adaptive behavior in auctions are rare (see e.g., [8] for an exception). Moreover, the existing work in this regard deals with the adaptation of the buyers' strategy given a particular auction, rather than with the dynamics of auction selection by sellers.

In this paper, we propose an adaptive systems, evolutionary dynamics approach, to address the problem of auction choice. We use a population dynamics perspective on market design, where auction mechanisms are "naturally selected" based on some performance criteria, e.g., maximization of revenue. The dynamics we study assumes as little as possible on the sellers' rationality in auction selection, as well as on the syntactic representation of information. We will assume that there are two types of auctions available, and that the population of sellers may choose one of these auctions. The way that sellers adapt their behavior is as follows: from time to time (modelled as generations) the result of two auction instances, one of each type, is communicated, and the population of sellers will adapt its behavior based on this input. Although the model we use in this paper is an evolutionary dynamics model, we won't make any syntactic assumptions about the way information is modelled (i.e., we will not make any assumptions on the structure of the so-called genetype structure); our analysis will be based only on the most basic evolutionary rule: the number of sellers that use a particular auction will be changed from generation to generation based on the relative success of these auctions in maximizing the sellers' revenue.

The main contributions of this paper are as follows:

1. We introduce an evolutionary dynamics model for the context of mechanism choice.
2. We show that although the famous revenue equivalence theorem in economics tells us that the expected revenues of all of the classical auctions are identical for any fixed number of potential buyers (given the independent private value model we consider), an evolutionary process will prefer one auction upon another. In particular, Dutch auction (or first-price auction) will be selected for, rather than English auction (or second-price auction), when the system stabilizes.
3. We show that attracting even one additional potential buyer to an auction, this auction will be selected for by the evolutionary process. In particular, if an English auction is able to attract even one more buyer than the Dutch auction (e.g., due to its simplicity or familiarity) then the English auction is the one that will be selected for.

The structure of this paper is as follows. In section 2 we present some preliminaries of auction theory. In section 3 we present an evolutionary dynamics model for the problem of auction selection. In section 4 we present our basic results. Section 5 concludes with further discussion.

2. Auctions

Consider a seller who wishes to sell a particular good where there are n potential buyers, denoted by $i = 1, 2, \ldots, n$, who wish to purchase this good. An auction is a procedure in which participants submit messages, typically monetary bids regarding the good. The auction's rules specify the type of possible messages. Based on the submitted messages, these rules also specify the payment each participant pays as well as the winner of the auction.

Classical auction theory associates a (Bayesian) game with each auction procedure and analyzes the behavior of the participants under the equilibrium assumption. Equilibrium is defined when each participant submits a message which maximizes its expected utility, given its knowledge of the distribution of possible messages submitted by all other participants. A detailed description of the equilibrium concept is beyond the scope of this paper (for formal definitions and analysis see [5]). Here we assume that the participants' valuations for the good, $(r_i)_{i=1}^n$, are independently drawn from a uniform distribution on the interval $[0, 1]$.[1] Each participant knows its own valuation, in addition, each participant knows that the valuations of the others are drawn from the same distribution, but possess no information of their actual valuations. Furthermore, each participant knows the total number of participants in the auction, and his behavior is "risk-neutral."[2]

Given an auction procedure, and under the above assumptions, one can compute the messages to be transmitted at equilibrium for the corresponding game. Here we limit our analysis to some classical auction types to be discussed below.

First-price auction: One of the most popular auction mechanisms is the first-price, sealed bid auction. In such an auction, each participant submits a bid in a sealed envelope. The agent with the highest bid wins the object and pays his bid; all other participants pay nothing. Ties are broken with some lottery mechanism.

Second-price auction: The second-price auction, or the Vickrey auction, is, again, a sealed-bid auction, where each participant submits a bid in a sealed envelope. The winner is the one with the highest bid and he pays the second highest bid.

More generally, in a k**-price auction** each agent submits a sealed bid, and the winner is the one with the highest bid and he pays the kth highest bid. The set of possible messages in k-price auctions is taken to be the set of possible monetary bids.

In a k-price auction with n participants, the message $b(r_i)$ transmitted at equilibrium by participant i is (see [19])

$$b(r_i) = r_i \frac{n-1}{n-k+1}. \quad [1]$$

Note that the seller's revenue in an auction is a random variable and depends on the auction type, number of participants, and their behavior at equilibrium. The expected revenue (expectation is taken over multiple realizations of a particular auction type) and its variance for a k-price auction with n participants are

$$E[W^{n,k}] = \frac{n-1}{n+1} \quad [2]$$

and

$$Var[W^{n,k}] = \frac{k(n-1)}{(n-k+1)(n+1)^2(n+2)^2}, \quad [3]$$

respectively [19] where $W^{n,k}$ is a random variable representing the seller's' revenue in an auction. Note that under the assumptions described above, the expected revenue is independent of k, i.e., sellers get the same expected revenue in all k-price auctions.

English and Dutch auctions: Two other famous auction types are the English (ascending bid) and the Dutch (descending bid) auctions. Dutch auctions (see e.g., www.webswap.com) are strategically equivalent to first-price auctions. English auctions (see e.g., www.ebay.com) and second-price auctions have, under our model assumptions, identical expectation and variance of revenue. Thus, our results for the first- and second-price auctions can be directly extended to include the English and Dutch auctions as well.

3. An evolutionary dynamics model for auction choice

So far we have described the stochastic properties of an environment (i.e., expected revenue and its variance) that a seller is faced with when engaged in an auction process. Consider now a population of sellers, each with one of two possible market strategies in a repetitive auction situation. Sellers of the first type use a k-price auction in order to sell a product to one of n potential buyers, while sellers of the second type use an l-price auction in order to sell a product to one of m potential buyers. Again, we assume that the buyers' valuations for the product are independently drawn from the uniform distribution on the interval $[0, 1]$, and that they use the equilibrium strategies of the type of auction they are participating in, i.e., k-price or l-price auction.

We consider a evolutionary model where at each generation, j, a pair of sellers, one using k-price and the other using l-price auction, are chosen at random to sell their goods. The revenues they obtain serve as the relative fitness associated with each seller's type. Other models, such as frequency-dependent models where the number of sellers executing an auction of a given type depends on their relative abundance in the population, are under consideration.

To study the dynamics of the sellers' population, we present the auction process in an evolutionary framework. That is, we study the stability of a homogeneous population of k-price sellers with n participants each, against an invasion by a *"mutant"* seller whose strategy is an l-price auction with m participants, $l \neq k$. Though, we are especially interested in the case where $k = 1$ and $l = 2$, we analyze and present results for the general case.

As described before, we consider a population genetics model where at each generation, selection is based upon the revenues obtained by a randomly chosen pair of k- and l-price sellers.

Revenues, $W^{n,k}$ and $W^{m,l}$, of the two types of sellers are stationary stochastic processes with means and variances as described above, i.e.,

$$E[W^{n,k}] = \frac{n-1}{n+1} \quad \text{and} \quad Var[W^{n,k}] = \frac{k(n-1)}{(n-k+1)(n+1)^2(n+2)^2},$$

respectively, and similarly for the second type of sellers.

The frequency of sellers associated with l-price and m participants at generation j, denoted by x_j, satisfies the stochastic recursion equation

$$x_{j+1} = \frac{x_j W_j^{m,l}}{x_j W_j^{m,l} + (1 - x_j) W_j^{n,k}} \quad [4]$$

where $W_j^{n,k}$ is the realization of the random variable $W^{n,k}$ at generation j. Given the above model for evolutionary dynamics in the context of market choice, we will be able to analyze the emergent behavior of auction selection by sellers. Our basic results are presented in the following section.

4. Analysis

We now present our results on market choice given the model presented in the previous section.

The resistance of a population of k-price sellers with n participants to the invasion of an l-price seller with m participants can be determined by the properties of the linear approximation of equation (4) at $x_j \approx 0$ (i.e., the population is near fixation for a k-price seller with n participants).

$$x_{j+1} \approx x_j \frac{W_j^{m,l}}{W_j^{n,k}}. \quad [5]$$

Using the transformation

$$W_j^{m,l} = 1 - Y_j^{m,l} \quad [6]$$

$$W_j^{n,k} = 1 - Z_j^{n,k} \quad [7]$$

we get

$$x_{j+1} \approx x_0 \prod_{i=1}^{i=j} \frac{1 - Y_i^{m,l}}{1 - Z_i^{n,k}}. \quad [8]$$

The simplex origin, $x \approx 0$, will be stochastically locally unstable, with respect to the stochasticly stationary revenue processes, $W_j^{n,k}, W_j^{m,l}$, i.e., an l-price with m participants will increase in frequency, if and only if $E[\prod_{i=1}^{i=j}(1 - Y_i^{m,l})/(1 - Z_i^{n,k})] > 1$, that is

$$\lambda = E\left[\ln \frac{1 - Y_j^{m,l}}{1 - Z_j^{n,k}}\right] > 0. \quad [9]$$

For $n \gg 1$ the stochastic effects are small, formally,

$$\forall \delta > 0 \lim_{m\to\infty} Pr\left[Y_j^{m,l} > \delta\right] = 0, \quad [10]$$

i.e., the random variables $Y_j^{m,l}$ is small compare to 1 (similarly for the random variable $Z_j^{n,k}$). Another way to explain this is that when the number of participants approaches infinity then the revenue approaches the highest possible agents' valuation which is 1 (see e.g., [14]). Given the above, inequality (9) can be approximated by

$$\lambda = E\big[Z_j^{n,k}\big] - E\big[Y_j^{m,l}\big] + \frac{1}{2}\Big(E\Big[\big(Z_j^{n,k}\big)^2\Big] - E\Big[\big(Y_j^{m,l}\big)^2\Big]\Big) > 0, \qquad [11]$$

ignoring third and higher terms, moments, of the Taylor expansion of $\ln[(1 - Y_j^{m,l})/(1 - Z_j^{n,k})]$.

Using (2) and (3) we obtain the following:

$$E[Z_j^{n,k}] = \frac{2}{n+1} \qquad [12]$$

$$E[Y_j^{m,l}] = \frac{2}{m+1} \qquad [13]$$

$$E\big[(Z_j^{n,k})^2\big] = Var\big[Z_j^{n,k}\big] + E^2\big[(Z_j^{n,k})\big] = \frac{k(n-1)}{(n-k+1)(n+1)^2(n+2)^2} + \frac{4}{(n+1)^2} \qquad [14]$$

$$E\big[\big(Y_j^{m,l}\big)^2\big] = Var\big[Y_j^{m,l}\big] + E^2\big[Y_j^{m,l}\big] = \frac{l(m-1)}{(m-l+1)(m+1)^2(m+2)^2} + \frac{4}{(m+1)^2}. \qquad [15]$$

It is interesting to note that even if the expected revenue of one type of sellers is smaller than that of the other type, the frequency of the former in the population can increase. This may occur when the revenue's variance of that type is sufficiently smaller than that of the other type (for detailed analysis see also [1, 6, 10]).

When the expected revenue of the two auction types are equal (i.e., when $n = m$) the variances determine the stochastic local stability of the simplex origin, $x = 0$,

$$\lambda = \frac{0.5(l-k)(1-n^2)}{(n+1)^2(n+2)^2(n+1-k)(n+1-l)}. \qquad [16]$$

It is easy to see that $\lambda < 0$ when $l > k > 0\ \forall k, l$, and $m = n > l$, that is, k-price sellers are stochastically locally stable against an invasion by l-price sellers.

Next we examine the case where $l > k\ \forall k, l$, and $m = n + s$, for $s \geq 1$, that is,

$$\lambda = \frac{2}{n+1} + \frac{2}{(n+1)^2} + \frac{0.5k(n-1)}{(n+1)^2(n+2)^2(n+1-k)} - \frac{0.5l(n+s-1)}{(n+s+1)^2(n+s+2)^2(n+s+1-l)} - \frac{2(n+s+2)}{(n+s+1)^2}. \qquad [17]$$

It is easy to show that $2/(n+1) > [2(n+s+2)/(n+s+1)^2]\ \forall n, s \geq 1$, and similarly $2/(n+1)^2 > 0.5l(n+s-1)/[(n+s+1)^2(n+s+2)^2(n+s+1-l)]$

$\forall\, n + s \geq l$. Thus, λ is positive $\forall\, l \geq k$ and $m = n + s$, $s \geq 1$. Therefore, it suffices for an l-price seller to attract one additional participant over the k-price sellers for it to invade. Similar observation about the power of an additional participant has been pointed out by Bulow and Klemperer [3]. Their result has been obtained under different model assumptions, in particular, their model is a static model, as oppose to the population dynamic model presented here.

From symmetry arguments, when the population is fixed on an l-price seller ($x \approx 1$), k-price, $k < l$, sellers will invade that population if the number of participants engaging in the k-price auction is greater than *or* equal to the number of participants in the l-price auction. In the model we presented and analyzed the *fundamental theorem of natural selection* holds [4], i.e., the population average fitness is a monotonically increasing function of time (generations), thus, there exist two possible equilibria the population can be at, $x = 0$ and $x = 1$, that is, a homogeneous population composed of either all k-price sellers or all l-price sellers, respectively. Thus, there are no internal equilibria. Using the more general results obtained by [7], the direction of change in the sellers' frequency will not change its sign during the evolutionary process and is independent of the composition of the population. Therefore, conditions for invasion are sufficient to determine domination as well. Extensions to more elaborated genetic models, such as multi-gene populations with recombination, where the fundamental theorem of natural selection no longer holds, as well as cases where individual sellers can choose among more than two different auction mechanisms, may result in a heterogeneous population at equilibrium, and will be discussed elsewhere.

Auction protocols are useful tools for task allocation in distributed systems. In a multi-agent system, when different parties (e.g., nodes, processors) may adopt different auction procedures, the study of the emergent properties of the distributed system is of considerable importance. Our work introduces a possible direction for tackling this basic issue. However, we would like to point out that we presented here a simplified model intended to illustrate basic phenomenon in the evolution of market choice. We conjecture that these results will hold when sellers are to be modeled more realistically. However, more accurate market situation may include the modelling of participants' choice among markets in addition to the market dynamics. This topic requires a closer look, and calls for further analysis.

One motivation for our study is to gain insight and understanding of market dynamics in the context of e-commerce. Needless to say, that the analytic model we present is too simple to cope with the complexities of real-world markets. Nevertheless, our results shed light on some aspects relevant to e-commerce. In particular, our analysis shows the effect of an additional potential buyer on the emergent properties of market choice. This illustrates the significant role of advertisement, by attracting additional buyers, in driving market evolution.

5. Discussion

In this paper, we have presented a population dynamics approach to a most basic problem of market choice. Notice that the model we studied in this paper, as captured in equations (4)–(11) presents a general approach to the study of the natural selection of market choice. In contrast to the approach taken in the context of learning in auctions and other games, where buyers are assumed to adopt strategies that have been beneficial in the past [15], or where buyers apply best response strategies based on past observations [8] (for an exhaustive overview see [11]), we consider an evolutionary model where a population of sellers evolve according to a fitness calculated based on their revenue. This approach enables us to study the dynamic properties of a mixed population of sellers. In particular, we studied the stability properties of a homogeneous population against the invasion of sellers with a different auction mechanism. Our results highlight the important role of the variance of the revenue in the case where different mechanisms attract the same number of participants. In addition, we have shown the overwhelming effect that an additional participant has on the outcome of the evolutionary process.

Notes

1. The basic analysis presented in this paper holds for general distributions. The basic message of this paper however is best communicated in our opinion by considering a well-understood standard distribution.
2. A risk neutral agent is defined as a participant who is indifferent to the choice between participating in a lottery and getting the expected value of that lottery.

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