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Fuzzy observer-based fuzzy control design for nonlinear systems with persistent bounded disturbances

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Abstract

To date, nonlinear L_∞-*gain* control problems have not been solved by the conventional control methods for the nonlinear dynamic systems with persistent bounded disturbances. This study introduces a fuzzy control design to deal with the nonlinear L_∞-*gain* output feedback control problem. First, the Takagi and Sugeno (T–S) fuzzy model is employed to approximate the nonlinear dynamic system. Next, based on the fuzzy model, a fuzzy observer-based fuzzy controller is developed to minimize the upper bound of L_∞-*gain* of the closed-loop system under some linear matrix inequality (LMI) constraints. Therefore, the nonlinear L_∞-*gain* control problem is transformed into a suboptimal control problem, i.e., to minimize the upper bound of the L_∞-*gain* of the closed-loop system subject to some LMI constraints. In this situation, the nonlinear L_∞-*gain* output feedback control problem can be easily solved by a LMI-based optimization method. The proposed methods, which efficiently attenuate the peak of perturbation due to persistent bounded disturbances, extend the L_∞-*gain* control problems from linear dynamic systems to nonlinear dynamic systems.

Keywords: L_∞-*gain* control; T–S fuzzy model; Output feedback fuzzy control; Fuzzy observer-based fuzzy controller; Persistent bounded disturbances; LMI constraints

1. Introduction

The L_2-*gain* (H_∞) optimal control design [13,1,14] has been employed to achieve the robust stabilization of nonlinear dynamic systems from the L_2 control point of view, i.e., the external disturbance should be with finite energy or bounded L_2-*norm*. In the past few years, L_2-*gain* control techniques have been widely employed to stabilize the nonlinear systems with an external disturbance belonging to L_2 space. However, L_2-*gain* robust control scheme cannot be applied to the nonlinear system with persistent bounded disturbance, which does not belong to L_2 space. In the last decade, L_∞-*gain* (L_1) optimal control has been introduced to reject the bounded disturbance with concern of minimizing the maximum (peak) amplitude of the tracking error in the time domain [23].

In the L_∞-*gain* (L_1) optimal control design for the linear systems [23,24,7,19,8,6], the duality theory and linear programming techniques have been proposed to deal with this optimal design problem by solving a set of linear

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equations. This approach is very complicated, especially for the high-order MIMO systems [8], and may not lead easily to a finite-dimensional linear programming problem for some cases [24,8]. On the other hand, for the nonlinear systems, the L_∞-*gain* control problem is more difficult to be solved [16,9,17]. At present, there are no efficient algorithms to solve the L_∞-*gain* control problem for the nonlinear dynamic systems. However, in practical control cases, the systems are inherently nonlinear and the external disturbances are persistent. In this situation, it is more appealing to develop a feasible method to solve the L_∞-*gain* optimal control for the nonlinear systems with persistent bounded disturbances.

Since it is not easy to extend the conventional L_∞-*gain* optimal control design from linear dynamic systems to nonlinear dynamic systems, in this study, the fuzzy approximation technique will be employed to deal with the L_∞-*gain* optimal control design for the nonlinear dynamic system from the suboptimal perspective. In the last decade, there has been rapidly growing interest in fuzzy control of the nonlinear systems and there have been many successful applications [3–5,12,15,22,11]. When dealing with the output feedback stabilization, a fuzzy observer is involved, for example, in [4,5] bilinear matrix inequality (BMI) conditions and a two steps algorithm based on two LMI problems are proposed. More recently, sufficient conditions in terms of LMI for the output feedback stabilization with a more general case are proposed by Guerra et al. [11]. Unlike the L_2-*gain* control case, which reduces the influence of the energy of external disturbance on the energy of output signal as small as possible, in this study, a L_∞-*gain* fuzzy observer-based fuzzy controller is proposed to reduce the influence of the peak (i.e., L_∞-*norm*) of external disturbance on the peak (i.e., L_∞-*norm*) of the output signal as small as possible. Therefore, the L_∞-*gain* fuzzy observer-based fuzzy controller may be more suitable for some practical applications because control engineers may be concerned more about the peak (amplitude) than the total energy of the perturbed signal.

In the proposed L_∞-*gain* fuzzy control design, the Takagi and Sugeno (T–S) fuzzy model is employed to approximate the nonlinear dynamic system first [20]. Next, based on the T–S fuzzy model, the upper bound of L_∞-*gain* of the nonlinear closed-loop system can be obtained under some LMI constraints. Therefore, the nonlinear L_∞-*gain* optimal control problem can be transformed into a suboptimal control problem, i.e., to minimize the upper bound of L_∞-*gain* of the closed-loop system subject to some LMI constraints. In this situation, L_∞-*gain* optimal control problem is simplified significantly and is approached by minimizing the upper bound of L_∞-*gain* of the closed-loop system until the LMI constraints are violated.

The main contributions of this paper are stated as follows:

(1) The L_∞-*gain* output feedback control of nonlinear dynamic systems is dealt with for the first time via a fuzzy observer-based fuzzy controller.
(2) A novel separation method is provided to solve the control gains and the observer gains simultaneously.
(3) Both the stability and L_∞-*gain* disturbance rejection performance are guaranteed for the fuzzy control systems.
(4) The fuzzy L_∞-*gain* output feedback control problem of nonlinear dynamic systems is transformed into a LMI problem, which can be efficiently solved and is suitable for practical application.

The paper is organized as follows: some notations and definitions are given in Section 2. In Section 3, L_∞-*gain* fuzzy observer-based fuzzy control design for nonlinear dynamic systems with persistent bounded disturbances is introduced, while some simulation results are presented in Section 4. Finally, concluding remarks are made in Section 5. In what follows,

$$\begin{bmatrix} M_{11} & * & * \\ M_{21} & M_{22} & * \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \triangleq \begin{bmatrix} M_{11} & M_{21}^{\mathrm{T}} & M_{31}^{\mathrm{T}} \\ M_{21} & M_{22} & M_{32}^{\mathrm{T}} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}.$$

2. Notation and definition

Before the description of the fuzzy control problem, some notations and definitions are stated below:

$$|x(t)| \triangleq \sqrt{x^{\mathrm{T}}(t)x(t)} \quad \text{for } x(t) \in R^n,$$

$$\|x(t)\|_\infty \triangleq \sup_t |x(t)| \quad \text{for } x(t) \in R^n,$$

$$x(t) \in L_\infty \quad \text{if } \|x(t)\|_\infty < \infty.$$

Remark 1. For the linear systems [7,18],

$$\|H\|_1 \triangleq \sup_{v\in L_\infty} \frac{\|x(t)\|_\infty}{\|v(t)\|_\infty} \quad \text{where } H : L_\infty \to L_\infty.$$

The physical meaning of $\|H\|_1$ (L_1-*norm* or L_∞-*gain*) is the worst ratio of the peak of the output signal $x(t)$ to the peak of all possible bounded input signal $v(t)$. If we can make $\|H\|_1 \leqslant \rho$, then the peak of the output signal $x(t)$ can be attenuated by a level ρ under the effect of the peak of the persistently bounded input signal $v(t)$.

3. L_∞-*gain* fuzzy observer-based fuzzy control design for nonlinear dynamic systems

A fuzzy linear dynamic model has been proposed by Takagi and Sugeno [20] to represent local linear input/output relations of nonlinear systems. The fuzzy linear model is described by fuzzy If-Then rules and will be employed here to deal with the L_∞-*gain* control problem for the nonlinear system. The *i*th rule of the fuzzy linear model for a nonlinear system is of the following form [3–5,12,15,22]:

Plant Rule i :

$$\begin{aligned} &\text{If} && z_1(t) \text{ is } F_{i1} \text{ and } z_2(t) \text{ is } F_{i2} \ldots \text{ and } z_g(t) \text{ is } F_{ig}, \\ &\text{Then} && \dot{x}(t) = A_i x(t) + B_{1i} u(t) + B_{2i} w(t)\ , \\ & && y(t) = C_{1i} x(t) + C_{2i} v(t) \text{ for } i = 1, 2, \ldots, L, \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^{\mathrm{T}} \in R^{n\times 1}$ denotes the vector of the states, $u(t) = [u_1(t), \ldots, u_m(t)]^{\mathrm{T}} \in R^{m\times 1}$ denotes the vector of the control inputs; $w(t) = [w_1(t), w_2(t), \ldots, w_p(t)]^{\mathrm{T}} \in R^{p\times 1}$ denotes the vector of the bounded external disturbances; $y(t) = [y_1(t), \ldots, y_r(t)]^{\mathrm{T}} \in R^{r\times 1}$ denotes the vector of the outputs; $v(t) = [v_1(t), \ldots, v_q(t)]^{\mathrm{T}} \in R^{q\times 1}$ denotes the vector of the bounded measurement disturbances; F_{ij} is the fuzzy set, $A_i \in R^{n\times n}$, $B_{1i} \in R^{n\times m}$, $B_{2i} \in R^{n\times p}$, $C_{1i} \in R^{r\times n}$, $C_{2i} \in R^{r\times q}$; L is the number of If-Then rules; and $z_1(t), z_2(t), \ldots, z_g(t)$ are the premise variables.

The final outputs of the fuzzy system are inferred as follows [3–5,12,15,22]:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^{L} \mu_i(z(t))(A_i x(t) + B_{1i} u(t) + B_{2i} w(t))}{\sum_{i=1}^{L} \mu_i(z(t))} \\ &= \sum_{i=1}^{L} h_i(z(t))[A_i x(t) + B_{1i} u(t) + B_{2i} w(t)], \\ y(t) &= \frac{\sum_{i=1}^{L} \mu_i(z(t))(C_{1i} x(t) + C_{2i} v(t))}{\sum_{i=1}^{L} \mu_i(z(t))} \\ &= \sum_{i=1}^{L} h_i(z(t))[C_{1i} x(t) + C_{2i} v(t)], \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mu_i(z(t)) &= \prod_{j=1}^{g} F_{ij}(z_j(t)), \\ h_i(z(t)) &= \frac{\mu_i(z(t))}{\sum_{i=1}^{L} \mu_i(z(t))}, \\ z(t) &= [z_1(t), z_2(t), \ldots, z_g(t)], \end{aligned} \tag{3}$$

and $F_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in F_{ij}.

It is assuming that

$$\mu_i(z(t)) \geqslant 0$$

and

$$\sum_{i=1}^{L} \mu_i(z(t)) > 0 \quad \text{for } i = 1, 2, \ldots, L$$

for all t.

Therefore, we get

$$h_i(z(t)) \geqslant 0 \quad \text{for } i = 1, 2, \ldots, L \tag{4}$$

and

$$\sum_{i=1}^{L} h_i(z(t)) = 1. \tag{5}$$

Suppose the following fuzzy observer is proposed to deal with the state estimation of nonlinear system (1).

Observer Rule i :

$$\begin{array}{ll} \text{If} & z_1(t) \text{ is } F_{i1} \text{ and } z_2(t) \text{ is } F_{i2} \ldots \text{ and } z_g(t) \text{ is } F_{ig}, \\ \text{Then} & \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_{1i}u(t) + L_i(y(t) - \hat{y}(t)), \end{array} \tag{6}$$

where $\hat{x}(t)$ is an estimate of $x(t)$, $\hat{y}(t) = \sum_{i=1}^{L} h_i(z(t))C_{1i}\hat{x}(t)$, and L_i is the observer gain for the ith observer rule.

The overall fuzzy observer is represented as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{L} h_i(z(t))(A_i\hat{x}(t) + B_{1i}u(t) + L_i(y(t) - \hat{y}(t))). \tag{7}$$

Hence, the fuzzy observer-based fuzzy controller is proposed as

$$u(t) = \sum_{j=1}^{L} h_j(z(t))K_j\hat{x}(t), \tag{8}$$

where K_j is the control gain for the jth controller rule.

Remark 2. In this study, the premise variables depend on state variables. In other words, some state variables should be measurable to construct the proposed fuzzy observer-based controller. In general, there are two cases for fuzzy observer-based controller design. For case A, the premise variables do not depend on the state variables estimated by a fuzzy observer. For case B, the premise variables depend on the state variables estimated by a fuzzy observer [11]. In general, case B is more complex than case A. In this paper, only case A is studied. The details can be found in [21,11].

Let us denote the estimation errors as

$$e(t) = x(t) - \hat{x}(t). \tag{9}$$

By differentiating (9), we get

$$\begin{aligned} \dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i x(t) + B_{1i}u(t) + B_{2i}w(t) \end{aligned}$$

$$-[A_i\hat{x}(t) + B_{1i}u(t) + L_iC_{1j}(x(t) - \hat{x}(t)) + L_iC_{2j}v(t)])$$

$$= \sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))[(A_i - L_iC_{1j})e(t) - L_iC_{2j}v(t) + B_{2i}w(t)]. \tag{10}$$

After manipulation, the augmented system can be expressed as the following form:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t)) \left(\begin{bmatrix} A_i + B_{1i}K_j & -B_{1i}K_j \\ 0 & A_i - L_iC_{1j} \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B_{2i} & 0 \\ B_{2i} & -L_iC_{2j} \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} \right). \tag{11}$$

Let us denote

$$\eta(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} A_i + B_{1i}K_j & -B_{1i}K_j \\ 0 & A_i - L_iC_{1j} \end{bmatrix},$$

$$B_{ij} = \begin{bmatrix} B_{2i} & 0 \\ B_{2i} & -L_iC_{2j} \end{bmatrix}, \quad d(t) = \begin{bmatrix} w(t) \\ v(t) \end{bmatrix}, \tag{12}$$

for all $i, j = 1, 2, \ldots, L$, then the augmented system in (11) can be expressed as the following form:

$$\dot{\eta}(t) = \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))[A_{ij}\eta(t) + B_{ij}d(t)]. \tag{13}$$

The optimal L_∞ fuzzy control problem is to specify the control input u in (8) such that the following minimax problem is achieved for the nonlinear systems in (13) [7,9,18,16].

$$\min_u \sup_{d(t)\in L_\infty} \frac{\|\eta(t)\|_\infty}{\|d(t)\|_\infty}. \tag{14}$$

However, it is very difficult to solve the minimax problem in (14) for the nonlinear system in (13) directly, the suboptimal approach via minimizing the upper bound of the L_∞-*gain* is proposed in this study.

Let us consider the L_∞ performance as follows: given a disturbance attenuation level ρ, the L_∞ control problem is said to be solved if there exists a control law such that the following L_∞ performance with $\eta(0) = 0$.

$$\frac{\|\eta(t)\|_\infty}{\|d(t)\|_\infty} \leqslant \rho, \quad \forall d(t) \in L_\infty \tag{15}$$

or

$$\|\eta(t)\|_\infty \leqslant \rho\|d(t)\|_\infty, \quad \forall d(t) \in L_\infty \tag{16}$$

can be achieved.

Remark 3. If we can make $\|\eta(t)\|_\infty / \|d(t)\|_\infty \leqslant \rho$, then the peak of the state $\eta(t)$ can be attenuated by a level ρ under the effect of the peak of the persistently bounded input signal $d(t)$.

If the initial condition is considered, i.e., $\eta(0) \neq 0$, the L_∞-*gain* in (16) is modified as follows:

$$\|\eta(t)\|_\infty \leqslant \beta|\eta(0)| + \rho\|d(t)\|_\infty, \tag{17}$$

where β and ρ are some positive scalars.

Remark 4. The L_∞-*gain* performance in (16) is equivalent to the bounded input bound (error) state stability (Moreover, The L_∞-*gain* performance in (17) is called L_∞-*stable* with finite gain).

The purpose of this study is to determine a fuzzy controller in (8) for the closed-loop system in (13) with the guaranteed L_∞-*gain* performance in (17) for all $d(t) \in L_\infty$. Thereafter, the attenuation level ρ can be minimized so that the L_∞-*gain* performance in (17) is reduced as small as possible at steady-state. First, one useful Lemma is stated below.

Lemma 1. *If a real scalar function* $\omega(t)$ *satisfies the following differential inequality*

$$\dot{\omega}(t) \leqslant -\varsigma\omega(t) + \kappa v(t), \tag{18}$$

where $\varsigma > 0$ *and* $\kappa > 0$ *then*

$$\omega(t) \leqslant \mathrm{e}^{-\varsigma t}\omega(0) + \kappa \int_0^t \mathrm{e}^{-\varsigma\tau} v(t-\tau)\,\mathrm{d}\tau. \tag{19}$$

Proof. The proof is trivial. □

Then, we obtain the following main result:

Theorem 1. *In the augmented nonlinear system* (13), *if the following matrix inequalities*

$$\begin{bmatrix} \left(\frac{A_{ij}+A_{ji}}{2}\right)^{\mathrm{T}} P + P\left(\frac{A_{ij}+A_{ji}}{2}\right) + \alpha P & * \\ \left(\frac{B_{ij}+B_{ji}}{2}\right)^{\mathrm{T}} P & -cI \end{bmatrix} < 0 \tag{20}$$

hold for all $i \leqslant j$ $(i, j = 1, 2, \ldots, L)$ *and some positive scalars* α *and* c, *where* P *is a symmetric positive-definite matrix, then the* L_∞*-gain control performance in* (17) *is guaranteed for an attenuated level* $\rho = \sqrt{c/\alpha\lambda_{\min}(P)}$, *and* $\beta = \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}$, *where* $\lambda_{\max}(P)$ *and* $\lambda_{\min}(P)$ *denote the maximal and minimal eigenvalues of* P, *respectively.*

Proof. From (13), we obtain

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}|P^{1/2}\eta(t)|^2 &= \dot{\eta}^{\mathrm{T}}(t)P\eta(t) + \eta^{\mathrm{T}}(t)P\dot{\eta}(t) \\
&= \left[\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))(A_{ij}\eta(t) + B_{ij}d(t))\right]^{\mathrm{T}} P\eta(t) \\
&\quad + \eta^{\mathrm{T}}(t)P\left[\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))(A_{ij}\eta(t) + B_{ij}d(t))\right] \\
&= \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t))[(A_{ij}\eta(t))^{\mathrm{T}} P\eta(t) + \eta^{\mathrm{T}}(t)PA_{ij}\eta(t)) \\
&\quad + d^{\mathrm{T}}(t)B_{ij}^{\mathrm{T}}P\eta(t) + \eta^{\mathrm{T}}(t)PB_{ij}d(t)] \\
&= \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(z(t)) \begin{bmatrix} \eta(t) \\ d(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} A_{ij}^{\mathrm{T}}P + PA_{ij} + \alpha P & * \\ B_{ij}^{\mathrm{T}}P & -cI \end{bmatrix} \begin{bmatrix} \eta(t) \\ d(t) \end{bmatrix} \\
&\quad - \alpha\eta^{\mathrm{T}}(t)P\eta(t) + cd(t)^{\mathrm{T}}d(t)
\end{aligned}$$

$$= \left\{ \sum_{i=1}^{L} h_i^2(z(t)) \begin{bmatrix} \eta(t) \\ d(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \left(\frac{A_{ii}+A_{ii}}{2}\right)^{\mathrm{T}} P + P\left(\frac{A_{ii}+A_{ii}}{2}\right) + \alpha P & * \\ \left(\frac{B_{ii}+B_{ii}}{2}\right)^{\mathrm{T}} P & -cI \end{bmatrix} \begin{bmatrix} \eta(t) \\ d(t) \end{bmatrix} \right.$$

$$\left. + 2\sum_{i=1}^{L} h_i(z(t)) \sum_{i<j} h_j(z(t)) \begin{bmatrix} \eta(t) \\ d(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \left(\frac{A_{ij}+A_{ji}}{2}\right)^{\mathrm{T}} P + P\left(\frac{A_{ij}+A_{ji}}{2}\right) + \alpha P & * \\ \left(\frac{B_{ij}+B_{ji}}{2}\right)^{\mathrm{T}} P & -cI \end{bmatrix} \begin{bmatrix} \eta(t) \\ d(t) \end{bmatrix} \right\}$$

$$- \alpha\eta^{\mathrm{T}}(t)P\eta(t) + cd^{\mathrm{T}}(t)d(t),$$

where α and c are positive scalars.

By the matrix inequalities in (20), we obtain

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}|P^{1/2}\eta(t)|^2 &\leqslant -\alpha\eta^{\mathrm{T}}(t)P\eta(t) + cd^{\mathrm{T}}(t)d(t) \\ &= -\alpha|P^{1/2}\eta(t)|^2 + c|d(t)|^2. \end{aligned} \tag{21}$$

By Lemma 1, we obtain

$$\begin{aligned} |P^{1/2}\eta(t)|^2 &\leqslant \mathrm{e}^{-\alpha t}|P^{1/2}\eta(0)|^2 + c\int_0^t \mathrm{e}^{-\alpha\tau}|d(t-\tau)|^2\,\mathrm{d}\tau \\ &\leqslant \sup_{\tau\in[0,t]} \{\mathrm{e}^{-\alpha t}|P^{1/2}\eta(0)|^2 + c|d(t-\tau)|^2 \int_0^t \mathrm{e}^{-\alpha\tau}\,\mathrm{d}\tau\} \\ &= \sup_{\tau\in[0,t]} \{\mathrm{e}^{-\alpha t}|P^{1/2}\eta(0)|^2 + \frac{c}{\alpha}|d(t-\tau)|^2(1-\mathrm{e}^{-\alpha t})\}. \end{aligned}$$

From above, we have

$$\lambda_{\min}(P)|\eta(t)|^2 \leqslant |P^{1/2}\eta(t)|^2 \leqslant \sup_{\tau\in[0,t]} \{\mathrm{e}^{-\alpha t}|P^{1/2}\eta(0)|^2 + \frac{c}{\alpha}|d(t-\tau)|^2(1-\mathrm{e}^{-\alpha t})\}.$$

Performing "$\sup_{t\in[0,\infty)}$" to both sides of the above inequality, we obtain

$$\begin{aligned} \sup_{t\in[0,\infty)} \lambda_{\min}(P)|\eta(t)|^2 &\leqslant \sup_{t\in[0,\infty)} \sup_{\tau\in[0,t]} \{\mathrm{e}^{-\alpha t}|P^{1/2}\eta(0)|^2 + \frac{c}{\alpha}|d(t-\tau)|^2(1-\mathrm{e}^{-\alpha t})\} \\ &\leqslant |P^{1/2}\eta(0)|^2 + \frac{c}{\alpha}\sup_{t\in[0,\infty)}|d(t)|^2. \end{aligned} \tag{22}$$

The above inequality implies that

$$\begin{aligned} \lambda_{\min}(P)\|\eta(t)\|_\infty^2 &\leqslant |P^{1/2}\eta(0)|^2 + \frac{c}{\alpha}\|d(t)\|_\infty^2 \\ &\leqslant \lambda_{\max}(P)|\eta(0)|^2 + \frac{c}{\alpha}\|d(t)\|_\infty^2. \end{aligned} \tag{23}$$

Hence,

$$\begin{aligned} \|\eta(t)\|_\infty^2 &\leqslant \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}|\eta(0)|^2 + \frac{c}{\alpha\lambda_{\min}(P)}\|d(t)\|_\infty^2 \\ &\leqslant \left(\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}|\eta(0)| + \sqrt{\frac{c}{\alpha\lambda_{\min}(P)}}\|d(t)\|_\infty\right)^2 \end{aligned}$$

or

$$\|\eta(t)\|_\infty \leqslant \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}|\eta(0)| + \sqrt{\frac{c}{\alpha\lambda_{\min}(P)}}\|d(t)\|_\infty \tag{24}$$

for all $d(t) \in L_\infty$. This completes the proof. □

To obtain better performance at steady-state, the control problem can be formulated as the following minimization problem so that the L_∞ performance in (17) is reduced as small as possible:

$$\begin{aligned} \min \quad & \frac{c}{\alpha\lambda_{\min}(P)} \\ \text{s.t.} \quad & \alpha > 0,\ c > 0,\ P = P^{\mathrm{T}} > 0 \text{ and (20).} \end{aligned} \tag{25}$$

For the convenience of design, let

$$P = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}. \tag{26}$$

Note that, by substituting (26) into (20), the matrix inequalities in (20) are equivalent to the following matrix inequalities:

$$\begin{bmatrix} \Psi_{11} + \alpha P_{11} & * & * & * \\ -\Psi_{12}^{\mathrm{T}} & \Phi_{22} + \alpha P_{22} & * & * \\ \Psi_{13}^{\mathrm{T}} & \Phi_{23}^{\mathrm{T}} & -cI & * \\ 0 & -\Phi_{24}^{\mathrm{T}} & 0 & -cI \end{bmatrix} < 0, \tag{27}$$

where

$$\begin{aligned}
\Psi_{11} &= \Xi_{11} + \Xi_{11}^{\mathrm{T}}, \\
\Xi_{11} &= \frac{P_{11}(A_i + B_{1i}K_j) + P_{11}(A_j + B_{1j}K_i)}{2}, \\
\Psi_{12} &= \frac{P_{11}B_{1i}K_j + P_{11}B_{1j}K_i}{2}, \\
\Psi_{13} &= \frac{P_{11}B_{2i} + P_{11}B_{2j}}{2}, \\
\Phi_{22} &= \Theta_{22} + \Theta_{22}^{\mathrm{T}}, \\
\Theta_{22} &= \frac{(P_{22}A_i - Y_iC_{1j}) + (P_{22}A_j - Y_jC_{1i})}{2}, \\
\Phi_{23} &= \frac{P_{22}B_{2i} + P_{22}B_{2j}}{2}, \\
\Phi_{24} &= \frac{Y_iC_{2j} + Y_jC_{2i}}{2},
\end{aligned}$$

and $Y_i = P_{22}L_i$

for all $i \leqslant j$ $(i, j = 1, 2, \ldots, L)$.

Since six parameters $\alpha,\ c,\ P_{11},\ P_{22},\ K_i$, and L_i should be determined from (27), there are no effective algorithms for solving them simultaneously. In the following, a novel separation method is provided to solve $\alpha,\ c,\ P_{11},\ P_{22},\ K_i$, and L_i simultaneously. The following useful lemma is introduced first.

Lemma 2. *If*

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} < 0 \quad and \quad \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} < 0, \tag{28}$$

then

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & b_{11} & b_{12} & \cdots & b_{1n} \\ 0 & b_{21} & b_{22} & \cdots & b_{2n} \\ 0 & \vdots & \vdots & \ddots & \vdots \\ 0 & b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} < 0. \tag{29}$$

Proof. The proof is straightforward and therefore is omitted. □

Note that (27) can be decoupled as follows:

$$\begin{bmatrix} \Psi_{11} + \alpha P_{11} & * & * & * \\ -\Psi_{12}^{\mathrm{T}} & \Phi_{22} + \alpha P_{22} & * & * \\ \Psi_{13}^{\mathrm{T}} & \Phi_{23}^{\mathrm{T}} & -cI & * \\ 0 & -\Phi_{24}^{\mathrm{T}} & 0 & -cI \end{bmatrix}$$

$$= \begin{bmatrix} \Psi_{11} + \alpha P_{11} & * & * & * \\ -\Psi_{12}^{\mathrm{T}} & -\varsigma_1 P_{11} & * & * \\ \Psi_{13}^{\mathrm{T}} & 0 & -\frac{c}{2} I & * \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & * & * & * \\ 0 & \Phi_{22} + \alpha P_{22} + \varsigma_1 P_{11} & * & * \\ 0 & \Phi_{23}^{\mathrm{T}} & -\frac{c}{2} I & * \\ 0 & -\Phi_{24}^{\mathrm{T}} & 0 & -cI \end{bmatrix} < 0, \tag{30}$$

where ς_1 is some positive scalar.

By Lemma 2, it is obvious that if

$$\begin{bmatrix} \Psi_{11} + \alpha P_{11} & * & * \\ -\Psi_{12}^{\mathrm{T}} & -\varsigma_1 P_{11} & * \\ \Psi_{13}^{\mathrm{T}} & 0 & -\frac{c}{2} I \end{bmatrix} < 0 \tag{31}$$

and

$$\begin{bmatrix} \Phi_{22} + \alpha P_{22} + \varsigma_1 P_{11} & * & * \\ \Phi_{23}^{\mathrm{T}} & -\frac{c}{2} I & * \\ -\Phi_{24}^{\mathrm{T}} & 0 & -cI \end{bmatrix} < 0, \tag{32}$$

then (27) (or (20)) holds.

Remark 5. Note that (31) is related to the controller part (the parameters are P_{11} and K_i) and (32) is related to the observer part (the parameters are P_{22}, L_i, and P_{11}), respectively. Although the parameter P_{11} is still included in (32), four parameters P_{11}, P_{22}, K_i, and L_i can be determined simultaneously by the following arrangement.

Note that (31) is equivalent to

$$\begin{bmatrix} \Omega_{11} + \alpha X_{11} & * & * \\ -\Omega_{12}^{\mathrm{T}} & -\varsigma_1 X_{11} & * \\ \Omega_{13}^{\mathrm{T}} & 0 & -\frac{c}{2} I \end{bmatrix} < 0, \tag{33}$$

where

$$X_{11} = P_{11}^{-1},$$

$$\Omega_{11} = \Pi_{11} + \Pi_{11}^{\mathrm{T}},$$

$$\Pi_{11} = \frac{(A_i X_{11} + B_{1i} W_j) + (A_j X_{11} + B_{1j} W_i)}{2},$$

$$\Omega_{12} = \frac{B_{1i} W_j + B_{1j} W_i}{2},$$

$$\Omega_{13} = \frac{B_{2i} + B_{2j}}{2},$$

and $W_i = K_i X_{11}$.

And, by the Schur complements [2], (32) is equivalent to

$$\begin{bmatrix} \Phi_{22} + \alpha P_{22} & * & * & * \\ \Phi_{23}^{\mathrm{T}} & -\frac{c}{2} I & * & * \\ -\Phi_{24}^{\mathrm{T}} & 0 & -cI & * \\ I & 0 & 0 & -\frac{1}{\varsigma_1} X_{11} \end{bmatrix} < 0. \tag{34}$$

Therefore, if (33) and (34) hold, then (27) (or (20)) holds. Based on the analysis above, the minimization problem in (25) is rewritten as follows:

$$\min \quad \frac{c}{\alpha \lambda_{\min}(P)}$$

$$\text{s.t.} \quad X_{11} = X_{11}^{\mathrm{T}} > 0,\ P_{22} = P_{22}^{\mathrm{T}} > 0,\ W_i,\ Y_i,\ \varsigma_1 > 0,\ \alpha > 0,\ c > 0, (33),\ \text{and } (34). \tag{35}$$

Since X_{11} instead of P_{11} is formulated in (35), the term $c/\alpha\lambda_{\min}(P)$ should be modified as follows to solve the minimization problem in (35). Note that $c/(\alpha\lambda_{\min}(P)) < \varepsilon^2$, if $(c/\alpha)I < \lambda_{\min}(P)\varepsilon^2 I < \varepsilon^2 P$, i.e.,

$$\begin{bmatrix} \frac{c}{\alpha} I & 0 \\ 0 & \frac{c}{\alpha} I \end{bmatrix} < \begin{bmatrix} \varepsilon^2 P_{11} & 0 \\ 0 & \varepsilon^2 P_{22} \end{bmatrix}, \tag{36}$$

which is equivalent to

$$X_{11} < \frac{\varepsilon^2 \alpha}{c} I \tag{37}$$

and

$$\frac{c}{\varepsilon^2 \alpha} I < P_{22}. \tag{38}$$

Finally, the minimization problem in (35) can be reformulated as follows:

$$\min_{\{X_{11}, P_{22}, W_i, Y_i, \varsigma_1, \alpha, c\}} \quad \varepsilon^2$$

$$\begin{aligned}
\text{s.t.} \quad & X_{11} = X_{11}^{\mathrm{T}} > 0,\ P_{22} = P_{22}^{\mathrm{T}} > 0,\ W_i,\ Y_i,\ \varsigma_1 > 0,\ \alpha > 0,\ c > 0 \\
& \begin{bmatrix} \Omega_{11} + \alpha X_{11} & * & * \\ -\Omega_{12}^{\mathrm{T}} & -\varsigma_1 X_{11} & * \\ \Omega_{13}^{\mathrm{T}} & 0 & -\frac{c}{2} I \end{bmatrix} < 0, \\
& \begin{bmatrix} \Phi_{22} + \alpha P_{22} & * & * & * \\ \Phi_{23}^{\mathrm{T}} & -\frac{c}{2} I & * & * \\ -\Phi_{24}^{\mathrm{T}} & 0 & -cI & * \\ I & 0 & 0 & -\frac{1}{\varsigma_1} X_{11} \end{bmatrix} < 0, \\
& X_{11} < \frac{\varepsilon^2 \alpha}{c} I, \\
& \text{and } \frac{c}{\varepsilon^2 \alpha} I < P_{22}.
\end{aligned} \tag{39}$$

Consequently,

$$\|\eta(t)\|_\infty \leqslant \beta|\eta(0)| + \rho\|d(t)\|_\infty < \beta|\eta(0)| + \varepsilon\|d(t)\|_\infty.$$

For the steady-state response, we obtain

$$\frac{\|\eta(t)\|_\infty}{\|d(t)\|_\infty} \leqslant \rho < \varepsilon.$$

According to the analysis above, the output feedback L_∞-*gain* control problem via fuzzy control scheme is summarized as follows:

Design procedure:

Step 1. Construct the fuzzy plant rules in (1).

Step 2. Given a set of $\alpha > 0$, $c > 0$, and $\varsigma_1 > 0$ iteratively.

Step 3. Given an initial ε^2.

Step 4. Solve the following linear matrix inequality problem (LMIP)

$$\begin{aligned}
& \begin{bmatrix} \Omega_{11} + \alpha X_{11} & * & * \\ -\Omega_{12}^{\mathrm{T}} & -\varsigma_1 X_{11} & * \\ \Omega_{13}^{\mathrm{T}} & 0 & -\frac{c}{2} I \end{bmatrix} < 0, \\
& \begin{bmatrix} \Phi_{22} + \alpha P_{22} & * & * & * \\ \Phi_{23}^{\mathrm{T}} & -\frac{c}{2} I & * & * \\ -\Phi_{24}^{\mathrm{T}} & 0 & -cI & * \\ I & 0 & 0 & -\frac{1}{\varsigma_1} X_{11} \end{bmatrix} < 0, \\
& X_{11} < \frac{\varepsilon^2 \alpha}{c} I, \\
& \text{and } \frac{c}{\varepsilon^2 \alpha} I < P_{22},
\end{aligned} \tag{40}$$

to obtain X_{11}, P_{22}, W_i, and Y_i (thus $K_i = W_i X_{11}^{-1}$ and $L_i = P_{22}^{-1} Y_i$ can also be obtained).

Step 5. Decrease ε^2 and repeat Steps 4–5 until $X_{11} > 0$ and $P_{22} > 0$ cannot be found. If $X_{11} > 0$ and $P_{22} > 0$ can not be found for all possible ε^2, try another set of $\alpha > 0$, $c > 0$, and $\varsigma_1 > 0$ iteratively and repeat Steps 3–5.

Step 6. Construct the fuzzy observer (7).

Step 7. Construct the fuzzy controller (8).

Remark 6. Note that (40) is a standard LMIP, if $\varepsilon^2 > 0$, $\alpha > 0$, $c > 0$, and $\varsigma_1 > 0$ are given in advance. Therefore, four parameters $P_{11} = X_{11}^{-1}$, P_{22}, $K_i = W_i X_{11}^{-1}$, and $L_i = P_{22}^{-1} Y_i$ can be determined simultaneously from (40).

Remark 7. Software packages such as LMI optimization toolbox in Matlab [10] have been developed for this purpose and can be employed to easily solve the LMIP.

4. Simulation example

To illustrate the proposed fuzzy control approach, a control problem of balancing an inverted pendulum on a cart is considered in this study. For this example, the state equations of the inverted pendulum are given by

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= \frac{-f(M+m)x_2 - (mlx_2)^2 \sin x_1 \cos x_1 + (M+m)mgl \sin x_1 - ml \cos x_1 u}{[(M+m)(J+ml^2) - (ml\cos x_1)^2]} + w,\\
y &= x_1 + 0.01v,
\end{aligned} \tag{41}$$

where x_1 denotes the angle (rad) of the pendulum from the vertical, x_2 is the angular velocity (rad/s), $g = 9.8\,\mathrm{m/s^2}$ is the gravity constant, m is the mass (kg) of the pendulum, M is the mass (kg) of the cart, f is the friction factor (N/rad/s) of the pendulum, l is the length (m) from the center of mass of the pendulum to the shaft axis, J is the moment of inertia (kg m^2) of the pendulum, u is the force (N) applied to the cart, and w is persistent bounded external disturbance. The parameters in this example are assumed to be $m = 0.3$, $M = 15$, $l = 0.3$, $J = 0.005(\mathrm{kg\,m^2})$, and $f = 0.007(\mathrm{N/rad/s})$. The external disturbance w and the measurement disturbance v are assumed to be square wave with magnitude ± 0.5.

In this example, the state x_1 (the angle of an inverted pendulum) is assumed to be measurable. To minimize the design effort and complexity, we try to use as few rules as possible. Hence, we approximate the system by the following four-rule fuzzy model.

Rule 1 : IF x_1 is about 0
THEN $\dot{x} = A_1 x + B_{11} u + B_{21} w$, and $y = C_{11} x + C_{21} v$.
Rule 2 : IF x_1 is about $\pm\pi/9$
THEN $\dot{x} = A_2 x + B_{12} u + B_{22} w$, and $y = C_{12} x + C_{22} v$.
Rule 3 : IF x_1 is about $\pm 2\pi/9$
THEN $\dot{x} = A_3 x + B_{13} u + B_{23} w$, and $y = C_{13} x + C_{23} v$.
Rule 4 : IF x_1 is about $\pm\pi/3$
THEN $\dot{x} = A_4 x + B_{14} u + B_{24} w$, and $y = C_{14} x + C_{24} v$,
where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 28.0262 & -0.2224 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0 \\ -0.1869 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 27.4065 & -0.2220 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0 \\ -0.1753 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ 25.6263 & -0.2209 \end{bmatrix}, \quad B_{13} = \begin{bmatrix} 0 \\ -0.1422 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 1 \\ 22.8887 & -0.2197 \end{bmatrix}, \quad B_{14} = \begin{bmatrix} 0 \\ -0.0923 \end{bmatrix},$$

$$B_{2i} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_{1i} = [1\ 0] \quad \text{and} \quad C_{2i} = 0.01 \quad (i = 1, \ldots, 4).$$

Triangle type membership functions are used for Rules 1–4. According to the Design procedure, the LMIP in (40) with $\alpha = c = 3.6$ and $\varsigma_1 = 15$ is solved using the LMI optimization toolbox in Matlab. In this case, $\varepsilon^2 = (0.71)^2$,

$$P = \begin{bmatrix} 73.123 & 6.1628 & 0 & 0 \\ 6.1628 & 2.5177 & 0 & 0 \\ 0 & 0 & 2072.5 & -154.08 \\ 0 & 0 & -154.08 & 14.189 \end{bmatrix},$$

$$\rho = \sqrt{\frac{1}{\lambda_{\min}(P)}} = \sqrt{\frac{1}{1.9839}} = 0.70997 < \varepsilon = 0.71,$$

$$\text{and } \beta = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} = \sqrt{\frac{2084}{1.9839}} = 32.411.$$

For the steady-state response (i.e., $t \geqslant 3$),

$$\frac{\|\eta(t)\|_\infty}{\|d(t)\|_\infty} = \frac{0.2386}{0.7071} = 0.3374 < \rho = 0.70997.$$

The control gains are found to be

$$K_1 = \begin{bmatrix} 417.01 & 79.352 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 444.5555 & 85.2978 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 526.9350 & 103.0242 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 690.2393 & 138.4399 \end{bmatrix},$$

and the observer gains are found to be

$$L_1 = \begin{bmatrix} 86.3577 \\ 944.1885 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 86.3835 \\ 944.3999 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 86.4575 \\ 945.0079 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 86.5716 \\ 945.9464 \end{bmatrix}.$$

The trajectories of x_1, $\hat{x}_1$, and measurement disturbance v are shown in Fig. 1, while the trajectories of x_2, $\hat{x}_2$, and external disturbance w are shown in Fig. 2. The control signal is shown in Fig. 3. From the simulation results, the proposed output feedback fuzzy control clearly results in desired L_∞-*gain* performance.

Remark 8. For simplicity, $\alpha = c$ can be considered in the design procedure. From the simulations in this example, if $\alpha = c = 1.1$ is set, $14 \leqslant \varsigma_1 \leqslant 18$ is the feasible range for the LMIP in (40). If $\varsigma_1 = 15$ is set, $1.1 \leqslant \alpha = c \leqslant 3.6$ is the feasible range for the LMIP in (40). The better performance can be obtained for the larger value of α and c (i.e., $\alpha = c = 3.6$), however, the larger control input should be paid.

Alternatively, the minimization problem in (25) can also be solved using the two-step procedure proposed by Chen et al. [4,5]. The idea of the two-step method is to solve P_{11} and K_i first from the entry (1, 1) of (27) and then by substituting them into (27) to solve P_{22} and L_i. Following the two-step procedure, the following parameters can be obtained by letting $\alpha = c = 3.6$ and $\varepsilon^2 = (0.71)^2$:

$$P = \begin{bmatrix} 59.245 & 14.795 & 0 & 0 \\ 14.795 & 6.1296 & 0 & 0 \\ 0 & 0 & 2900.5 & -246.35 \\ 0 & 0 & -246.35 & 23.279 \end{bmatrix},$$

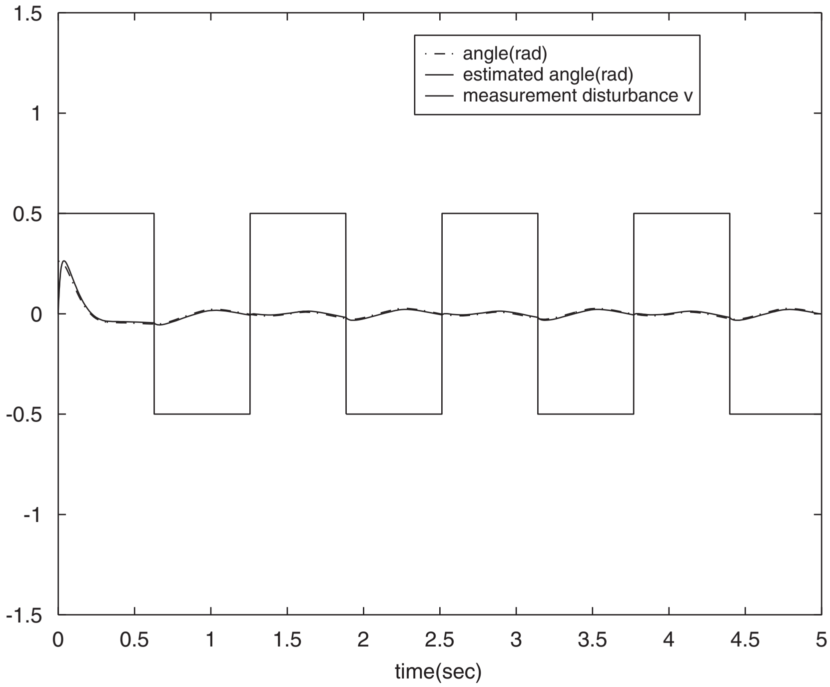

Fig. 1. The trajectories of x_1 (dash-dot line), $\hat{x}_1$ (solid line), and measurement disturbance v (square wave with period $2\pi/5$).

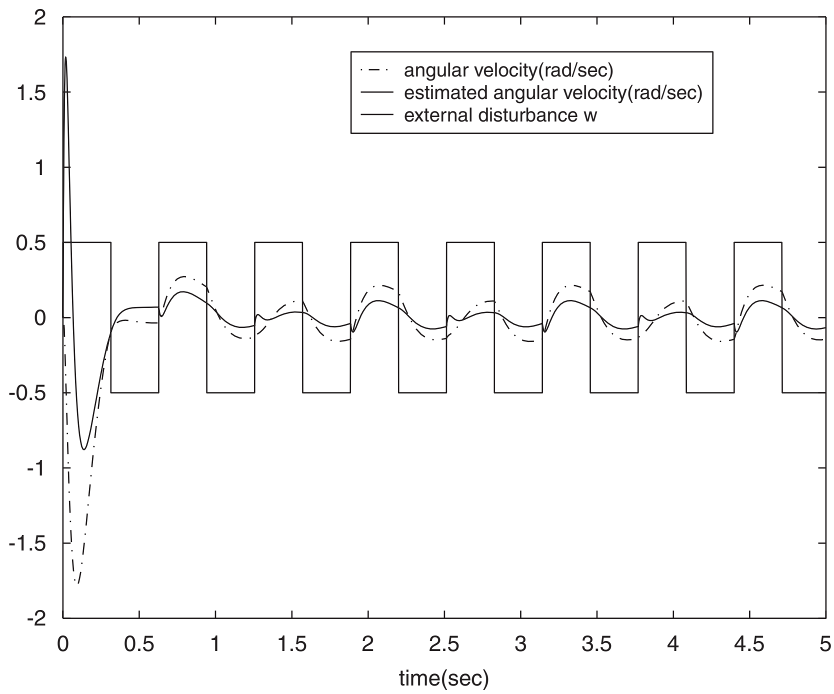

Fig. 2. The trajectories of x_2 (dash-dot line), $\hat{x}_2$ (solid line), and external disturbance w (square wave with period $\pi/5$).

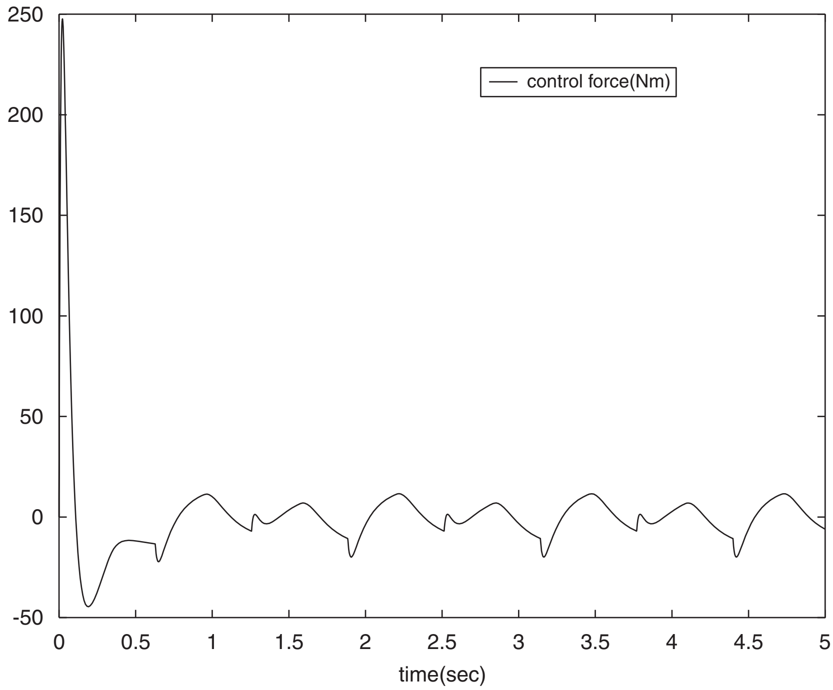

Fig. 3. The control signal.

the control gains are found to be

$$K_1 = \begin{bmatrix} 381.5911 & 95.7524 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 403.7620 & 97.2794 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 486.5864 & 124.4424 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 600.7120 & 146.2048 \end{bmatrix},$$

and the observer gains are found to be

$$L_1 = \begin{bmatrix} 102.95 \\ 1116.81 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 102.82 \\ 1114.77 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 102.82 \\ 1113.12 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 103.86 \\ 1122.54 \end{bmatrix}.$$

However, sometimes one can solve P_{11} and K_i from the first step but cannot solve P_{22} and L_i from the second step. The following example illustrates the case. In the case of $\alpha = c = 1.1$ and $\varepsilon^2 = (0.71)^2$, one can solve

$$P_{11} = \begin{bmatrix} 30.0526 & 12.5479 \\ 12.5479 & 8.0486 \end{bmatrix}$$

and the control gains

$$K_1 = [213.6589 \;\; 34.3263], \quad K_2 = [226.4445 \;\; 36.6610],$$

$$K_3 = [259.5527 \;\; 41.9007], \quad K_4 = [306.5351 \;\; 50.3899],$$

in the first step; however, there is no feasible solution for P_{22} and L_i in (27) by substituting above P_{11} and K_i into (27) in the second step. Note that feasible solution can be found for this case by the proposed method in this study.

5. Conclusions

In this study, L_∞-*gain* output feedback control problem for a nonlinear dynamic system is solved for the first time by the fuzzy control scheme. A novel separation method is provided to solve the control gains and the observer gains simultaneously. The problem is transformed into a L_∞-*gain* suboptimal problem with LMI constraints and can be efficiently solved with LMI toolbox in Matlab [10]. Both the stability and L_∞-*gain* disturbance rejection performance are guaranteed for the fuzzy control system. Since the peak of the system is of greater concern than the energy in some design cases, the proposed L_∞-*gain* fuzzy observer-based fuzzy control for nonlinear dynamic systems is more appealing in some practical applications. Several simulation results have confirmed the elimination of the peak of the output signal against the persistent bounded disturbances for the nonlinear system by the proposed fuzzy L_∞-*gain* suboptimal control scheme.

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