Comments on "HEED：A Hybrid, Energy-Efficient, Distributed Clustering Approach for Ad Hoc Sensor Networks"

Chia-Hung Lin and Ming-Jer Tsai

Department of Computer Science
National Tsing Hua University
Hsing Chu, Taiwan, ROC
E-mail: mjtsai@cs.nthu.edu.tw

Abstract

We provide a better sufficient condition for the connectivity of cluster heads asymptotically almost surely (a.a.s.) and a tighter bound on the number of cluster heads in HEED [1].

Index Terms－Sensor networks, clustering, network lifetime, distributed algorithm.

1. Introduction

In network field $[0, L]^2$, where $L \to \infty$, Younis and Fahmy [1] show each $\frac{R_c}{\sqrt{2}} \times \frac{R_c}{\sqrt{2}}$ cell contains at least one node a.a.s. in Lemma 6. They claim that cluster heads are connected a.a.s. in HEED if $R_t \geq \sqrt{5}\left(2 + \frac{1}{\sqrt{2}}\right) R_c \approx 6 R_c$ in Theorem 1. Besides, they show any $\left(2 + \frac{1}{\sqrt{2}}\right) R_c \times \left(2 + \frac{1}{\sqrt{2}}\right) R_c$ area contains at least one cluster head a.a.s. in HEED in Lemma 7, implying HEED has at least $\frac{2}{4\sqrt{2}+9} \times \frac{L^2}{R_c^{\,2}} \approx 0.137 \frac{L^2}{R_c^{\,2}}$ cluster heads a.a.s. in network field $[0, L]^2$. In this paper, Theorem 1 shows cluster heads are connected a.a.s. in HEED if $R_t \geq \left(1 + \frac{\sqrt{26 + 16\sqrt{2}}}{4}\right) R_c \approx 2.75 R_c$ in Section 2, and Theorem 2 shows HEED has at least $\frac{2}{3\left(4\sqrt{2} - 1\right)} \times \frac{L^2}{R_c^{\,2}} \approx 0.144 \frac{L^2}{R_c^{\,2}}$ cluster heads a.a.s. in network field $[0, L]^2$ in Section 3.

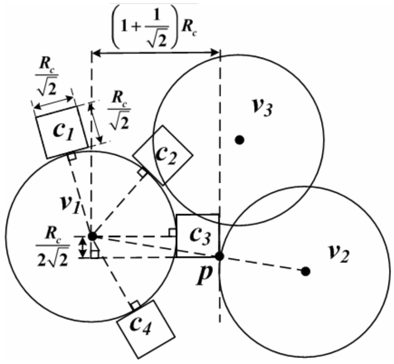

Fig. 1. Uncovered neighboring cells c_1, c_2, c_3, and c_4 and a neighboring cluster head v_3 of cluster head v_1, where the circle represents the coverage of the cluster head. v_2 is a cluster head, where the straight line between v_1 and v_2 passes through c_3's down-right corner P and the distance between P and v_2 is R_c.

2. The Better Sufficient Condition

The following notations and lemmas are used in the proof of Theorem 1.

Definition 1. *Let c be a $\frac{R_c}{\sqrt{2}} \times \frac{R_c}{\sqrt{2}}$ cell and let v be a cluster head. c is said to be a v's uncovered neighboring cell if (1) c is not covered or partially covered*[i] *by v, and (2) the center of one c's border is the point of tangency with v's coverage.*

Definition 2. *Let v_1 and v_2 be two cluster heads. v_2 is said to be v_1's neighboring cluster head if v_2 covers or partially covers at least one of v_1's uncovered neighboring cells.*

Take Fig. 1, for example. c_1, c_2, c_3, and c_4 are some v_1's uncovered neighboring cells, and v_3 is v_1's neighboring cluster head. Note that v_2 is not v_1's neighboring cluster head since v_2's coverage does not cover or partially cover any v_1's uncovered neighboring cell.

Lemma 1. *Two neighboring cluster heads can communicate if* $R_t \geq \left(1+\frac{\sqrt{26+16\sqrt{2}}}{4}\right)R_c$.

Proof. The maximum distance between any two neighboring cluster heads is less than the distance between v_1 and v_2 in Fig. 1, which is $\left(1+\sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2}\right)R_c$. □

i. A cell c is partially covered by v_1 if some subarea of c is within v_1's coverage.

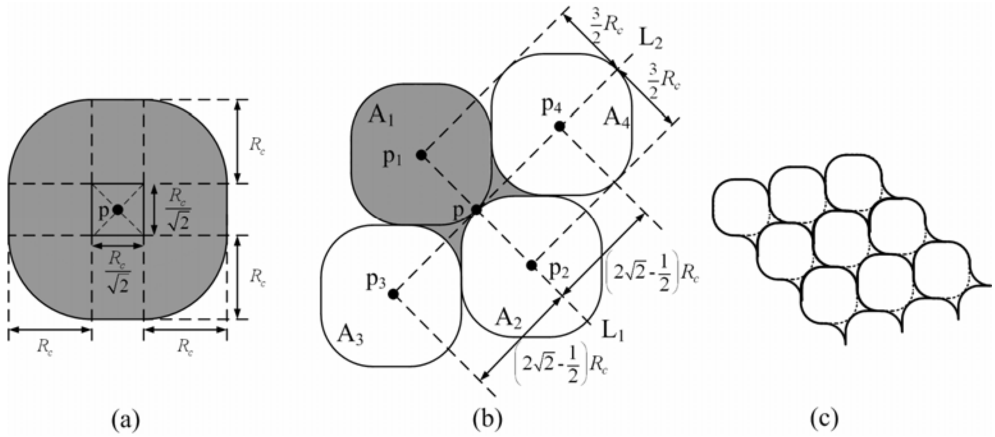

Fig. 2. Areas θ and Ω. (a) shows a θ area shown in gray area. (b) shows a Ω area shown in gray area. (c) shows the network field $[0, L]^2$ divided into several Ω areas.

Lemma 2. *Assume that HEED produces two isolated connected components of cluster heads G_1 and G_2 and that G_2 lies on the right side of G_1. Then any cluster head v_1 in G_1 has one neighboring cluster head on its right side a.a.s.*

Proof. Let c be the rightmost one of v_1's uncovered neighboring cells. So, c cannot be covered or partially covered by any cluster head on v_1's left side. Since at least one node is in c a.a.s. by Lemma 6 in [1], c is covered or partially covered by at least one cluster head, say v_2, on v_1's right side a.a.s. Clearly, v_2 is v_1's neighboring cluster head. □

Theorem 1. *HEED produces a connected multihop network a.a.s. if* $R_t \geq \left(1 + \frac{\sqrt{26+16\sqrt{2}}}{4}\right) R_c$.

The proof of Theorem 1 is similar to that of Theorem 1 in [1], and hence is omitted here.

3. The Tighter Bound

The following notation and lemma are used in the proof of Theorem 2.

Definition 3. *Area A is said to be a θ area (as shown in Fig. 2a) if a $\frac{R_c}{\sqrt{2}} \times \frac{R_c}{\sqrt{2}}$ cell c exists such that c is only covered or partially covered by any cluster head inside A, where c is said to be centrally enclosed by A. In addition, the intersection point p of two diagonals of c is said to be A's core.*

Definition 4. *Let* p_1, p_2, p_3 *and* p_4 *be the cores of four non-overlapping* θ *areas* A_1, A_2, A_3 *and* A_4, *respectively, where* A_1 *tangent to* A_2 *in one point* p, $D(p_1, p_2) = 3R_c$ ($D(u,v)$ *denotes the distance between two points* u *and* v), $D(p_3, p) = D(p_4, p) = \left(2\sqrt{2} - \frac{1}{2}\right)R_c$, p_1, p_2, p *lie in line* L_1, p_3, p_4, p *lie in line* L_2, *and* L_1 *is perpendicular to* L_2. *Then an area is a* Ω *area (as shown in Fig. 2b) if it contains* A_1, *the area surrounded by* A_1, A_2, A_3, *and the area surrounded by* A_1, A_2, A_4.

Lemma 3. Any θ area contains at least one cluster head a.a.s.

Proof. Let A be a θ area centrally enclosing $\frac{R_c}{\sqrt{2}} \times \frac{R_c}{\sqrt{2}}$ cell c. Since at least one node is in c a.a.s. by Lemma 6 in [1] and c cannot be covered or partially covered by any cluster head outside A, there is at least one cluster head inside A a.a.s. □

Theorem 2. In network field $[0, L]^2$, where $L \to \infty$, the number of cluster heads in HEED is at least $\frac{2}{3\left(4\sqrt{2}-1\right)} \times \frac{L^2}{R_c^2}$ a.a.s.

Proof. The network field $[0, L]^2$ can be divided into several Ω areas as shown in Fig. 2c. Because a Ω area contains a θ area, any Ω area contains at least one cluster head a.a.s. by Lemma 3. The measurement of a Ω area is $\left(6\sqrt{2} - \frac{3}{2}\right)R_c^2$. Thus there is at least $\frac{2}{3\left(4\sqrt{2}-1\right)} \times \frac{L^2}{R_c^2}$ cluster heads a.a.s. in network field $[0, L]^2$. □

References

[1] O. Younis and S. Fahmy, "HEED: A Hybrid, Energy-Efficient, Distributed Clustering Approach for Ad Hoc Sensor Networks," *IEEE Trans. Mobile Computing*, vol. 3, no. 4, pp. 366-379, Oct.-Dec. 2004.