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NEW CLASSES OF SIGNATURE SEQUENCES FOR DS/CDMA SYSTEMS

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Abstract

In this paper, we propose three classes of multi-valued signature sequences for direct sequence code division multiple access (DS/CDMA) systems. The first class derived from the two-band perfect reconstruction quadrature mirror filter (PR-QMF) bank has perfect correlation properties when the number of users is two in the DS/CDMA system. The first two classes have perfect correlation properties for synchronous DS/CDMA systems. In addition, the third class in general has better correlation properties than other well-known sequences also in asynchronous DS/CDMA systems.

Keywords

DS/CDMA, signature sequence, PR-QMF, correlation

1 Introduction

The acceptable number of users depends on many aspects including the correlation properties of the signature sequences in direct sequence code division multiple access (DS/CDMA) systems. The odd correlation (OC) function affects the output of the correlator when the information symbols change over one integration interval, while the even correlation (EC) function affects the output when the information symbols do not change.

The autocorrelation (AC) property is said to be ideal when the AC value is N for the inphase ($\tau = 0$) component and zero for the out-of-phase ($\tau \neq 0$) components, where N is the length of the sequence. This property can support fast synchronization and low multipath interference (MPI). The crosscorrelation (CC) property of signature sequences is linked directly with the multiple access interference (MAI), and consequently with the capacity of the system. Thus, for low MAI and large channel capacity, it is desired that the CC value is always zero. However, the design of signature sequences with near-perfect correlation properties is not an easy task [1]-[4].

Recently, in order to obtain sequences having better correlation properties, a number of non-binary signature sequences have been suggested with various techniques. Among them, multi-valued signature sequences based on the perfect reconstruction quadrature mirror filter (PR-QMF) have been proposed in [5]-[7]. These signature sequences have been generated using the impulse responses of the subbands of a PR-QMF bank. In this paper, three classes of signature sequences are proposed based on the PR-QMF banks for DS/CDMA systems.

2 The First Class of Sequences

The M analysis filters $\{h_0(n)\}$, $\{h_1(n)\}$, $\cdots$, $\{h_{M-1}(n)\}$ represent a PR-QMF bank if and only if

$$\sum_{n=0}^{L} h_i(n)h_i(n+Mk) = \delta(k) \tag{1}$$

and

$$\sum_{n=0}^{L} h_i(n)h_j(n+Mk) = 0 \tag{2}$$

for $i, j \in \{0, \cdots, M-1\}$, $i \neq j$, and any integer k. Here, L is the order of the filters, M is the number of subbands of the PR-QMF bank, $\delta(\cdot)$ is the Kronecker delta function,

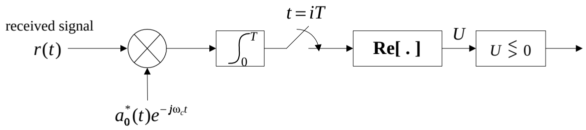


Figure 1. Receiver model for the reference user (user index $= 0$) when the signature sequence is in the form (5).

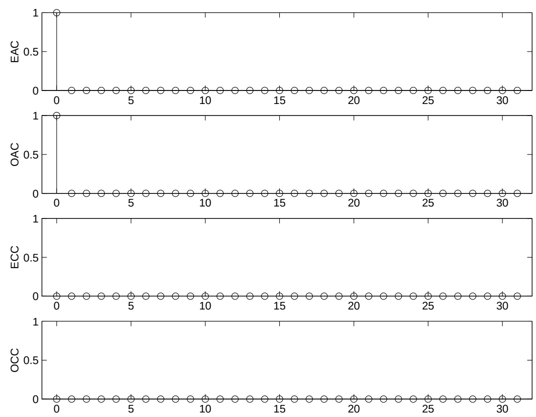


Figure 2. Correlation values of the modified sequences

and $h_i(n) = h_i(n + l(L+1))$ for any arbitrary integer l is the impulse response of the ith subband filter. Equation (1) implies that the impulse response $h_i(n)$ is normalized to have unit energy $\sum_{n=0}^{L} |h_i(n)|^2 = 1$ and is orthogonal to its translations by non-zero multiples of M. Equation (2) implies that $h_i(n)$ is orthogonal to $h_j(n)$, $j \neq i$, translated by any multiple of M.

Let us consider the two-band ($M = 2$) PR-QMF bank with analysis filters $H_0(z) = \sum_{n=0}^{L} h_0(n)z^{-n}$ and $H_1(z) = \sum_{n=0}^{L} h_1(n)z^{-n}$. Then, from (1) and (2) with $M = 2$, it is easy to see that the magnitudes of the AC function of $h_i(\cdot)$ for $i = 0, 1$ are zero when the non-zero chip-delay is even, and those of the CC function between $h_0(\cdot)$ and $h_1(\cdot)$ are zero at all even chip-delay. However, the magnitudes of the AC and CC functions are not always zero when the chip-delay is odd. In [6], a pair of signature sequences is suggested through an optimization method: the CC functions still have non-zero magnitudes when the chip-delay is odd although the AC functions of the sequence are perfectly optimized.

To eliminate the effect of the non-zero magnitudes which occur at the odd chip-delay of the CC functions, we propose a set of sequences $\{\mathbf{c}_i\}$ defined by

$$\mathbf{c}_i = [u_{i,0} + jv_{i,0} \quad u_{i,1} + jv_{i,1} \quad \cdots \quad u_{i,N-1} + jv_{i,N-1}]^T, \tag{3}$$

where $N = \lfloor \{L(M-1)+1\}/2 \rfloor$ is the length of the sequence, $u_{i,n} = h_i(2n)$, and $v_{i,n} = h_i(2n+1)$, for $i = 0, 1, \cdots, M-1$. Then, when $M = 2$, the undesired (non-zero) components in the CC function of the original coefficients become the imaginary parts of the correlation outputs. We can eliminate the undesired components by taking the real part of the correlation output. Figure 1 shows the receiver model of the reference user when a signature sequence of the form (3) is used, where $r(t)$ is the received signal, T is the symbol duration, ω_c is the common carrier frequency, and $a_i(t) = \sum_{k=-\infty}^{\infty} c_{k,i} p_{T_c}(t - kT_c)$ is the ith signature waveform with $c_{k,i}$ the kth element of the ith signature sequence $\mathbf{c}_i$, T_c the chip duration, and p_A the unit rectangular pulse defined by $p_A(t) = 1$ for $0 \leq t < A$ and 0 elsewhere.

Figure 2 shows the correlations of the modified sequence when $N = 31$ and $M = 2$. From Figure 2, we can clearly see that the modified sequence has perfect correlation properties as expected: this is because the chip can take any value and the number of sequences is limited to two.

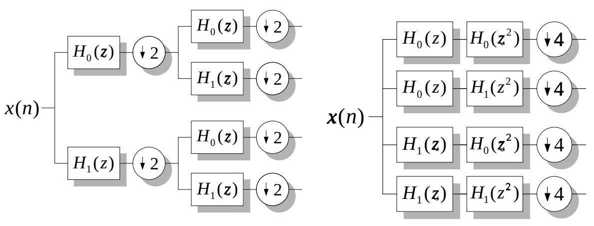
A set of a large number of signature sequences which have good correlation properties is required in practical DS/CDMA systems. In the next section, we propose two classes of signature sequences based on the multi-band PR-QMF bank with which we can allow a large number of users in DS/CDMA systems.

3 The Second and Third Classes of Sequences

3.1 Sequences based on tree structure PR-QMF bank

One of the simplest approaches for the design of a multi-band PR-QMF bank is to connect a number of two-band PR-QMF banks successively in a tree structure. Let $P_i(z)$ be the ith subband filter of a tree structure M-band PR-QMF bank without decimation and $p_i(n)$ be the impulse response of $P_i(z)$, for $i = 0, 1, \cdots, M-1$, where $M = 2^m$ with an integer depth $m \geq 1$. Then, the order of $P_i(z)$ is given by $L(M-1)$, where L is the order of the prototype low-pass filter (LPF) and high-pass filter (HPF). We now obtain a class of signature sequences by modifying $p_i(n)$ as in (3) with $u_{i,n} = p_i(2n)$ and $v_{i,n} = p_i(2n+1)$ for $n = 0, 1, \cdots, N-1$. Such sequences will be called the *tree-structure (TS)* sequences. The length N of a TS sequence is given by $N = \lfloor \{L(M-1)+1\}/2 \rfloor$.

As an example, a four-band tree structure of the PR-QMF bank is shown in Figure 3a. In this figure, $H_0(z)$ and $H_1(z)$ are the prototype LPF and HPF of order L in the analysis stage, respectively. Similarly, $F_0(z)$ and $F_1(z)$ are the prototype LPF and HPF in the synthesis stage, respectively. Using the noble identities [8] of multirate systems, the analysis stage of Fig. 3a can be represented equivalently as Fig. 3b. Then, the four subband filters $P_i(z)$ and their impulse responses $p_i(n)$, for $i = 0, 1, 2, 3$ and



(a) a four-band tree structure

(b) a structure equivalent to (a)

Figure 3. A four-band tree structure of the PR-QMF bank

$n = 0, 1, \cdots, 3L$, are represented as

$$\begin{aligned} P_0(z) &= H_0(z)H_0(z^2) = \sum p_0(n)z^{-n}, \\ P_1(z) &= H_0(z)H_1(z^2) = \sum p_1(n)z^{-n}, \\ P_2(z) &= H_1(z)H_0(z^2) = \sum p_2(n)z^{-n}, \end{aligned}$$

and

$$P_3(z) = H_1(z)H_1(z^2) = \sum p_3(n)z^{-n}.$$

We can then obtain four sequences $\mathbf{c}_0$, $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ by modifying $p_i(n)$ as in (3) with $u_{i,n} = p_i(2n)$ and $v_{i,n} = p_i(2n+1)$ for $n = 0, 1, \cdots, \lfloor (3L+1)/2 \rfloor$.

Let $\mathcal{A}$ be the class of all the TS sequences with size M. Then, the class $\mathcal{A}$ can be divided into two subclasses $\mathcal{A}_0$ and $\mathcal{A}_1$ each with size $M/2$. The subclass $\mathcal{A}_0$ is defined by the TS sequences for which the first filter of the subband used is $H_0(z)$. The subclass $\mathcal{A}_1$ is the complement of $\mathcal{A}_0$ in $\mathcal{A}$. Then, the correlation properties of the TS sequence can be characterized as follows:

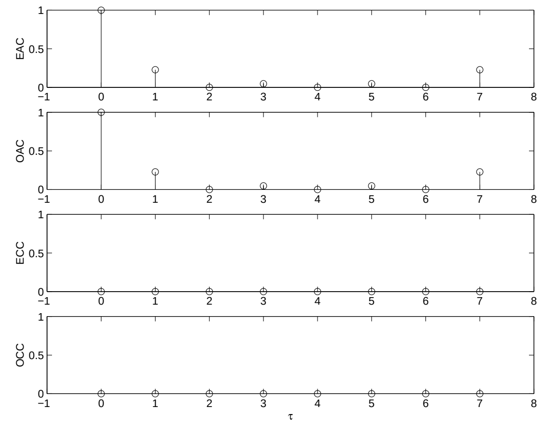
i) The magnitudes of the AC and CC are N and zero, respectively, when the chip-delay is 0.

ii) The magnitudes of the AC and CC are zero when the non-zero chip-delay is a multiple of $M/2$.

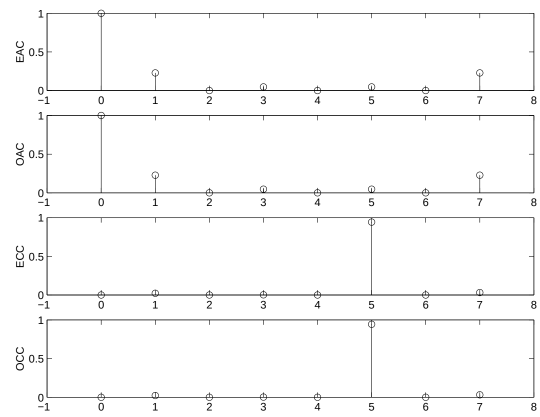
iii) The CC property between a sequence in $\mathcal{A}_0$ and one in $\mathcal{A}_1$ is perfect.

iv) The maximum magnitude of the CC between two sequences in the same subclass is very large.

Properties (i) and (ii) are induced from (1), (2), and (3). According to Properties (iii) and (iv), while $M/2 \times M/2$ pairs of sequences can be selected in a class of TS sequences so that their CC properties are perfect, the other $M/2 \times (M/2-1)$ pairs of sequences unfortunately have CC properties inappropriate to be applied for asynchronous multiple access systems. Thus, the enhanced correlation properties of the TS sequence can be fully exploited only in synchronous DS/CDMA systems.



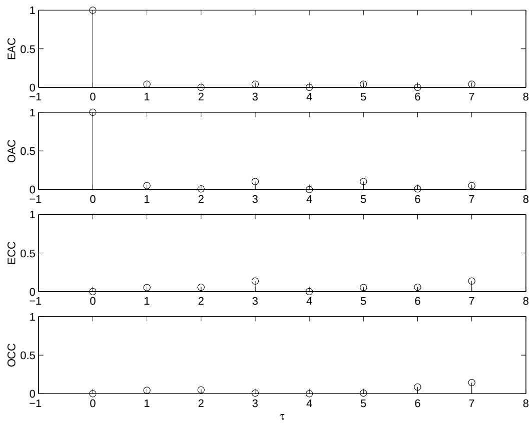
(a) in the different subclasses



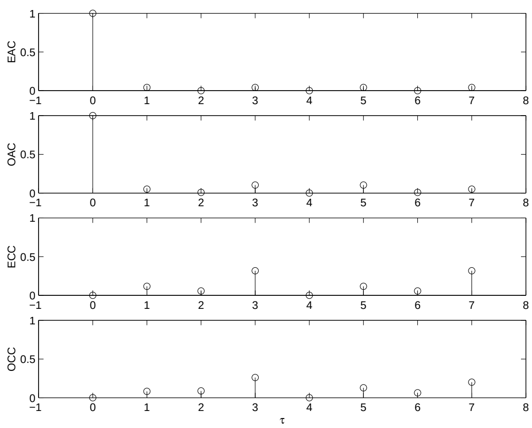
(b) in the same subclass

Figure 4. Correlation of two TS sequences

Figures 4a and 4b show the normalized absolute correlation of a pair of TS sequences in different subclasses and in the same subclass, respectively, when $L = 5$, $M = 4$, and $N = 8$. The maximum values of the out-of-phase AC in the two figures are both 0.1223: this value is satisfactory considering that, when the length of sequence is 7, the corresponding value of the Gold sequence is about 0.125. The CC values in Figure 4a are all zero as mentioned in Property (iii). The maximum CC value in Figure 4b is 0.9783, which is too large for the sequences to be used in the asynchronous CDMA systems. Thus, we have to consider another way to enlarge the number of users for asynchronous DS/CDMA systems. In the next subsection, we will find a solution using the lattice structure multi-band PR-QMF bank.

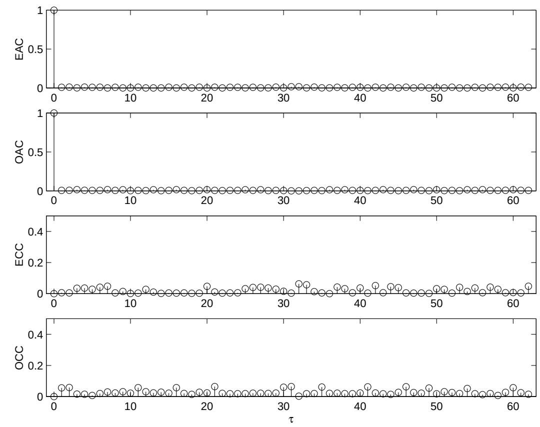


(a) the smallest maximum CC

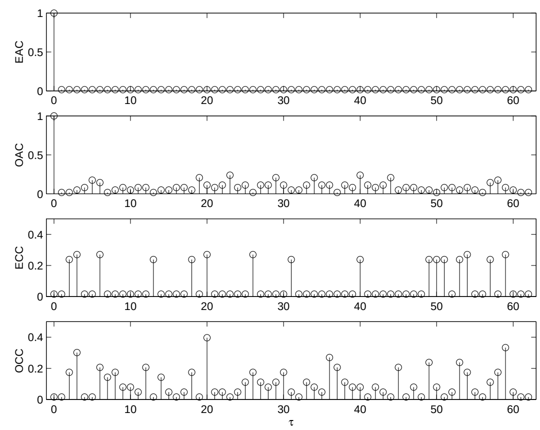


(b) the largest maximum CC

Figure 5. Correlation of two TS sequences



(a) the LS sequences



(b) the Gold sequences

Figure 6. Correlations of the LS and Gold sequences

3.2 Sequences based on lattice structure PR-QMF bank

Another approach for the design of multi-band PR-QMF bank is to use the lattice structure connecting the factorized paraunitary transfer matrices in cascade. Let us consider the transfer matrix

$$\mathbf{V}_m(z) \quad = \quad \mathbf{I} - \mathbf{v}_m {\mathbf{v}_m}^{\dagger} + z^{-1} \mathbf{v}_m {\mathbf{v}_m}^{\dagger}, \qquad (4)$$

where $\mathbf{v}_m$ is an $M \times 1$ column vector with unit-norm. Using the unit-norm property, it is easy to verify that $\tilde{\mathbf{V}}_m(z)\mathbf{V}_m(z) = \mathbf{I}$ so that $\mathbf{V}_m(z)$ is paraunitary: that is, $\tilde{\mathbf{V}}_m(z) = {\mathbf{V}_m}^{\dagger}(1/z^*)$, where * and † represent the conjugate and conjugate transpose, respectively. The matrix $\mathbf{V}_m(z)$ can be implemented using one delay, and therefore has order one. It can be shown that any causal paraunitary matrix $\mathbf{E}(z)$ with order J can be expressed as [8]

$$\mathbf{E}(z) \quad = \quad \mathbf{V}_J(z)\mathbf{V}_{J-1}(z)\cdots\mathbf{V}_1(z)\mathbf{U}, \qquad (5)$$

where $\mathbf{U}$ is a constant unitary matrix (that is, $\mathbf{U}^\dagger\mathbf{U} = d\mathbf{I}$). The analysis filter bank is then obtained as

$$\begin{aligned} \mathbf{h}(z) \quad &= \quad [H_0(z) \;\; H_1(z) \;\; \cdots \;\; H_{M-1}(z)]^T \\ &= \quad \mathbf{E}(z^M)\mathbf{e}(z), \qquad (6) \end{aligned}$$

where $\mathbf{e}(z) = [1 \; z^{-1} \; \cdots \; z^{-M+1}]^T$.

We can obtain a class of signature sequences by modifying the coefficients of the analysis filters of the lattice structure M-band PR-QMF bank using (3). Such sequences will be called the *lattice-structure (LS)* sequences in this chapter. The length N of an LS sequence is given by $N = \lfloor M(J+1)/2 \rfloor$.

Let us now consider the correlation properties of the LS sequence with some examples. Figures 5a and 5b show the correlations of the LS sequence when $N = 8$, $M = 4$, $J = 3$, and the maximum value of the CC is the smallest and largest, respectively. We can clearly see that Properties (i) and (ii) obtained for the TS sequence hold also for the LS sequence since we use the M-band PR-QMF bank for generating the LS sequence. We can also see that, although the smallest magnitude for the LS sequence is not zero (while that for the TS sequence is zero), the largest magnitude for the LS sequence is satisfactorily small: the maximum magnitude of the CC is 0.3472 in Figure 5b and it is smaller than that ($= 0.5345$) of the ECC of the Gold sequence with length 7.

Figures 6a and 6b show the correlations of the LS and Gold sequences, respectively, when $N = 63$. In these figures, the maximum magnitude of the normalized out-of-phase AC of the LS sequence is 0.0157 while that of the Gold sequence is 0.0159. The maximum value of the CC functions of the LS sequence is 0.0754 while that of the Gold sequence is 0.4182. Thus, we can expect that the LS sequence can be used in asynchronous DS/CDMA systems, resulting in a better performance than the Gold sequence.

4 Conclusion

In this paper, we have proposed three classes of signature sequences based on PR-QMF banks, investigated the correlation properties of the proposed sequences, and considered applications of the proposed sequences in DS/CDMA systems.

The sequences in the first class has perfect correlation properties, but accommodate only two users when applied in DS/CDMA system. To obtain signature sequences which can support more users in DS/CDMA systems, we have proposed the second and third classes. The second class obtained from the tree structure PR-QMF bank has perfect CC and good AC properties in synchronous DS/CDMA systems. The third class obtained from the lattice structure PR-QMF bank possesses enhanced AC and CC properties for asynchronous DS/CDMA systems also. The third class has been shown to have much better correlation properties than Gold sequence.

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