Efficient pricing of a bundled product of both real and reactive power

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This paper examines price mechanism with one price assigned for each level of bundled real and reactive power. This pricing approach is simulated on the simple 3-bus system power auction where generators provide their bids on the bundle of real and reactive power. System operator (SO) is able to dispatch generators efficiently when the generators bid competitively. Incentives to exercise market power with respect to reactive power are tested on the auction. In addition, 30-bus network was tested, with the purpose of identifying generators willing to raise reactive power bids.

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1 Introduction

An efficient transmission network plays a crucial role in growing power markets around the world. In the last five years generating capacity grew enormously; however, transmission investment has been declining for many years(US. Department of Energy). Many countries have already adopted competitive market programs where an Independent System Operator (ISO) or Regional Transmission Operator (RTO) is responsible for scheduling and dispatching generators on regional networks, implementing a market-based mechanism for allocating scarce transmission capacity Joskow(2003).

The direct current (DC) load model is a common approximation for estimating spot market prices under constrained network. The DC Load model is insufficient since it ignores reactive power effects on the production of real power, congestion of the line and voltage constraint.

Hogan(1993) created a separate price mechanism for reactive power in order to stimulate its production with a purpose of satisfying voltage constraints. Later, Kahn and Baldick (1994) demonstrated that although Hogan's pricing example for reactive power yielded a Pareto improving (more efficient) dispatch, it was not a solution of the formal optimal power flow problem.

Further theoretic studies of the reactive power were shifted into the engineering literature, focusing on how marginal reactive power should be determined and priced. Hao[1997, 2003] explored the technical and economic issues of determining reactive power structures and designed a practical solution for managing reactive power services. Singh[1999] discussed auction design for the ancillary services.

A great deal of engineering research centered on the technical side of the solution algorithm. Weber[1998] modified standard OPF to simulate real and reactive power prices. Gil[2000] proposed a theoretical approach of marginal pricing charge for reactive services. Alvarado[1996] suggested marginal costs for dynamic reactive power.

There is a clear gap in the game theoretic approach, and strategic bargaining in the area of the reactive power, comparing to the amount of work done by economists on the real power side with DC approximation.

This paper presents an alternative pricing mechanism where price for real and reactive power are merged into one bid, without distinct separation of value for each power component. While making production offer generators consider evaluate overall profit obtained from the production. In addition, separation of the reactive power is not very practical since it yields extremely volatile variable dependent on the real power output.

In the next chapter a simple 3 bus network OPF solution is presented as a starting point. The following chapter contrasts real and reactive power properties, in particular substitutability in production and cost effects. Chapter 4 and 5 introduces bundled pricing methods and its advantages for the power auction for the 3 bus system. Chapter 6 demonstrates that there are no efficiency loss for this pricing method in the general case. Chapter 7 examines a 30-bus power system, investigating incentives of generators to exercise market power with respect to reactive power.

2 Power Network Model

A simple 3-bus triangular AC network example is analyzed. There are two generators and a load connected by the transmission lines (see Figure 1). Numerical parameters of the transmission lines and generators are represented in the Appendix. Loadflow on Figure 1 represent economic dispatch of the generators. That is in order to satisfy LOAD (1500 MW; 200 MVR) in the cheapest possible way, first generator (G1) has to produce 847.1 MW of real and 8.1 MVR of reactive power; second generator(G2) has to produce remaining 721.6 MW and 371.1 MVR. The solution was found by solving formal Bergen(1986) non-linear cost minimization subject to transmission, voltage and generation constraints [1]

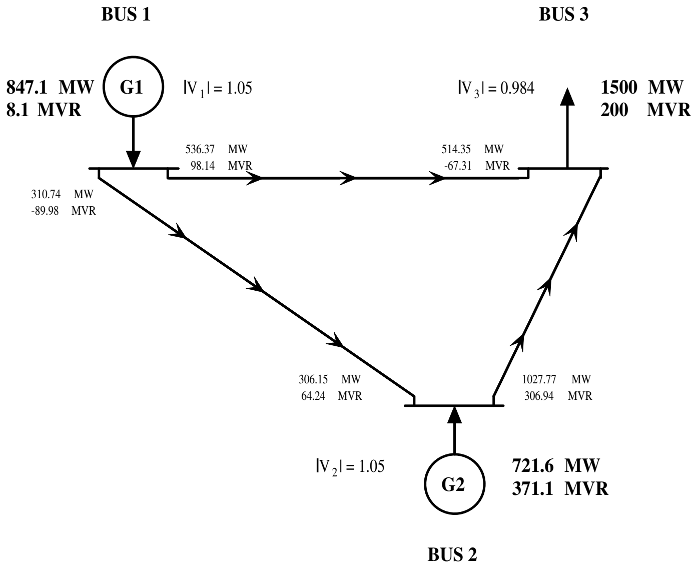

Figure 1: Triangular Network OPF example

It is important to notice that no prices were involved while the efficient allocation was calculated. This cost minimizing approach is referred to as the Social Planner Solution or alternatively in the power literature Wilson(2002) as a Regulated Market Equilibrium.

Competitive market is another way to obtain the efficient allocation. Efficiency is obtained by dispatching more expensive generators via a price mechanism. Competition in the power market is organized in the form of an auction, where cheaper generators presumably underbid more expensive generator and remain profitable. When competition eliminates expensive (inefficient) producers, power is produced efficiently (i.e. at the lowest possible auction reward).

There are some features about an electricity auction which makes it different from a regular tangible goods auction. First of all, generators and loads are located at different nodes of the transmission network implying that more distant generators can have substantial

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[1] See Appendix for details

transmission losses; second direction of power flow obeys Kirchkoff's law which means that in addition to the location, losses at one node depend on the power injection at every node as well. When prices are adjusted at each node to the extend of transmission losses, it maps the power network into the space where all generators are located directly at the load. Therefore, nodal prices allow the generator to outbid competitors when its "after-loss-delivered-power" is cheaper. Basically, location marginal prices (LMPs) provide fair competition among generators and ultimately yield efficient dispatch.

3 Reactive Power

Under existing technology power generation and distribution uses alternating current (AC). In contrast to direct current (DC), AC power is characterized by two components Real (megawatts MW) and imaginary Reactive (megavars MVR). Reactive power is used to define how far the current is out of phase with the voltage (Reason 1989). The combined complex power magnitude is denoted in volt amperes (VA) or usually in mega VA (MVA) [2].

Real Power vs. Reactive Power

Unlike real power reactive power is usually cheap to produce within certain range and therefore it is not priced. There are 3 major sources of reactive power production: capacitor banks, synchronous condensers, and generators. In this paper we consider a static model under normal operating conditions, with generators only capable to produce reactive power at no cost within a certain range.

Reactive power is usually generated close to the load, since it can not be transmitted efficiently over a long distance Joskow(2003). The example on Figure 1 is a clear evidence. Most of the reactive power is produced by the generator 2, located closer to the load.

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[2] $|\mathrm{MVA}| = \sqrt{\mathrm{MW}^2 + \mathrm{MVR}^2}$

Despite the fact that it can be generated cheaply, reactive power congests the line to the same extent as real power. Line capacity is usually measured in MVA.

The main reason why the reactive power should be valued in the competitive market (even though its production could be cheap) is to restrict its impact on real power production.

In Figure 2 I have shown the changes in the overall costs of power production when OPF solution is run and the reactive power of the second generator is fixed. The production of the reactive power would not influence efficiency of the system when shortage in the reactive power production by one generator could be substituted by higher reactive power production of the other generator without losses in the real power.[3] In other words, reactive power would not matter if its production level could vary for generation for the same amount of the real power. In that case cheap reactive power production could be substituted by any generator in the system keeping the expensive real power unchanged.

Economic incentive to produce reactive power inefficiently is justified, since making higher production costs for the competitor gives the advantage in bidding.

4 Pricing the Reactive Power

Hogan(1993) priced the reactive power based on voltage constraint and line congestion. In his work each generator had two distinct prices for real and reactive power. Kahn and Baldick (1994) found that although Hogan's pricing approach was Pareto improving, it was not Pareto optimal. They demonstrated a counterexample that allowed to produce power at the lower costs. So the question of how to price the reactive power remained open.

Opportunity costs of the reactive power is tightly related to the amount of real power. That is reactive power cost depends a lot on the output of real power. For example, 50 MVAR can

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[3] In the program code I have set the reactive power of the second generator fixed and then run OPF.

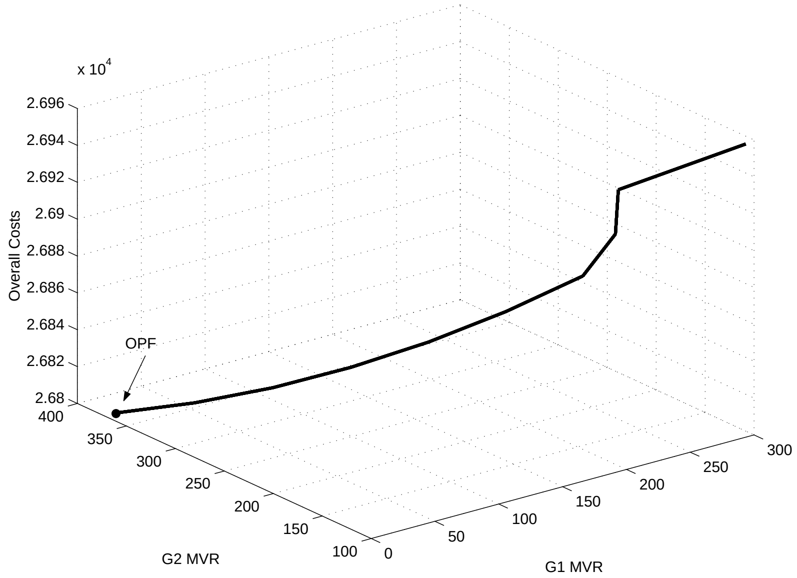

Figure 2: Overall costs for OPF solution when the reactive power is constrained

be produced at virtually no cost when the generator is producing 300 MW of real power or it can be generated by the synchronous condenser, consuming 1.5 MW of active power. Since overall generation costs depend on the bundle of active and imaginary power – generators should be paid for the bundle as well. Consequently, bids on the power market should be provided as a bundle with a single price assigned for each level of both real and reactive power. Graphically, both cost and bid functions can be presented as three dimensional surface, with each bid or cost level is characterized by two coordinates MW and MVAR.

In the simple 3 bus example (Fig 1.) generator 2 can produce reactive power at no cost, and its cost surface is shown on the Fig. 3(a). In the reality, reactive procurement outside the range of 0.95 leading and 0.9 lagging power factors [4] should be produced by the synchronous condenser at the expense of real power Fig 3(b).

It implies that cost of reactive power can be counted in the model as additional costs of real power. Therefore, pricing reactive power separately from real yields additional highly

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[4] power factor=$\frac{MW}{\sqrt{MW^2+MVAR^2}}$; leading implies $MVAR > 0$, lagging $MVAR < 0$.

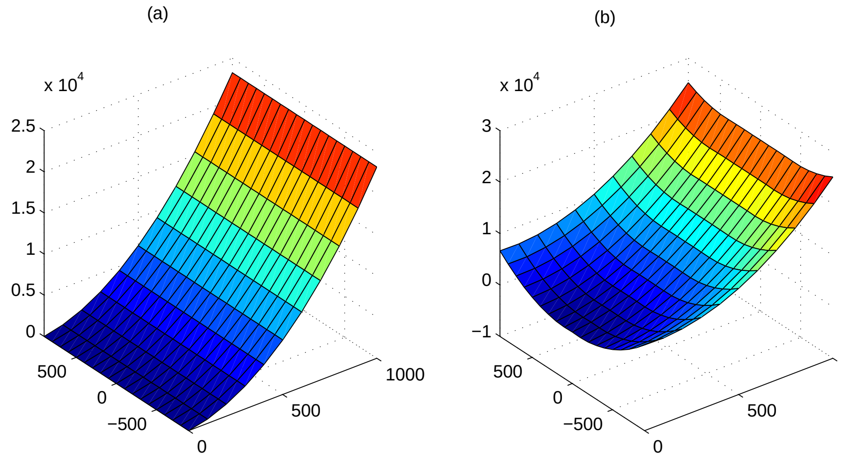

Figure 3: Cost surface example for (a) free reactive power and (b) costly reactive power

volatile variable and hence is not very practical [Hao 2003].

When power is sold on the competitive power auction generators bid there marginal cost. In case of bundled production generator's bid will be equal to the sum change in cost due to incremental change in real and reactive power.

$$\text{Cost of G}_i \ = \text{Cost}(\text{MW}_i) + \text{Cost}(\text{MVAR}_i(\text{MW}_i)) = \text{COST}(\text{MW}_i, \text{MVAR}_i)$$

$$MC_i = \text{COST}_{\text{MW}_i}(\cdot) + \text{COST}_{\text{MVAR}_i}(\cdot)\ \text{MVAR}'(\text{MW}_i)$$

5 Algorithm for Calculating Bundled LMPs

LMPs for bundled product of real and reactive power are assigned based on the generator's bids. When generators are price takers, they will bid their marginal costs.[5] The generator at the swing bus is paid $26,598 for 847.11 MW and 8.1 MVR. This amount implies that

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[5] $LMP_i = MC_i$ is a profit maximizing condition for the competitive market

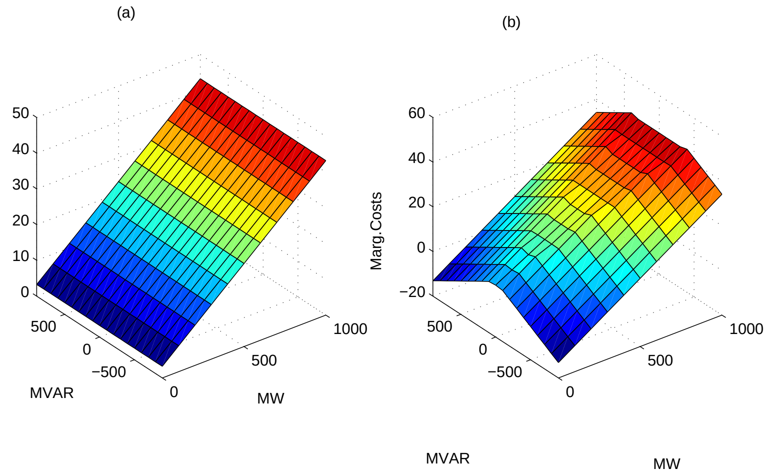

Figure 4: $MC_i = LMP_i$ surface for (a) free reactive power and (b) costly reactive power

the generator is paid \$31.4 per MW conditionally that generator produces 8.1 MVR of the reactive power. Assuming that generator is capable of producing 8.1 MVR at no cost the reward for the bundle of real and reactive power will be considered by generator as a reward for the costly real power (\$31.4 per MW). While calculating power flow for the simple triangular system production level at the swing bus is endogenous (derives from the solution of power flow equations). Without loss of generality \$31.4 per MW can be assigned to any production level at the swing bus. In the Table 1[6], calculations of the nodal price at the second bus are presented.

When the second generator produces 721.6 MW and 21.1 MVR – overall transmission losses of the real power are 68.73 MW. Corresponding amount of marginal loss -0.023 implies that overall system loss decrease by 0.023 MW when the second generator increases the real power output a little[7]. It happens because the second generator occupies better location at the network meaning that its power output increase is compensated by the substantial reduction

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[6] Note that columns 6,7 and 8 are rounded to integer precision.

[7] $\epsilon = 10^{-6}$ MW = 1 Watt

Table 1: Accounting of optimal power flow

	Gross Production (MW; MVR)	Overall Loss (MW)	Marginal Loss (MW)	Nodal price ($ per MW)	Gen. Reven. ($)	Gen. Cost ($)	Gen. Profit ($)
G1	(847.11; 8.1)	swing	swing	31.399	26,598	14,040	12,558
G2	(721.6; 371.2)	68.73	-0.023	32.114	23,173	12,759	10,414
				TOTAL	49,772	26,800	22,972

on the further located swing bus generator. Therefore, if nodal price at the swing bus is normalized to 1 (Hogan 1993), price at the second bus should be equal to 1.023. Price ratio is equal to the Marginal Rate of Substitution (MRS)= $0.9777 = \frac{1}{1.023} = \frac{31.4}{32.11}$.

It is important to mention that LMPs depend on the cost of reactive power. Consequently, graphically LMPs should be represented as a three dimensional surface where corresponding price is assigned for certain value of real and reactive power. On the Figure 4a one may see LMP surface calculated for the generator 2 .

Reactive power is cheap in the range of 0.95 leading and 0.9 lagging power factors. It implies that above 271 MVAR generator 2 would have to face additional costs of reactive power. That is generator 2 would have to use a synchronous machine using real power as input and reactive power as output. Synchronous machine consumes 3 MW for production of 100 MVAR PSERC[2001]. These will change the shape of cost function, marginal cost function and consequently LMPs (Figure 4b).

6 Efficiency of LMP

Mathematical appendix proves that when both generators are paid LMPs=MC at the power auction calculated for the bundle of real and reactive power, system operator is able to do efficient dispatch equivalent to OPF. When reactive power is cheap to produce marginal costs depend only on the active power production and SO dispatch efficiently based on active power bids only and enforcing reactive power output to the required level.

When reactive power output level is produced by synchronous machines generator's bid will raise at the cost of additional amount of active power consumed by a synchronous condenser required to produce reactive power (Figure 4b).

Figure 4[8] demonstrates overall profit function for both generators, calculated for given LMP's (Figures 3). Both generators' profit is maximized at the level of OPF which means that ISO's prices will give highest reward for both generators when they bid at the efficient level. Calculations of the profit for the OPF case (Figure 1) are shown in the last 3 columns of the Table 1.

It is crucial for the efficient dispatch that generators bid competitively (true marginal cost) without exercising market power. When the number of generators is large enough generator will not be able to raise bids above the level of marginal costs since cheaper generators will underbid.

Unfortunately, reactive power does not travel far, and its efficient production is usually done by the generators located closer to the load which reduces competition and motivates higher bids. Therefore it is important to identify those buses and stimulate competition among generators there.

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[8] The profit function was mapped in the production space of the second generator. It looks the same in the production space of the first generator.

Table 2: Strategic bidding with respect to reactive power for the generators on the 30 bus network

	Competitive Bids		Strategic Bids		Gain in
Gen.	MW	MVAR	MW	MVAR	Profit ($)
1	41.5298	-5.4198	41.5298	-5.4198	0
2	55.3872	1.7306	55.3872	1.7306	0
3	22.7352	34.2045	23.0284	18.4780	0.7967
4	39.9631	31.6563	41.8444	27.3382	9.7519
5	16.2545	6.9646	16.2955	4.3605	-0.5456
6	16.1883	35.9161	16.6685	19.3354	3.2930

7 Power Auction Simulations for the 30-bus network

In the simple three bus example generator 2 is producing most of the reactive power due to better location in the system. When it raises bid with respect to reactive power ISO will have to pay more to G2 despite the fact that reactive power can be produced for free in large quantities by the first generators. Reactive power substitution is not justified anyway since line congestion will increase real power losses.

Larger power system have many loads and lots of generators, therefore identifying market power for the reactive power is a lot more complicated. For this purpose 30-bus power network with six generators was tested.[9]

In Table 2 incentives to exercise market power are summarized. For simplicity it is assumed that the reactive power demand can be produced by the generators at no costs, however generators attempt increase bids by 3 % simulating use of the synchronous machine. Gener-

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[9] 30-bus network is a standard IEEE test system was a part of MatPower simulation package

ators are obliged to procure reactive power within normal normal reactive capabilities[10] for free. First two columns of the Table 2 represent equilibrium allocation of the ideal power auction that is all generators bid there true marginal costs. ISO is able to dispatch generators efficiently (OPF solution). Next three columns represent auction outcome when each generator attempts to bid above marginal costs.

First two generators are to procure reactive power without compensation since there production is within normal requirements.

Generators 3 and 4 gain a positive profit while bidding strategically. Each of them have reactive power level partially substituted by other generators and in addition this inefficiency requires more real power to be produced by both generators.

Generators 5 and 6 have identical cost function and power output characteristics. In addition, under ideal auction both generators are required to produce almost equal amounts of active power, meaning the same profit. Generator 6, however, is able to gain substantial profit from charging extra for the reactive power, while generator 5 gets loss.

Hence, competition or some other restrictive measures with respect to reactive power should be stimulated on the buses 3, 4 and 6. Generation on buses 1 and 2 remains within free capacity standards and therefore is forced to be procured. Finally, generator 5 is better off bidding true marginal costs.

8 Conclusions

In this paper a pricing mechanism for real and reactive power was proposed. One price is imposed for both real and reactive power production. As a result, the demand function for a generator becomes a surface in three-dimensional space. Prices for production are

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[10] 0.95 leading and 0.9 lagging power factor

assigned by the ISO based on the generators' bids. When generators bid competitively (true marginal cost for the bundle of real and reactive power) the ISO is able to do efficient dispatch equivalent to OPF solution. This power auction has been tested on the simple triangular network. Bid surface for the generator bidding competitively, and exercising market power was described. As a result a generator located closer to the load would benefit from raising bids with respect to reactive power, since its substitution from another generator is not justified due to high power losses. In addition, this power auction was tested on the 30-bus network with six generators. Generators willing to exercise reactive power were identified. Reducing high bids for the reactive power can be achieve by stimulating competition among generators.

9 Appendix

The parameters of the power network are given in per unit with respect to base of 100 MVA and 1.0 per unit Bergen(1986). Each line parameters and generators are represented in the table below.

Table 3: Electrical Parameters for the Triangle Load Flow (see Figure 1)

	Series Parameter		Shunt Parameters		Maximum
Bus	Resistance (R)	Reactance (X)	Susceptance (B)	Conductance (G)	capacity (MVA)
1-2	0.00500	0.02000	0.40000	0.00000	1500.00
1-3	0.00800	0.03000	0.50000	0.00000	1500.00
2-3	0.00400	0.01000	0.30000	0.00000	1500.00

Table 4: Generator's Data

		Power Production Constraint				
Generator	Bus	$P_{\min}$ (MW)	$P_{\max}$ (MW)	$Q_{\min}$ (MVR)	$Q_{\max}$ (MVR)	Cost function
G1	1	0.0000	1000.00	-900	900	$0.0175P^2 + 1.75P$
G2	2	0.0000	1000.00	-900	900	$0.02P^2 + 3.25P$

Optimal Power Flow Setup

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\begin{aligned}
\min_{\{P_1,P_2,Q_1,Q_2\}} \quad & 0.0175P_1^2 + 1.75P_1 + 0.02P_2^2 + 3.25P_2 && \text{(cost minimization)} \\
& P_1 + P_2 - P_1^{\text{loss}} - P_2^{\text{loss}} = P_D && \text{(load flow equations} \\
& f(Q_1, Q_2) = Q_D && \text{ are satisfied for both } P_D \text{ and } Q_D\text{)} \\
& MVA_{1-2}, MVA_{1-3}, MVA_{2-3} \leq 1500 && \text{(transmission constraints)} \\
& 0 \leq P_1, P_2 \leq 1000 && \text{(production constraints)} \\
& -900 \leq Q_1, Q_2 \leq 900 && \text{(reactive power constraints)} \\
& 0.95 \leq |V_1|, |V_2|, |V_3| \leq 1.05 && \text{(voltage constraints)}
\end{aligned}
$$

where

P_i, Q_i- real and reactive power produced by the generator i;

$P_i^{\text{loss}}-$ loss in the real power (out of P_i) produced by generator i;

MVA_{i-j} power flow from bus i to bus j;

P_D, Q_D- load demand for real and reactive power respectively.

Matpower [11] simulation package was used to solve the non-linear programming problems.

Marginal Prices Efficiency

$$\begin{aligned}
\min_{\{P_1,P_2,\dots P_n\}} \quad & \sum_{i=1}^{n} C_i(P_i, Q_i(P_i)) = \sum_{i=1}^{n} C_i(P_i) \\
\text{s.t.} \quad & \sum_{i=1}^{n} P_i - P^{\text{loss}}(P_2, \dots P_n, Q_2(P_2), \dots Q_n(P_n)) = P_D \\
& \text{(Voltage, transmission, capacity etc. constraints)}
\end{aligned}$$

$$Ł = \sum_{i=1}^{n} C_i(P_i) + \lambda \left(P_D - \sum_{i=1}^{n} P_i + P^{\text{loss}}(P_2, \dots P_n, Q_2(P_2), \dots Q_n(P_n)) \right)$$

F.O.C.

$$\frac{\partial Ł}{\partial P_i} = \frac{\partial C_i(P_i)}{\partial P_i} - \lambda \left(1 - \frac{\partial P^{\text{loss}}(\cdot)}{\partial P_i} \right) = 0$$

$$\frac{\partial Ł}{\partial \lambda} = \sum_{i=1}^{n} P_i - P^{\text{loss}}(P_2, \dots P_n, Q_2(P_2), \dots Q_n(P_n)) - P_D = 0$$

Denote $MC_i = \dfrac{\partial C_i(P_i)}{\partial P_i}$ and $ML_i = \dfrac{\partial P^{\text{loss}}(\cdot)}{\partial P_i}$

$$MC_1 = \lambda(1 - ML_1) = \lambda$$

$$\frac{MC_i}{MC_1} = 1 - ML_i$$

In the competitive market each generator is a profit maximizer.

Generators profit

$$\pi(P_i) = \text{LMP}_i \times P_i - C(P_i)$$

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[11] Matpower is an open code Matlab package developed by Cornell university http://blackbird.pserc.cornell.edu/matpower/

Profit maximizing condition

$$\mathrm{LMP}_i = \mathrm{MC}_i$$

When the ISO sets price ratio

$$\frac{\mathrm{LMP}_i}{\mathrm{LMP}_1} = 1 - ML_i = \frac{MC_i}{MC_1}$$

Therefore, LMPs are efficient.

10 References

Articles:

Alvarado F., Brehm R., Kirsch L., Panvini A., "Retail pricing of reactive power service" in Proc. EPRI Conf. Innovative Approaches to Electricity Pricing, La Jolla, CA, Mar. 1996.

Gil J,(1999) "Reactive Power Pricing: A Conceptual Framework for Remuneration and Charging Procedures"

Hao S. (2003) " A Reactive Power Management Proposal for Transmission Operators." *IEEE Transactions on Power Systems*, Vol. 18, NO. 4. November.

Hao S., Papalexopoulos A, "Reactive Power pricing and management", *IEEE Transactions on Power Systems*, vol. 12, Feb. 23-29

Hogan W. (1993)"Markets in Real Electric Networks Require Reactive Prices." *The Energy Journal* 14(3):211-242

Reason J (1989). "Reactive-Power Compensation Avoids New Line Construction," *Electrical World*, October, p.35

Kahn E., and Baldick, R. (1994) "Reactive Power is a Cheap Constraint" *The Energy Journal* 15(4):191-201

Singh H., Papalexopoulos A. (1999) "Competitive Procurement of Ancillary Services by an Independent System Operator" IEEE Transactions on Power Systems, Vol 14, No. 2 May, 498-504

Wilson Robert (2002), "Architecture of Power Markets" *Econometrica*, Vol. 70 NO.4- July.

Books:

Bergen, A.R. (1986). "Power System Analysis", Prentice Hall, Englewood Cliffs, New Jersey, Chapters 5-6.

Papers Presented at Conferences (Unpublished):

Joskow P (2003) "The Difficult Transition to Competitive Electricity Markets in the U.S." *Cambridge Working Papers in Economics*, July 2003. Prepared for the conference "Electricity Deregulation: Where From Here?" Texas A & M, April 4,2003

Weber J, (1998) " A Simulation Based Approach to Pricing Reactive Power" *Hawai International Conference on System Sciences*

Reports:

PSeRC (2001)-"Reactive Power Support Services in Electricity Markets"- - 2001 Final Report. www.pserc.wisc.edu/ecow/get/publicatio/ reports/2000report/