Paper D :

Attachment of Mating Faces - an Interrelational Feature Approach

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Attachment of Mating Faces
- An Interrelational Feature Approach

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ABSTRACT

Many engineering systems are characterized by a physical behavior that to a large extent is determined by complex interactions between subsystems. FE modeling and simulation of the physical behavior of complex systems is enabled by methods and technologies that allow subsystems to be modeled independently of other subsystems, and where systems models can be aggregated from a set of submodels, that that may be defined at various levels of abstraction, i.e. detailed, coarse, and condensed superelements. A method to attach pairs of nodally incompatible submodels is presented in this paper. A major benefit of the approach is that FE submodels and superelements are treated with the same formalism. The essence of the approach starts by first establishing a master-slave relation between sets of nodes on mating faces, followed by a Delaunay triangulation of the selected master node set, and finally a step where each slave nodal degree of freedom (DOF) is mapped to the DOFs of the three most appropriate master nodes. The method is robust and it has been developed as a support tool for FE modeling and simulation of multiphysics behavior of complex systems. It has been implemented in the general purpose FE software ANSYS as a macro library.

INTRODUCTION

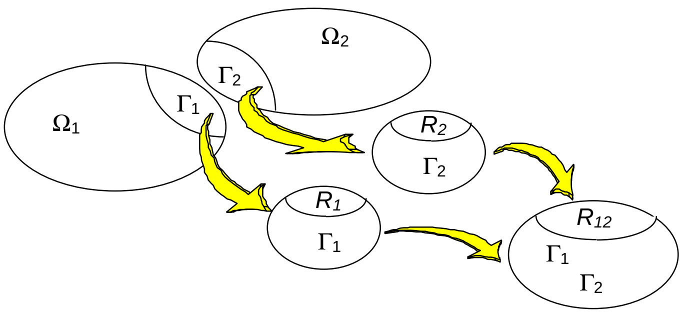

Figure 1. Two mating faces form a complex Relational object, i.e. an Interface.

A system can be defined as a finite set of subsystems, attributes, and relations. A subsystem is recursively defined as a system. Subsystems interact at common interfaces, where an interface is a pair of mating faces. Consider the geometric domains Ω_1 and Ω_2 of the two subsystems in figure 1. A unique region, a face, on each body, Γ_1 and Γ_2 respectively, have a mating relation. The geometric face object Γ_i and other relevant attributes can be viewed as a relational object

R_i. The two relational objects with mating faces Γ_i and Γ_j can be viewed as a complex relational object R_{ij}, i.e. an interface.

Material and shape are the major attributes that define the physical behavior of a single body. The physical behavior of a multi-body system is determined by the properties of the individual bodies, the interbody relationship, and the relationship between the bodies and the environment. It must be kept in mind that behavior is unique for each type of physics domain (e.g., mechanical, thermal, electrical, magnetic). Roy and Liu (1988) stated that there are three basic types of mating relations between engineering objects - *rigid attachment*, *constraint*, and *contact*. A rigid attachment, or attachment for short, relationship permits no relative motion between the mating faces. A constraint relationship constrains the physical behavior at one face to the behavior at the other mating face. A contact relationship permits separation and a relative sliding motion but no penetration between the two mating faces.

The finite element (FE) method is a general tool for numerical modeling and simulation of physical behavior of bodies with arbitrary shape. The geometric shape forms the basis for defining a FE mesh. There are many algorithms available that are capable of automatically or semi-automatically meshing general 2D and 3D geometric domains. Most of the algorithms are variants of the following five methods - mapping transformation, geometric decomposition, node insertion and connection, regular grid overlay, and paving or advancing front techniques (Saxena et al., 1995). Automatic meshing, with an associative one-to-many relation between the geometric model and the FE model, is a corner stone in any integrated CAD/CAE system.

A numerical model that is capable of describing the physical behavior of an engineering system is usually very large. Condensation of the full set of DOFs to a reduced set, as first proposed by Guyan (1965) and Irons (1965), is a standard method to enable the simulation of large systems. Condensation introduces some errors in the numerical model. Automatic selection of a specified number of internal master DOFs for dynamic models, as defined by Henshell and Ong (1975), is a standard tool in most FE systems. Thomas (1982) defined a priori error bounds for the computed eigenvalues. Techniques that enables efficient coupling of independently modeled subsystems have not received the same attention as condensation. Efficient systems modeling requires treatment of a mixture of condensed and ”full” FE models of subsystems and relationships between them. A method to attach independently discretized FE submodels in order to support efficient FE modeling of complex systems is presented in this paper. It has been implemented as a macro library in the commercial FE software ANSYS 5.2 from ANSYS Inc.

THE SELECTED METHOD FOR SUBMODEL ATTACHMENT

In the FE method, mating relationships can be treated as relationships between discrete nodal DOFs that are located on two different bodies. Two related domains that are individually discretized, i.e. two submodels, are either nodally compatible or incompatible at the interface. Nodally compatible faces are easily attached by merging nodes. Nodally incompatible submodels on the other hand can be attached to each other with sets of multi-point constraint (MPC) equations or with special purpose interface elements. Any approach to attach submodels that relies on the existence of nodally compatible faces requires complex mesh transitioning, which makes modeling of the various subsystems highly interdependent. Several commercial FE software have capabilities to couple DOFs at slave boundaries to master boundaries of nodally incompatible subdomains, e.g. Schaeffer (1979) and Kohnke (1994). Aminpour et al. (1995) presented a concept to couple nodally incompatible 2D finite element models by introducing pseudo nodes on the interface and formulating interface elements based on a hybrid variational formulation. This approach ensures an excellent compatibility between the solution fields in the two subdomains, and it is thus ideal for ”break-out” modeling, where a general desire is to keep the required detailed region as small as possible. In systems modeling, the size of the overall system is of a great concern. A concept that allows independently modeled regions to be coupled and that also reduces the number of DOFs at the mating faces has thus significant advantages.

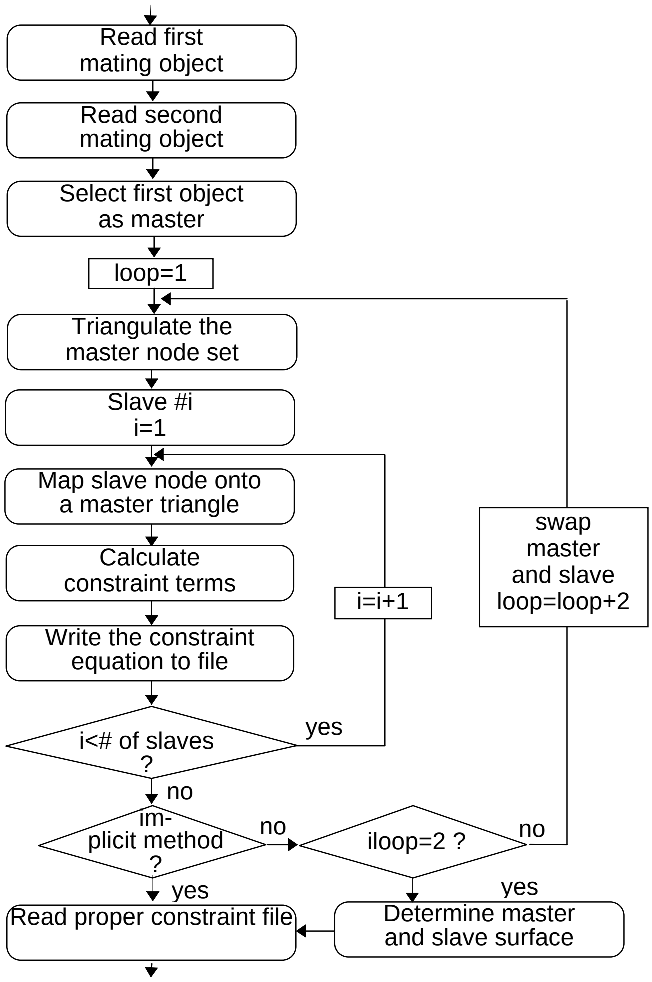

Figure 2. Logic for identifying and constraining slave DOFs.

The presented concept is based on a sequence of activities that includes pairing of the mating faces, an implicit selection of the master and slave relation at each interface, Delaunay triangulation of the master node set in each interface, and finally coupling of each slave DOF to a triangular set of master DOFs with an MPC equation. The master and slave sets in each complex relation are selected automatically, based on the properties of the actual discretization of the two related faces. The 3D algorithm for attaching the DOFs at an interface is briefly shown in figure 2.

For the case when two mating faces are geometrically compatible, e.g. Γ_{1A} and Γ_{2A} in figure 3, the more densely discretized face is selected as the slave face in the relationship. When the faces are geometrically incompatible, the master-slave selection is chosen to be the one that creates the largest number of constraint equations. The actual discretization of the two mating faces Γ_{1B} and Γ_{2B} in figure 3 requires that face Γ_{1B} be chosen as the master face, even though it has more nodes than face Γ_{2B}. With Γ_{1B} as master the coupling would be hinged in the mechanical domain, i.e. only three constraint equations for the translational DOFs of the central node of face Γ_{1B}. The chosen approach is to triangularize both faces, count the total number of constraint equations for both combinations, and based on that result keep the best solution. The automatic master-slave selection, which is the default method, is herein referred to as implicit mating. An optional method to explicitly define the master and slave objects, where the first face given in the relationship is treated as the master face, has also been implemented. Quiroz and Beckers (1995) proposed that in a mating situation like this, the constraints from the side of the interface with the fewer DOFs should be slightly relaxed. Relaxation has not been implemented in the presented method.

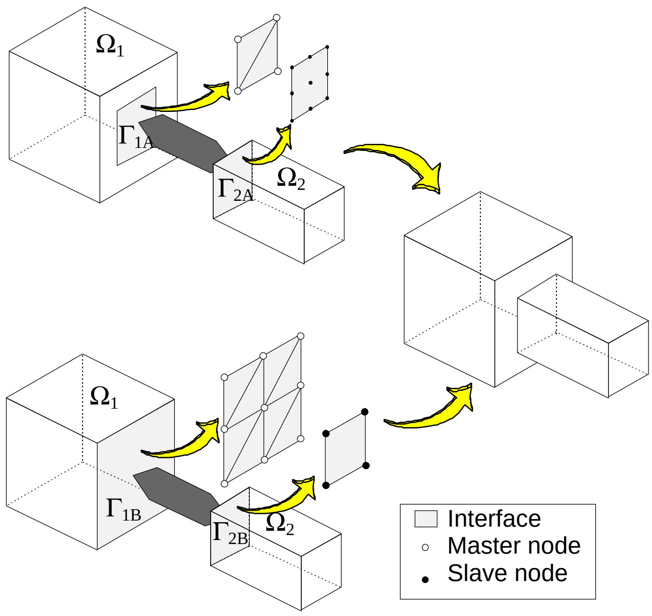

Figure 3. Master-slave relationship between geometrically compatible/incompatible faces.

TRIANGULATION

The first early implementations of triangulation algorithms were usually intended for contouring or surface generation. Triangulation of a given set of points was discussed by Green and Sibson (1978). A Delaunay triangle has the property that no point falls in the interior of the circumcircle of the triangle, i.e. the circle that passes through all three vertices. In 3D, the circumcircle is simply replaced by a circumsphere. The uniqueness of the Delaunay triangulation was proved by Sibson (1978). A relatively large number of algorithms for Delaunay triangulation of surfaces and volumes has been presented in the literature. Most of the algorithms are designed for good performance on uniformly distributed point sets. McLain (1976) and Sibson (1978) implemented algorithms for 2D interpolation and triangulation of random data. The advances in the FE method in the early 1980s resulted in a need for general mesh generation algorithms. Watson (1981) and Bowyer (1981) proposed algorithms for Delaunay triangulation of simple and convex domains. Turkiyyah et al. (1994) used Delaunay triangulation to generate skeletons, a class of geometric abstractions of solid models. Dirichlet/Voronoi/Thiessen tessellation, the geometric dual of the Delaunay triangulation (see figure 4), has been successfully applied to model crystal growth (Gilbert, 1962) and the interface network of polycrystalline materials (Watson, 1981).

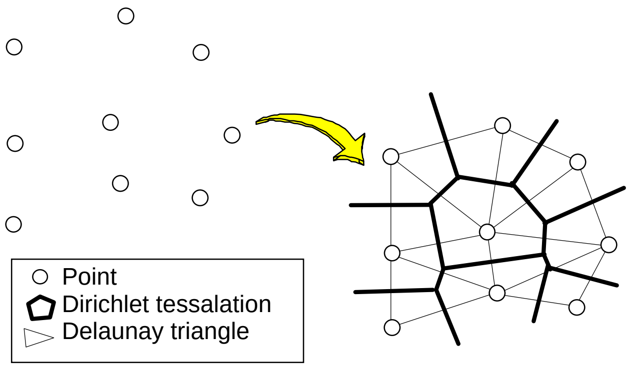

Figure 4. Dirichlet tessellation and Delaunay triangulation of a Finite point set.

The actual implementation of the presented method has not been focused on the computational efficiency of the triangulation algorithm. The main objective is to enable the submodeling activities to be as independent as possible. A secondary objective is to enable parallelization of the actual computations of the different complex relationships that are present in a complex engineering system. The triangulation algorithm used here, starts by creating one Delaunay triangle and then continues by incrementally creating valid Delaunay triangles until the complete set of master nodes is triangulated. Each new triangle is created from the two vertex points that a free edge of a previously discovered triangle and the unique unconnected master node that form a valid Delaunay, i.e. a triangle whose circumcircle is empty of any master node. Two generated triangles and the currently active free edge (active edge for short) connecting points *A* and *B* are shown in figure 5. Nodes *C*-*G* is the set of master nodes that are not presently part of any triangle definition. The subset of unconnected master nodes that are not colinear with the active edge can be considered as candidate nodes for creating a new triangle. The radius of the circle that circumscribes the triangle *ABC* is:

$$R_{ABC} = \frac{\left|\overline{AB}\right| \cdot \left|\overline{AC}\right| \cdot \left|\overline{BC}\right|}{4 \cdot Area_{ABC}} \tag{1}$$

where the area of the candidate triangle *ABC* is:

$$Area_{ABC} = 0.5\left|\overline{AB} \times \overline{AC}\right| \tag{2}$$

The position of the center point of the circumcircle is:

$$\overline{OL} = 0.5\left(\overline{OA} + \overline{OB}\right) + l_{min} \frac{\overline{AB} \times \overline{AC}}{\left|\overline{AB} \times \overline{AC}\right|} \times \frac{0.5\overline{AB}}{0.5\left|\overline{AB}\right|} \tag{3}$$

where the shortest distance between the currently active edge and the center point *L* of the circumcircle is:

$$l_{min} = \sqrt{R_{ABC}^2 - 0.25\left|\overline{AB}\right|^2} \tag{4}$$

The scrutinized triangle *ABC* is a Delaunay triangle if no point *Q* can be found within the circumcircle:

$$\left|\overline{LQ}\right| \geq R_{ABC} \tag{5}$$

If this is true, the triangle *ABC* can be stored as a new Delaunay triangle. At the same time, edges *BC* and *AC* are stored in the list of active edges, while edge *AB* is removed from this list. When there are no candidate points left, any valid Delaunay triangle that can be defined by the points on the set of active edges are created, i.e. the ”wrap up” triangles shown in the right portion of figure 5.

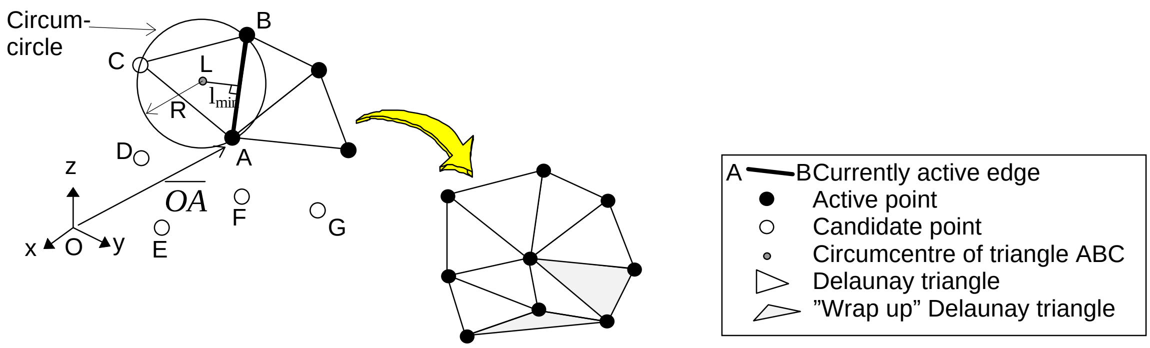

Figure 5. Delaunay triangulation of a prescribed node set.

SLAVE NODE MAPPING AND CONSTRAINT TERMS

Each slave node *S* is projected onto the closest master triangle *ABC*. The unit normal vector to the triangle *ABC* is:

$$\bar{n}_{ABC} = \frac{\overline{AB} \times \overline{AC}}{\left|\overline{AB} \times \overline{AC}\right|} \tag{6}$$

The distance between slave node *S* and the master triangle *ABC*, in the normal direction to the triangle plane is:

$$dist = \overline{AS} \bullet \bar{n}_{ABC} \tag{7}$$

The projected Cartesian location S_{ABC} of slave node *S* mapped on the triangle plane *ABC* is thus:

$$\overline{OS}_{ABC} = \overline{OS} - dist \cdot \bar{n}_{ABC} \tag{8}$$

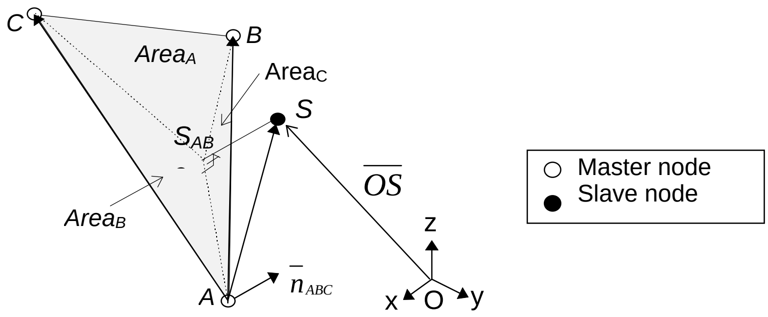

Figure 6. Master triangle and sub-triangles required for the constraint terms.

If the projection of the slave node falls outside all master triangles it is omitted. A small geometric tolerance is used to allow for situations where the slave is located exactly on a triangle edge or on a vertex point.

In equation 9, the slave DOF $q_{s,l}$ is constrained to a linear combination of the solution variables at the three generalized master DOFs $q_{m,n}$, where *n* is *A*, *B*, or *C*.

$$\beta_{s,l} q_{s,l} + \sum_{n=A,B.C} \beta_{m,n} q_{m,n} = \delta_l \tag{9}$$

The constant term δ_i is set equal to zero, the slave constraint term $\beta_{s,l}$ is set equal to 1.0, and the three constraint terms for the master DOFs β_{mn} are calculated from the relative area of the three subtriangles, that each is defined by the projected slave node and two of the vertices of the master triangle:

$$\beta_{m,A} = -\,Area_A / Area_{ABC} \tag{10}$$

ATTACHMENT EXAMPLES

Three examples are presented below. The first example shows attachment of two nodally incompatible submodels. The second case is a substructuring case, where nodally compatible superelements are coupled with the presented method. In this example, the number of DOFs at each face is also reduced with the same method. The third example is a real case, where the presented method is applied to attach submodels in the thermomechanical domain.

Attachment of Nodally Incompatible Faces

Consider the two independently discretized geometric domains, Ω_1 and Ω_2 in figure 7. Each of the domains is part of the definition of the complete FE model, C_1 and C_2 respectively. On a systems level, face Γ_1 located on Ω_1 has an attaching relation to face Γ_2 on Ω_2. The nodal numbers and other relevant attributes, such as the active DOFs and the intended relation to other potential bodies, are extracted from each mating face and stored as elementary relational features, R_i and R_2 (see table 1). Entering the macro command "mate,R1,R2" in ANSYS starts a routine that scrutinizes the two relational objects R_1 and R_2, determines the physical domain and the proper mating method. In the actual example, the mating routine implicitly selects one of the two node sets as the master set and triangulates the selected master node set. The slave nodes are then mapped onto the master triangles, and a proper set of equations, that constrain each slave DOF to a triangular set of master DOFs, are formed, i.e. a complex relational object is created ($R_1.R_2$ in figure 7). The complex relational object and the two complete submodels C_1 and C_2 form a systems model S_1.

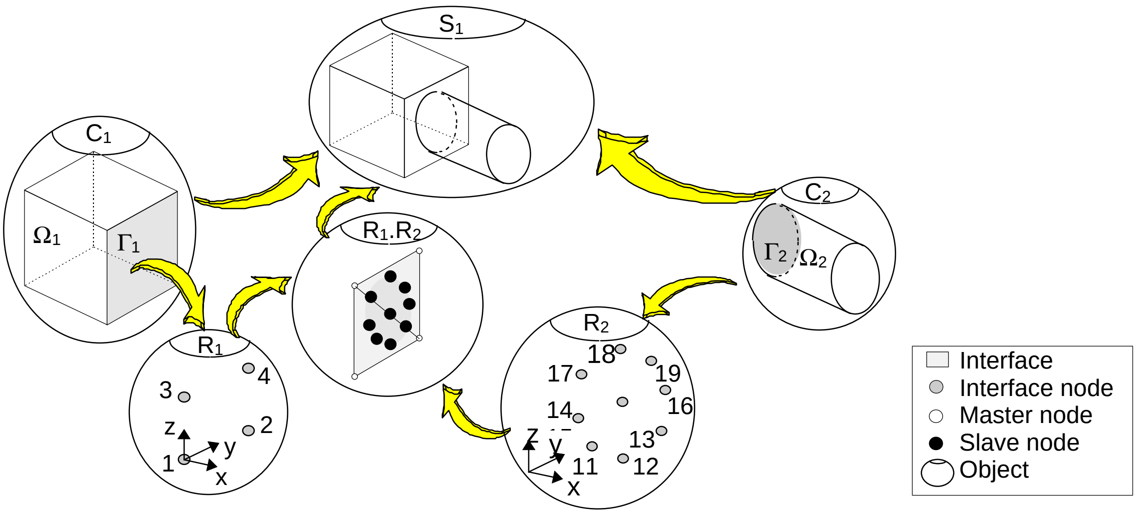

Figure 7. Master-slave relationship between two geometrically incompatible faces.

Attribute name	Attribute value	Attribute value	Attribute value	
name	R1	R2	R1.R2	
method	attach	attach	attach	
dimensions	'x','y','z'	'x','y','z'	'x','y','z'	
physical_dofs		‘ux’,’uy’,’uz’	‘ux’,’uy’,’uz’	‘ux’,’uy’,’uz’
node_list	1,2,3,4	11,12,13,14, 15,16,17,18, .. 19 ,,	11,1,2,3 12,1,2,3 .. 16,3,2,4 ..	

Table 1. Elementary relational objects R_1 and R_2 and the complex object $R_1.R_2$.

Constraint equation 1 in table 2 is in ANSYS input format:

ce,1,0,11,'ux',1,1,'ux',-.5668
ce,1,,2,'ux',-.217,3,'ux',-.217

Eq. no.	Slave DOF	Term	Masters DOF	Term	DOF	Term	DOF	Term
1	11,ux	1.0	1,ux	-0.566	2,ux	-0.217	3,ux	-0.217
2	11,uy	1.0	1,uy	-0.566	2,uy	-0.217	3,uy	-0.217
3	11,uz	1.0	1,uz	-0.566	2,uz	-0.217	3,uz	-0.217
4	12,ux	1.0	1,ux	-0.4	2,ux	-0.5	3,ux	-0.1
..	.	.	.	.	.	.	.	.
27	19,uz	1.0	3,uz	-0.217	2,uz	-0.217	4,uz	-0.566

Table 2. MPC equations created from the complex relational object $R_1.R_2$.

Results from a FE analysis of the two attached domains are visualized in figure 8. One face on Ω_1 was clamped and one face on Ω_2 was given a distributed shear force.

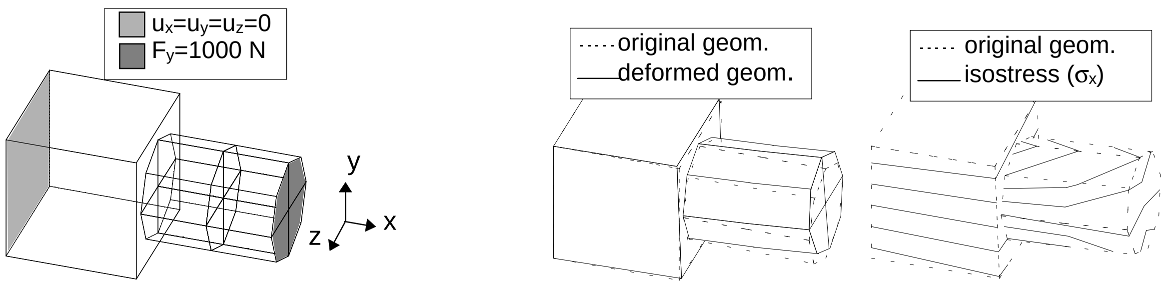

Figure 8. Loading and boundary condition, deformed shape, and bending stress.

Attachment of Nodally Compatible Superelements

In this example, a 2m long cantilever beam, clamped at one end and loaded with a shear force at the free end (see figure 9), was assembled from four instances of a 24 DOF superelement. The superelement was generated from a submodel composed of 16 isoparametric SOLID45 elements. The four nodally compatible instances were attached with constraint equations generated with the presented method, which in this case is similar to using the standard ANSYS command *cpintf*.

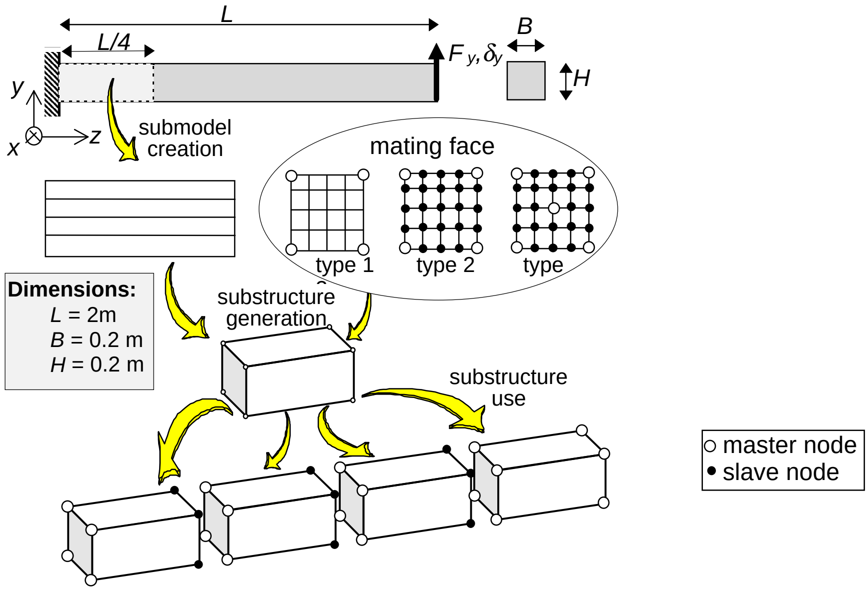

Figure 9. A substructuring example.

Small errors in the submodels can have a cascading effect on the total systems error. Error assessment is thus very important in systems modeling. We can define the error ε in a calculated parameter λ as:

$$^{\lambda}\varepsilon = \left(\lambda_{calculated} - \lambda_{exact}\right)/\lambda_{exact} \tag{11}$$

It is advantageous if the total error ε_{tot} embedded within a model is decomposed into components that can be evaluated and controlled individually, e.g. to distinguish between idealization error ε_{ide}, discretization error ε_{dis}, condensation error ε_{con}, and connection error ε_{int}:

$$\varepsilon_{tot} = \varepsilon_{ide} + \varepsilon_{dis} + \varepsilon_{con} + \varepsilon_{int} \tag{12}$$

If we exclude the idealization and discretization errors, by comparing the results from the substructure use pass with the results from a full FE model with 16x4 linear brick elements, and recognized that a static condensation introduces no additional error in the global stiffness, we can get a good estimate of the total connection error.

Condensing the 75 nodal DOFs at each mating face to 12 master DOFs, indicated as a type 1 mating face in figure 9, gives a highly underconstrained connection. The displacement at the free tip is about 4 times too large, compared with the results from the full model, i.e. the connection error is 3.0.

The connection error can be reduced by partitioning the nodes on each mating face of the submodel into a master node set and a slave node set and introducing constraints for each mating face in the substructure generation phase. These constraint terms can also be generated with the presented method. In figure 9 we can see two such examples, referred to as type 2 and type 3 respectively. The Delaunay triangles generated from the type 2 and type 3 master node sets are shown in figure 10. The connection error in the type 2 case is -0.10, i.e. each connection is slightly overconstrained. The asymmetries introduced by the two triangles (see figure 10) gives a slightly asymmetric bending of the beam. Introducing an internal master node on each mating and reduces the accumulated connection error to -0.05. The additional master node face increases the number of triangles to four and resolves all asymmetries in the calculated displacements.

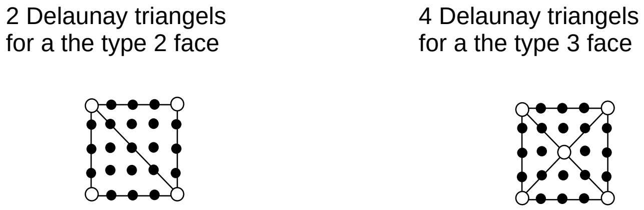

Figure 10. Triangulation for constraint term calculation.

Coupling of a Complex System

A systems model with mating relationships between the submodels of the thermomechanical behavior of the major components in an indexing module of a turn key grinding machine is shown in figure 11. The machines are adapted to specific customer requirements and manufactured in small numbers. The architecture is modularized and the physical behavior of a specific candidate configuration is preferably checked numerically against the technical requirements before the final configuration is chosen.

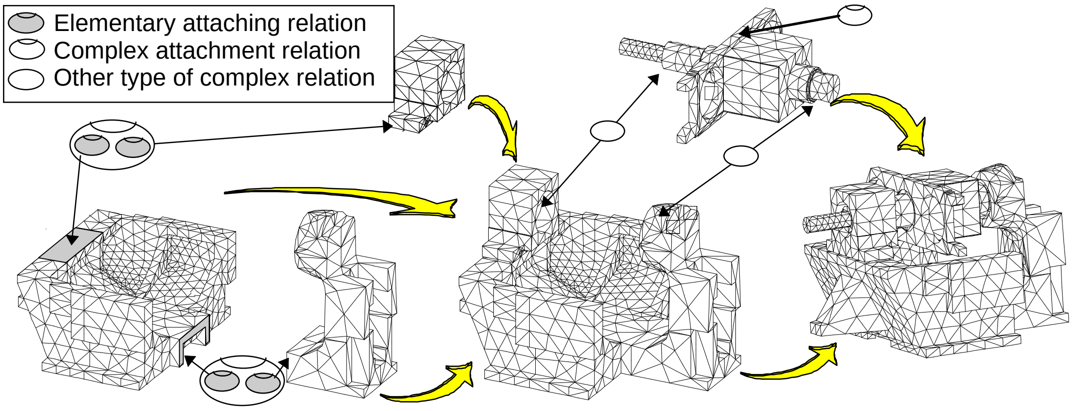

Figure 11. a mixture of thermal interface couplings at different levels of abstraction.

Results from a thermomechanical simulation with ANSYS, in terms of the positioning error caused by a sudden increase of the temperature of the cooling liquid, is shown in figure 12.

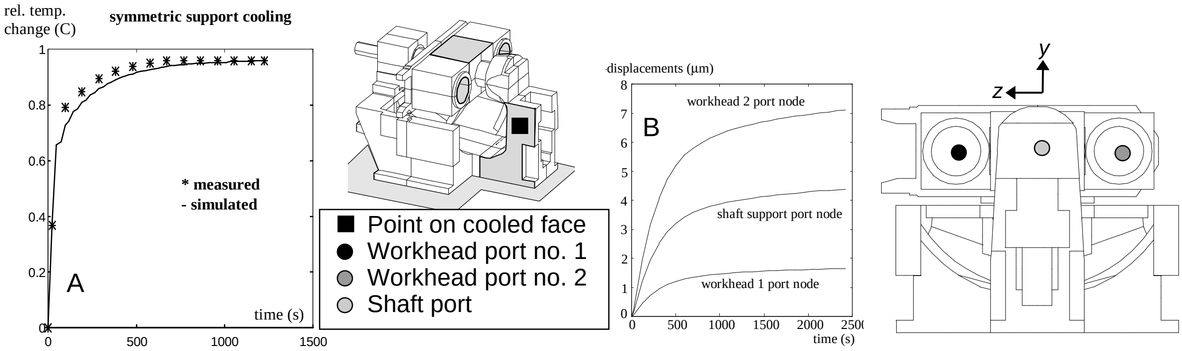

Figure 12. Simulated and measured effect of a cooling temperature step change in A, and workhead thermoelastic movements in B.

CONCLUSIONS AND FUTURE WORK

Because of the complex behavior of most engineering systems, the engineering process benefits from methods and technologies that allow systems models to be aggregated from a mixture of detailed, coarse, and condensed submodels.

Implicit mating of individually modeled bodies is one way to make modeling and simulation of systems behavior more flexible and thus more process oriented. A master-slave approach reduces the total number of DOFs at the mating faces. A Delaunay triangulation of the node set on each master face is a robust approach in the Euclidean as well as in many non-Euclidean spaces, e.g., spheres and cylinders. Since the presented method treats nodal data and not geometric data, it treats complete FE submodels and condensed FE models, i.e. superelements, in a similar way. The presented approach can also be used to reduce the number of DOFs of a mating face by constraining a set of slave nodes located on the face to a set of face master nodes.

There is a general need to incorporate the presented method to attach submodels in a general interface class structure which would also cover general contact and field transfer relations. This issue is targeted in an ongoing project, which also is investigating methods to, for a given simulation task, automatically select the most suitable set of connection masters for a general mating face.

For performance reasons it would be highly advantageous to convert the macro code to a compiled version. This can easily be done with the ANSYS user programming feature (UPF), i.e., the evolving application programming interface.

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REFERENCES

Aminpour, M.A.; Ransom; J.B.; McCleary, S.L. (1995) A coupled analysis method for structures with independently modelled finite element subdomains, Int. J. Num. Methods Eng., vol. 38, pp. 3695-3718.

Bowyer, A. (1981) "Computing Dirichlet tessalations", Comp. J., vol. 24 (2), pp. 162-166.

Gilbert, E.N. (1962) "Random subdivisions of space into crystals", Ann. Math. Stat., vol. 33, pp. 958.

Green, P.and Sibson, R. (1978) "Computing dirichlet tessalations in the plane", Comp. J., vol. 21 (2), pp. 168-173.

Guyan, R.J. (1965) "Reduction of Stiffness and Mass Matrices", AIAA J., vol. 3 (2), pp. 380.

Henshell, R.D and Ong, J.H. (1975) "Automatic masters for eigenvalue economization", Earthquake Eng. Struct. Dyn., vol. 3, pp. 375-383.

Irons, B. (1965) "Structural Eigenvalue Problems: Elimination of Unwanted Variables", AIAA J., vol. 3 (5), pp. 961-962.

Kohnke, P. (1994) "ANSYS Theory Manual", ANSYS Inc.,

McLain, D.H. (1978) "Two dimensional interpolation from random data", Comp. J., vol. 19 (2), pp. 78-181.

Quiroz, L. and Beckers,P. (1995) "Non-Conforming Mesh Gluing in the Finite Element Method", Int. J. Num. Meth. Eng., vol. 38, pp. 2165-2185.

Roy, U.and Liu, C.R. (1988) "Establishment of functional relationships between product components in assembly database", Computer Aided design, vol. 20 (10), pp. 570-580.

Saxena, M., Finnigan, P.M., Graichen, C.M., Hathaway, A.F. and Parthasarathy, V.N. (1995) "Octree-based automatic mesh generation for non-manifold domains", Engineering with Computers, vol. 1, pp. 1-14.

Schaeffer, H.G. (1979) "MSC/NASTRAN Primer, Static and Normal Modes Analysis", Schaeffer Analysis. Mont Vernon, New Hampshire, pp. 262-265.

Sibson, J. (1978) "Locally equiangular triangulations", Comp. J., vol. 21 (3), pp. 243-245.

Thomas, D.L. (1982) "Errors in natural frequency calculations using eigenvalue economization", Int. J. Num. Methods. Eng., vol. 111, pp. 169-183.

Turkiyyah, G.M., Storti, D.W., Ganter, M., Chen, H. and Vimawala, M. (1994) "An accelerated triangulation method for computing the skeletons of free-form solid models", Computer-Aided Design, vol. 29 (1), pp. 5-19.

Watson, D.F. (1981) "Computing the n-dimensional Delaunay tessalation with application of Voronoi polytopes", Comp. J., vol. 24 (2), pp. 167-172.