Joint estimation of path delay and complex gain for coded systems using the EM algorithm

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***Abstract*— We investigate the performance of a turbo estimation scheme based on the Expectation Maximization (EM) algorithm for M-PSK modulation and a convolutionally coded system. Estimates of the path delay and the complex gain are updated jointly by combining the training sequence and the soft information provided by the MAP channel decoder. Initial estimates may be obtained through either a Data Aided or a Non Data Aided algorithm. We show through computer simulation that the iterative process between turbo detection and estimation leads to performances close to those obtained for Perfect Channel State Information in terms of Bit Error Rate (BER). On the other hand, the Mean Square Estimation Errors (MSEE) of path delay and the complex gain reach the classical Modified Cramér-Rao Bound. Comparisons with conventional estimation schemes are carried out.**

I. Introduction

The excellent performance of turbo codes [1] has induced a lot of research dealing with more complex digital communication systems. One of the important topics is turbo channel estimation and/or turbo synchronization. The performance of coded communication systems is very dependent on the quality of the channel estimates. In classical systems, the channel estimator can be either Data-Aided (DA), Decision-Directed (DD) or Non Data Aided (NDA). A new alternative for coded systems is called Soft-Decision Directed (SDD): instead of hard data decisions, soft information based on the A Posteriori Probabilities (APP) of the data symbols is used in the estimation process. These APP are computed by the turbo detector. Such ideas were explored in [2]–[4] for phase recovery. In [2], the SDD mode in a turbo coded scheme using the APP of data symbols is considered to refine the phase estimate at each iteration of the turbo decoder for 16-QAM. In [4], the so-called extrinsic information, extracted from the turbo decoder, is used by the carrier phase synchronizer. The same ideas can be found in [5]: the authors consider a synchronous DS-CDMA system with iterative turbo processing that includes a soft interference canceler, a Soft-In Soft-Out (SISO) single user decoder and EM-based channel tap weight estimator. The turbo estimator combines the soft estimates of BPSK symbols and the training sequence for multiuser complex gain estimation. A more general theoretical framework for Maximum-Likelihood (ML) estimation in coded systems is provided in [6]. Simulations were reported for carrier phase estimation. In [7], the same techniques were used to tackle the problem of timing estimation. Although [2], [6] and [7] reported very good performance both in terms of mean-square estimation error (MSEE) and BER, no comparisons with conventional estimation algorithms were performed.

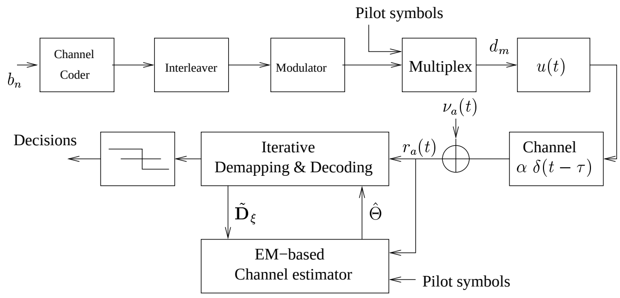

Fig. 1. transmitter and receiver front-ends

In this paper, the issue of joint estimation of the complex gain, α, embedding the phase (θ) and the amplitude (A) of the useful part of the received signal, and the timing (τ) is considered in an Additive White Gaussian Noise (AWGN) channel. Hence, the set of channel parameters to be estimated is $\{A, \theta, \tau\}$. Soft information is exchanged between the demapper, the BCJR-based [8] decoder and EM-based channel estimator. Bit-interleaved M-PSK coded modulation is assumed, though the proposed scheme may be easily altered for other constellations. The soft information required by the EM estimator is obtained by the turbo detector [9], performing joint demapping and decoding. We compare the MSEE of the complex gain, the carrier phase and the time delay to the corresponding Modified Cramér-Rao Bound (MCRB). We show that although EM turbo channel estimation may reduce the MSEE as compared to a conventional estimation scheme, for certain channel parameters this does not translate in a BER performance gain.

II. System model

The communication system, as depicted in Fig. 1, consists of the following blocks: a frame of M_b information bits enters a convolutional code of rate R and constraint length ν. The $M_c = M_b/R$ coded bits, after random interleaving, are mapped to complex symbols belonging to the alphabet $\{\Omega_1, \dots, \Omega_Q\}$ where Q is the size of the constellation and $q = \log_2(Q)$ is the number of coded bits per symbol. Hence, the number of complex data symbols per frame equals $M_d = M_c/q$. The total number of symbols per frame is $M_s = M_t + M_d$ where M_t is the number of pilot symbols. The complex symbols d_n are shaped by a normalized transmit pulse $u(t)$. The frame propagates through a channel with an impulse response $h(t) = \alpha\,\delta(t-\tau)$ where α and τ are the complex gain and the path delay, respectively. The transmitted frame is corrupted by a low-pass AWGN process, $\nu_a(t)$, with power spectral density $2N_0$. The received signal can be expressed as

$$\begin{aligned} r_a(t) &= \alpha \sum_{m=-M_t}^{M_d-1} d_m u(t - mT_d - \tau) + \nu_a(t) \\ &= s(t, \mathbf{D}, \mathbf{P}, \boldsymbol{\Theta}) + \nu_a(t) \end{aligned} \quad (1)$$

where $\mathbf{D}$ and $\mathbf{P}$ are the set of data symbols and the set of pilot symbols, respectively. $\boldsymbol{\Theta}$ is the set of complex gain and path delay, i.e., $\boldsymbol{\Theta} = \{\alpha, \tau\}$. To decode the received signal and obtain the information bits b_n, we have to estimate the channel parameters $\boldsymbol{\Theta}$. This can be done by maximizing, with respect to $\boldsymbol{\Theta}$, the average likelihood function (averaged with respect to $\mathbf{D}$) [6]:

$$\begin{aligned} \hat{\boldsymbol{\Theta}} &= \arg\max_{\boldsymbol{\Theta}} \mathrm{E}_{\mathbf{D}}\left[\mathrm{p}\left(r_a(t)|\,\boldsymbol{\Theta}, \mathbf{D}\right)\right] \\ &= \arg\max_{\boldsymbol{\Theta}} \mathrm{E}_{\mathbf{D}}\left[e^{\Lambda_L(\mathbf{D},\boldsymbol{\Theta})}\right] \end{aligned} \quad (2)$$

where

$$\Lambda_L(\mathbf{D}, \boldsymbol{\Theta}) = \frac{1}{N_0}\int_{-\infty}^{+\infty} \Re\left[r_a(t)\, s^*(t, \mathbf{D}, \mathbf{P}, \boldsymbol{\Theta})\right] dt - \frac{1}{2N_0}\int_{-\infty}^{+\infty} \left|s(t, \mathbf{D}, \mathbf{P}, \boldsymbol{\Theta})\right|^2 dt. \quad (3)$$

Note that the last term in (3) does not depend on θ or τ, only on $A = |\alpha|$. The iteration process starts using an initial estimate of the channel parameters $\boldsymbol{\Theta}$. This estimate is then fed to the turbo detector. There, soft tentative data decisions based on the marginal a posteriori probabilities of the coded bits are computed. The tentative soft decisions are combined with the training sequence to refine the estimates of the channel parameters. Hence, we iterate between turbo detection and estimation processes. In the next section, we will describe the iterative channel estimator based on the EM algorithm for our specific problem.

III. The EM turbo estimator

A. Iterative estimation

The derivation of the maximum in (2) is often very difficult to compute. An algorithm that can maximize (2) iteratively with less computational complexity is the EM algorithm [10]. It is based on the fact that if so-called complete data $\mathbf{z}$ were available, the estimation process would be easier. This complete data consists of the observable data $\mathbf{r}$ (where $\mathbf{r}$ is the projection of $r_a(t)$ onto a suitable basis) and the probabilistic missing data $\mathbf{D}$, i.e., $\mathbf{z} = [\mathbf{r}, \mathbf{D}]$. The estimation of $\boldsymbol{\Theta}$ can be performed iteratively, where each iteration breaks up into two steps: in the Expectation step (E-step), we compute the average log-likelihood of the complete data while in the Maximization step (M-step), we maximize the average log-likelihood of the complete data. After ξ iterations between turbo detection and estimation, this means:

$$\text{E-step: } U\left(\boldsymbol{\Theta}, \hat{\boldsymbol{\Theta}}(\xi)\right) = \mathrm{E}_{\mathbf{D}|\mathbf{r},\hat{\Theta}(\xi)}\left\{\log \mathrm{p}\left(\mathbf{z}|\,\boldsymbol{\Theta}\right)\right\}$$

$$\text{M-step: } \hat{\boldsymbol{\Theta}}(\xi+1) = \arg\left\{\max_{\Theta} U\left(\boldsymbol{\Theta}, \hat{\boldsymbol{\Theta}}(\xi)\right)\right\}.$$

The data $\mathbf{D}$ is treated as a probabilistic missing data in the E-step using the marginal a posteriori probability $\mathrm{p}\left(d_n|\,\mathbf{r}, \hat{\boldsymbol{\Theta}}(\xi)\right)$ [6]. The expectation in the presence of random parameters $\mathbf{D}$ can be written [11]:

$$\begin{aligned} U\left(\boldsymbol{\Theta}, \hat{\boldsymbol{\Theta}}(\xi)\right) &= \mathrm{E}_{\mathbf{D}|\mathbf{r},\hat{\Theta}(\xi)}\left\{\log \mathrm{p}\left(\mathbf{z}|\,\boldsymbol{\Theta}\right)\right\} \\ &= \mathrm{E}_{\mathbf{D}|\mathbf{r},\hat{\Theta}(\xi)}\left\{\log \mathrm{p}\left(\mathbf{r}|\,\boldsymbol{\Theta}, \mathbf{D}\right)\right\} \\ &+ \mathrm{E}_{\mathbf{D}|\mathbf{r},\hat{\Theta}(\xi)}\left\{\log \mathrm{p}\left(\mathbf{D}\right)\right\}. \end{aligned} \quad (4)$$

Substituting equation (3) in (4) and, as this will not affect the maximization step, dropping terms that do not depend on $\boldsymbol{\Theta}$, this yields:

$$\begin{aligned} &U\left(\boldsymbol{\Theta}, \hat{\boldsymbol{\Theta}}(\xi)\right) \\ &= \frac{1}{N_0}\int_{-\infty}^{+\infty} \Re\left[r_a(t)\, s^*\left(t, \tilde{\mathbf{D}}\left(\hat{\boldsymbol{\Theta}}(\xi)\right), \mathbf{P}, \boldsymbol{\Theta}\right)\right] dt \\ &- \frac{1}{2N_0}\left|\alpha\right|^2 M_s \end{aligned} \quad (5)$$

where $\tilde{\mathbf{D}}\left(\hat{\boldsymbol{\Theta}}(\xi)\right) \doteq \tilde{\mathbf{D}}_\xi$ denotes the soft tentative data decisions after ξ iterations between the estimation and turbo detection stages. Using the marginal a posteriori distributions $\mathrm{p}\left(d_n|\,\mathbf{r}, \hat{\boldsymbol{\Theta}}(\xi)\right)$ delivered by the SISO turbo detector, the soft estimates $\tilde{d_n}\left(\hat{\boldsymbol{\Theta}}(\xi)\right)$ can be expressed as $\tilde{d_n}\left(\hat{\boldsymbol{\Theta}}(\xi)\right) = \sum_{\omega\in\Omega_Q} \omega\, \mathrm{p}\left(d_n = \omega \,\middle|\, \mathbf{r}, \hat{\boldsymbol{\Theta}}(\xi)\right)$. The maximization of $U\left(\boldsymbol{\Theta}, \hat{\boldsymbol{\Theta}}(\xi)\right)$ can be performed as follows:

$$\hat{\tau}(\xi+1) = \arg\max_{\tau} \left|\psi(\tau)\right| \quad (6)$$

$$\hat{\alpha}(\xi+1) = \frac{\psi\left(\hat{\tau}(\xi+1)\right)}{M_s}. \quad (7)$$

where $\psi(\tau) = \sum_m \tilde{d}_m^* \int_{-\infty}^{+\infty} r_a(t) u(t - mT_d - \tau)\, dt$.

B. Initialization

Although the EM algorithm converges to a local maximum under fairly general conditions [10], the initial estimate, $\hat{\boldsymbol{\Theta}}(0)$, must be sufficiently accurate in order to reach the global maximum of (2), i.e., the ML estimate. We consider two initialization schemes: under the first scheme we use (6) and (7) exploiting only the pilot symbols (i.e., with $\tilde{\mathbf{D}} = \mathbf{0}$ and $M_s = M_t$). Consequently, this scheme is purely DA. Under the second scheme we break up the complex gain into a phase and an amplitude component: $\alpha = Ae^{j\theta}$, $A > 0$. An initial estimate of the delay may be obtained through the Oerder & Meyr (O&M) algorithm [12]. Once the signal is reconstructed according to this delay estimate, the matched filter outputs, sampled at the symbol rate $1/T_d$, can be used to

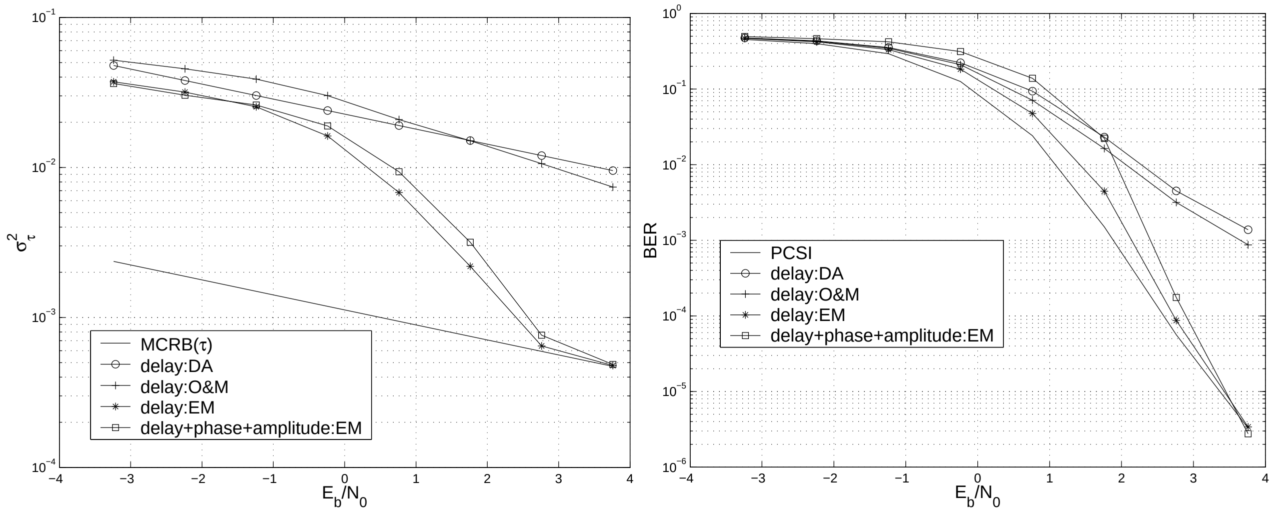

Fig. 2. The MSEE of the delay estimate (left) and corresponding BER (right) for DA, NDA and SDD delay estimation as well as joint delay, phase and amplitude estimation. A frame size of 202 QPSK symbols is assumed, including 10 pilot symbols.

provide an initial NDA estimate for the phase through the Viterbi&Viterbi (V&V) estimator [13]. Since there are no known simple and accurate schemes for NDA amplitude estimation [14], A is estimated using only the pilot symbols.

IV. Numerical results

In this section we will provide numerical results to evaluate the performance of the proposed iterative SISO receiver. The constraint length and the code rate of the convolutional code are $\nu = 9$ and $R = 1/3$ respectively. The polynomial generators are $(577, 663, 711)_8$. Frames consist of 192 data symbols and 10 pilot symbols using QPSK signalling. The transmit pulse $u(t)$ is a square-root raised-cosine pulse with roll-off factor 0.11. The path delay and the complex gain are fixed to $\tau = T_d/2$ and $\alpha = 1e^{j\pi/9}$. Timing correction is performed by means of a digital polynomial interpolator with 7 taps, operating at a rate $1/T_s = 4/T_d$. For the maximization of (6), we employ the Newton-Raphson algorithm. To obtain a DA initial estimate of the delay, (6) is maximized by performing a line search with a granularity of $T_d/100$. We have performed the EM algorithm until convergence (i.e., until $\hat{\mathbf{\Theta}}(\xi+1) \equiv \hat{\mathbf{\Theta}}(\xi)$).

The figures below show the estimation performance in terms of mean-square estimation error (MSEE) and BER. We denote by $x : X$, the estimation of a certain parameter $x \in \mathbf{\Theta}$ using algorithm X, where all remaining parameters in $\mathbf{\Theta} \setminus \{x\}$ are assumed to be perfectly known. In particular, $x : EM$ refers to the performance of the EM channel estimator after convergence. Note that this performance is independent of the type of initial estimate (i.e., DA or NDA) as long as the initial estimate is sufficiently accurate. For the MSEE plots we also include as a point of reference the MCRB corresponding to a known sequence of 202 symbols. We first investigate delay estimation in Fig. 2. Both the NDA and DA initial estimates give rise to a significant performance degradation in terms of MSEE and BER. After applying the EM algorithm the MSEE improves considerably and even reaches the MCRB for SNR above 2.5 dB. In terms of BER this translates to a low degradation (less than 0.25 dB) for all considered SNR when using the EM algorithm. In the same figure we also show BER performance of the joint estimation of delay, phase and amplitude. If we use only the initial channel estimates, high BER degradations are noticeable for all SNR. Application of the EM algorithm results in BER gains for SNR above 1 dB. As the SNR increases we can achieve very low BER degradations. In the left part of Fig. 2, it can be seen that jointly estimating phase, amplitude and delay has little impact on the delay EM MSEE.

Due to a lack of space we are not able to show simulation results for phase estimation. However, phase estimation suffers from the same problems as delay estimation: both DA and NDA estimates result in high BER degradations. Application of the EM estimator improves MSEE and BER. The conclusions are the same as for delay estimation.

When we evaluate the MSEE for amplitude estimation, in the left part of Fig. 3, we notice that even for moderate SNR (above -1 dB) we again reach the MCRB. Since the combination of a convolutional code with M-PSK mapping is not very sensitive to the estimate of the amplitude [15], only a very small degradation will occur, even when we do no apply the EM algorithm. Indeed, when we observe the right part of Fig. 3, which shows BER performance, the BER degradation corresponding to the DA algorithm is very low for all considered SNR. Hence, it is not necessary to apply the EM algorithm as far as amplitude estimation is concerned. This clearly illustrates that it is important to verify the necessity of applying the EM algorithm on a case by case basis. Depending on the system parameters (such as roll-off, block length, length of pilot sequence, type of code,...) a conventional algorithm may be acceptable.

V. Conclusion

In this paper we have investigated the performance of an iterative turbo detector and channel parameter estimator for a single user in a single path channel. The

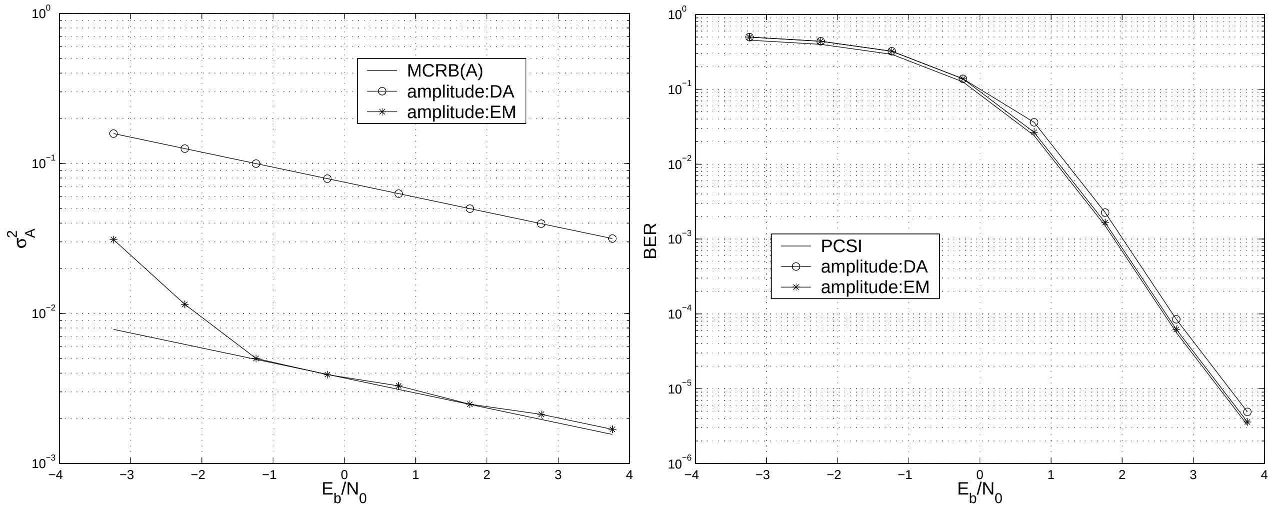

Fig. 3. The MSEE of the amplitude estimate (left) and corresponding BER (right) for DA and SDD amplitude estimation. A frame size of 202 QPSK symbols is assumed, including 10 pilot symbols.

performance degradation due to imperfect amplitude, carrier phase and/or path delay estimation was tackled. The iterative turbo estimation scheme presented here is based on the EM algorithm and combines SDD and DA operating modes. We show that the proposed EM algorithm attains MSEE performances of the estimated parameter close to the MCRB, even for moderate SNR. Through computer simulations, we have demonstrated that this gain in MSEE does not necessarily translate in a gain in terms of BER: depending on the system parameters, the BER degradation due to a conventional (N)DA algorithm may be acceptable. The proposed scheme is therefore well suited when conventional estimation algorithms fail to provide accurate channel parameter estimates.

ACKNOWLEDGEMENT

This work has been supported by the Interuniversity Attraction Poles Program P5/11 - Belgian State - Federal Office for Scientific, Technical and Cultural Affairs.

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