BLIND IMAGE RESTORATION FOR ULTRASONIC C-SCAN USING CONSTRAINED 2D-HOS

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ABSTRACT

A new approach is presented in this paper to use Higher Order Statistics (HOS) to deconvolve the effects of blurring in ultrasonic C-Scans. The quantization effects of the mechanical scan-grid and due to the conversion of 1-D A-scan into one pixel on the image cause blurring. The proposed approach is completely blind to the source or the type of distortion and the formulation is purely two-dimensional. When the blurring function is modeled as an AR process, the image is restored recursively with the application of the inverse filter based on the AR estimate. A significant improvement in the image quality has been demonstrated. Especially, the edges are detected more prominently than present in the original image.

1. INTRODUCTION

Ultrasonic C-Scans are usually constructed from the pulse-echo A-Scans which are predominantly Gaussian in nature. However, the medium characteristics incorporate distortions, which are essentially due to a non-minimum phase system. When these A-Scans are converted to C-Scan by mapping the spatial 1-D waveforms into spatial energy values, the statistical characteristics are completely unpredictable. The usual Second Order Statistics (SOS) may not work very well in this case.

The image formation process to construct the C-Scan in ultrasonic Nondestructive Evaluation (NDE) applications is equivalent to mapping a 1D signal into different energy levels in certain finite number of levels. In one level case, the whole waveform (A-Scan) is mapped into one point. Hence, a data cube of $m \times n$ A-Scans, each with p samples, can be converted into $m \times n \times k$ levels of energy values using k windows in the A-Scan axis. This mapping process is highly nonlinear and causes blurring of the image thus obtained. The A-Scans are pre-dominantly Gaussian but the C-Scans may not follow any particular form of distribution. This motivates the use of restoration methods using Higher-order Statistics (HOS) which can capture the non-Gaussian effects in a signal.

HOS are extensions beyond the well-known second order statistics (SOS), i.e. correlation and covariance functions. Cumulants are one form of HOS that are specific combinations of higher order statistical moments.

The theory behind 1D HOS has been established very well during past two decades [1,2], however, the extension to 2D has recently been started [3]. In this work a constrained subset for cumulant calculation is used, based on the 4-Neighbor model for pixels in an image. Cumulants are the most commonly used HOS measures. These are some forms of algebraic combinations of the moments of the signal under study. For a finite sample set of a random signal, the definitions of Moments M_i, and Cumulants C_i are defined as follows for $i = 2,3,4$:

$$M_1(m,n)=\frac{1}{N}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N}x(k_1,k_2) \qquad (1)$$

$$M_2(m,n,\sigma_1,\sigma_2)=\frac{1}{N^2}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N}x(k_1,k_2)x(\sigma_1-k_1,\sigma_2-k_2) \qquad (2)$$

$$M_3(m,n,k_1,k_2,\tau_1,\tau_2)=\frac{1}{N^3}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N}x(k_1,k_2)x(\sigma_1-k_1,\sigma_2-k_2)\,x(\tau_1-k_1,\tau_2-k_2) \qquad (3)$$

$$M_4(m,n,\sigma_1,\sigma_2,\tau_1,\tau_2,\rho_1,\rho_2)=\frac{1}{N^4}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N}x(k_1,k_2)x(\sigma_1-k_1,\sigma_2-k_2)\,x(\tau_1-k_1,\tau_2-k_2)x(\rho_1-k_1,\rho_2-k_2) \qquad (4)$$

The cumulants can be expressed in terms of moments as

$$c_2(m,n,k_1,k_2)=m_2(m,n,\sigma_1,\sigma_2)-[m_1(m,n)]^2 \qquad (5)$$

$$\begin{aligned}c_3^x(m,n,\sigma_1,\sigma_2,\tau_1,\tau_2)&=m_3(m,n,\sigma_1,\sigma_2,\tau_1,\tau_2)\\&-m_1(m,n)[m_2(m,n,\sigma_1,\sigma_2)-m_2(m,n,\tau_1,\tau_2)\\&+m_2(m,n,\sigma_1-\tau_1,\sigma_2-\tau_2)]+2[m_1(m,n)]^3\end{aligned} \qquad (6)$$

2. HOS-BASED IDENTIFICATION ALGORITHM

In this work, 3rd order cumulants are used with Auto Regressive (AR) representation for the C-Scan image enhancement. In order to utilize the time samples' analogy with the positions of pixels in the c-scan image, the causal 4-Neighbor model for pixels in an image has been used, as shown in Figure 1. The AR model for such arrangement can be written as follows:

$$\begin{aligned}y(m,n)&=-[a_1y(m-1,n-1)+a_2y(m-1,n)\\&+a_3y(m-1,n+1)+a_4y(m,n-1)]+r(m,n)\end{aligned} \qquad (7)$$

where *y(m,n)* is the measured pixel *y* at position *(m,n)* for the image under study, a_i are the unknown parameters for AR coefficients, and *r(m,n)* represents the superimposed noise pixel.

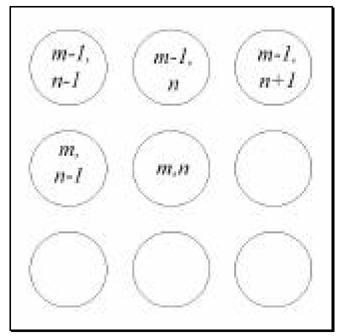

Fig. 1. The 4-Neighbor Causal model.

The system in (7) can be compactly represented as

$$y(m,n)=-[y(m-1,n-1)\quad y(m-1,n)\quad y(m-1,n+1)\quad y(m,n-1)]\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix}+r(m,n) \qquad (7)$$

$$y = z\,\theta + r \qquad (8)$$

3. RESULTS

Using approaches similar to [4], and assuming *y*(*m,n*) to be a zero-mean, 3rd order stationary, and non Gaussian random process, independent of *r*(*m,n*), following definitions can be introduced,

$$
\begin{aligned}
\Phi &= E\{d_1 z d_2 z \;\; z^T\} \\
Y &= E\{d_1 z d_2 z \;\; y^T\} \\
R &= E\{d_1 z d_2 z \;\; r^T\}
\end{aligned} \qquad (9)
$$

Where $d_1 z$ represents the sample structure z shifted by d_1 samples and $d_2 z$ represents the shift by d_2. Multiplying (8) with $d_1 z d_2 z$ then arranging as per (9), we get:

$$Y = \Phi q + R \qquad (10)$$

Under the conditions of assumptions, $d_1 z d_2 z$ and r are uncorrelated, and hence the solution for (10) can be given as follows:

$$\hat{q} = \Phi^{-1} \; Y \qquad (11)$$

The right hand side of equations in 9 represents the 3rd order cumulants. Hence the least square estimator of (11) estimates the blur model using the 3rd order cumulants. Once the model is obtained, the inverse filtering is performed after converting the low-pass (LP) structure of (7) into high-pass form.

The estimate $\hat{q}$ can be obtained by using other estimators, like RLS (Recursive Least Square), LMS (Least Mean Square), MLE (Maximum Likelihood Estimator), etc…

The estimation and filtering operations are performed recursively on the C-Scan. To quantify the enhancement, entropy is calculated for each image. However, in order to reduce the computational load, the probabilities are not calculated for the gray-scaled image but a binary edged image is calculated. The edges are calculated at each iteration. For each binary edge image, entropy (E) is calculated and a knee point is identified to stop the iterations. The entropy is calculated as:

$$E = -\frac{1}{N}\sum_{i=1}^{M} p_i \log_2 p_i \qquad (12)$$

Where, N is the number of pixels in the image, p_i is the probability of the i^{th} bin out of M bins in the histogram of the image. Once the iterative procedure is stopped, further improvement can be achieved by masking out the background using the final edges. These results are shown in Figure 2. As can be seen from Figure 2(f), the image has appreciably enhanced to fulfill most of the practical applications.

4. CONCLUSION

In this paper, a new method for image enhancement by blur deconvolution is presented. The process is carried out using 3rd order constrained cumulants, as only few options of delays (s_1, s_2, t_1, t_2) are used.

The algorithm works blindly to the blurring source and inverse filtering results a better image representation with recursive application of Equations (7-11). Other estimators can also be implemented for (10), for instance Least Mean Square (LMS) or Maximum Likelihood (ML) estimators.

5. REFERENCES

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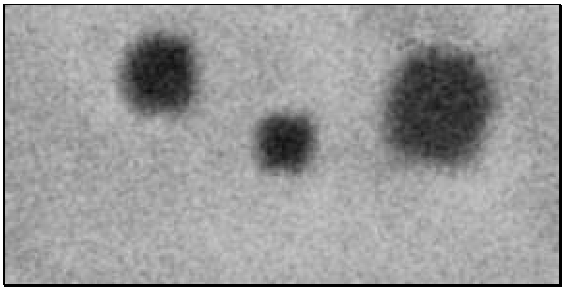
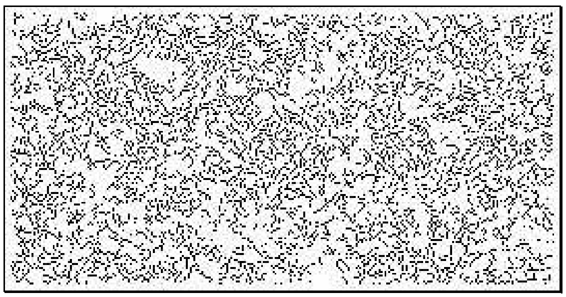
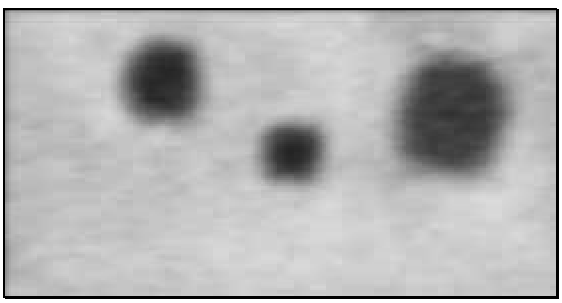


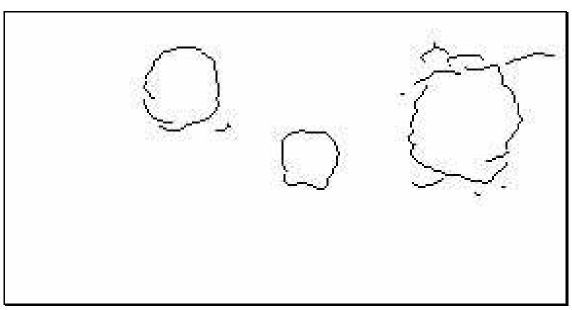
(a)



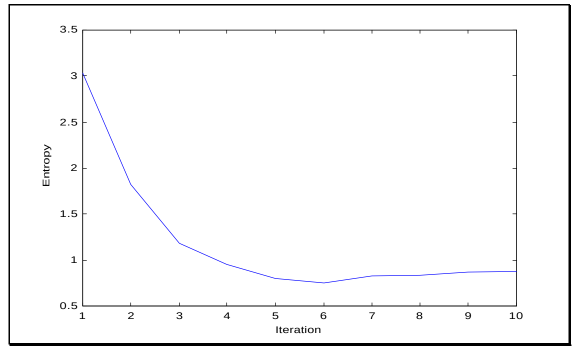
(b)



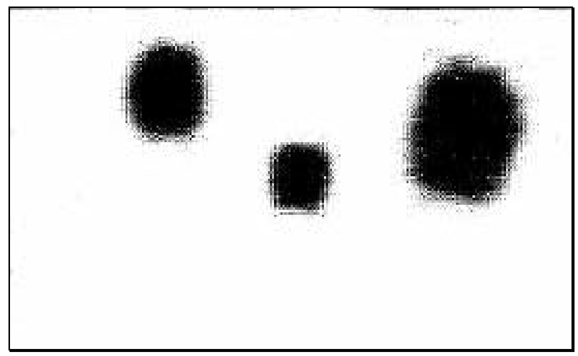
(c)



(d)



(e)



(f)

Fig. 2. (a) Original defect image, (b) Original edges, (c) Image after 6th iteration, (d) edges corresponding to (c), (e) Entropy curve, and (f) Final image after masking out the background.