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NEW ESTIMATIONS OF THE RELATION BETWEEN THE ALPHA-EFFECT AND MAGNETIC FIELD FLUCTUATIONS

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RESUMEN

Se muestra que la fórmula Diamond-Vainshtein "exacta", que relaciona el coeficiente del efecto alfa y la correlación de fluctuaciones del campo magnético, no es válida. Se propone, como una aproximación cruda, la fórmula análoga con difusividad turbulenta. De las ecuaciones de la magnetohidrodinámica se derivan relaciones integrales exactas que relacionan al campo magnético promedio y su parte fluctuante en estado estacionario con la velocidad turbulenta. Estas relaciones pueden ser usadas en simulaciones numéricas para verificar la exactitud de los cálculos. La nueva estimación del coeficiente alfa permite que el mecanismo de dinamo aumente el campo magnético medio en un medio conductor turbulento, inclusive a un nivel moderado de fluctuaciones magnéticas.

ABSTRACT

It is shown that the "exact" Diamond-Vainshtein formula connecting the alpha-effect coefficient with the correlator of magnetic field fluctuations is not valid. Instead, the analogous formula with turbulent diffusivity is proposed but only as a crude approximate relation. Two exact integral relations connecting the mean magnetic field and its fluctuating part in steady state between the magnetic field and a turbulent velocity field are derived from the equations of magnetohydrodynamics. These relations may be used by numerical simulations of the magnetic field evolution to check the accuracy of the calculations. The new estimation of the alpha-coefficient permits the effective dynamo mechanism of enhancement of the mean magnetic field in turbulent conducting media, even for a moderate level of magnetic fluctuations.

Key Words: **MAGNETIC FIELDS — MHD**

In Gruzinov & Diamond (1994) and Vainshtein (1998) the following remarkable formula

$$\alpha = -\eta \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle / B_0^2 \tag{1}$$

was derived. Here $\mathbf{B}_0$ is the mean magnetic field considered as permanent, $\mathbf{b}$ is the fluctuating part of the magnetic field ($\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, $\langle \mathbf{B} \rangle = \mathbf{B}_0$, $\langle \mathbf{b} \rangle = 0$), and η is the ohmic diffusivity. The α-coefficient describes the enhancement of the mean magnetic field in a turbulent helical medium. Equation (1) denotes that the enhancement is very small under normal cosmic conditions, when the magnetic Reynolds number $R_m \gg 1$. The level of magnetic fluctuations must be of the order $b^2 \simeq R_m B_0^2$ to ensure that the usual α-dynamo is an effectively working mechanism. For the Sun such large fluctuations are not observed and one needs to seek a new dynamo mechanism, if indeed equation (1) is valid. The authors insist that equation (1) is an *exact* relation (known now as Diamond-Vainshtein theorem). Here we shall show that this relation does not exist at all.

Seehafer (1996) derived two exact equations describing the evolution of the mean magnetic helicity

$$\langle H_m \rangle = \langle \mathbf{A} \cdot \mathbf{B} \rangle = \mathbf{A}_0 \cdot \mathbf{B}_0 + \langle \mathbf{a} \cdot \mathbf{b} \rangle \quad ,$$

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where $\mathbf{A} = \mathbf{A}_0 + \mathbf{a}$ is the vector potential:

$$\frac{\partial \mathbf{A}_0 \cdot \mathbf{B}_0}{\partial t} = -2\eta \mathbf{B}_0 \cdot \nabla \times \mathbf{B}_0 + 2\mathbf{B}_0 \cdot \langle \mathbf{v} \times \mathbf{b} \rangle - \nabla \cdot (\mathbf{E}_0 \times \mathbf{A}_0) \quad , \tag{2}$$

$$\frac{\partial \langle \mathbf{a} \cdot \mathbf{b} \rangle}{\partial t} = -2\eta \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle - 2\mathbf{B}_0 \cdot \langle \mathbf{v} \times \mathbf{b} \rangle - \nabla \cdot \langle \mathbf{e} \times \mathbf{a} \rangle \quad . \tag{3}$$

Here $\mathbf{E} = \mathbf{E}_0 + \mathbf{e} = \eta \nabla \times \mathbf{B} - \mathbf{v} \times \mathbf{B}$ is the electromotive force. The sum of equations (2) and (3) gives the equation describing total mean magnetic helicity $\langle H_m \rangle$. It does not depend on the term $\langle \mathbf{v} \times \mathbf{b} \rangle$ describing the α-effect. Therefore, considering the relation between the α-effect and the magnetic field fluctuations we should take into account *both* equations (2) and (3). Equation (1) comes from equation (3) if one assumes a *stationary* and *homogeneous* ensemble of fluctuations. But, for the case $\mathbf{B}_0 =$ const required for equation (1), equations (2) and (3) give $\alpha = 0$ and $\langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle = 0$. Thus, the exact, equally important equations (2) and (3) do not confirm the existence of equation (1) in a nontrivial form. In real situations stationary and homogeneity are valid only locally and the relation between the α-effect and the fluctuations $\mathbf{b}$ depends on the particular values of the derivatives in equations (2) and (3). In Gruzinov & Diamond (1994) equation (3) was considered without equation (2). This gave the erroneus derivation of equation (1).

For realistic cases, when the magnetic field tends to zero outside the finite volume, for a stationary state turbulent magnetized medium, integration of equations (2) and (3) gives the exact relations:

$$\int dV \left\{ \mathbf{B}_0 \cdot \nabla \times \mathbf{B}_0 + \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle \right\} = 0 \quad , \tag{4}$$

$$\int dV \left\{ \alpha B_0^2 + (\eta + \eta_T) \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle \right\} = 0 \quad . \tag{5}$$

Here we have used the known representation $\langle \mathbf{v} \times \mathbf{b} \rangle = \alpha \mathbf{B}_0 - \eta_T \nabla \times \mathbf{B}_0$, where η_T is the turbulent diffusivity. The second integral claims that for some characteristic values there exists the estimate

$$\alpha = -(\eta + \eta_T) \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle / B_0^2 \tag{6}$$

which, due to the evident inequality $\eta_T \gg \eta$, differs radically from equation (1). Vainshtein (1998) does not present an "exactly solvable model of nonlinear dynamo", as claimed in the title of that paper, because he had omitted the nonlinear fluctuating term in the Navier-Stokes equation. His solution does not permit, even in principle, any true relations concerning quadratic fluctuations like those in equation (1) to be found, since the omitted term has an unknown contribution to equation (1). For this reason his derivation of equation (1) is also wrong.

The integral relations (eqs. 4 and 5) may be used to check the correctness of any numerical simulations of magnetic field evolution in a turbulent medium. The calculated values must obey equations (4) and (5) by tending to a stationary state.

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