AN ADAPTIVE IMAGE ESTIMATE FRAMEWORK FOR LOW ORDER DYNAMIC MAGNETIC RESONANCE IMAGING

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ABSTRACT

Low order imaging techniques use signal processing concepts to improve magnetic resonance imaging (MRI) hardware performance. Building upon a linear MRI system model, this paper proposes an adaptive framework to efficiently estimate image changes in a dynamic MRI sequence. We show that our approach provides significant improvement to the current state of the art. The adaptive framework also suggests an MRI pulse sequence selection method that gives estimation error lower than that of other methods. Furthermore, we propose a minimization problem to enable the determination of optimal image encoding/reconstruction vectors as the image sequence progresses.

1. INTRODUCTION

Magnetic resonance imaging (MRI) has become a powerful tool for non-invasive imaging of body tissue, providing high resolution images of internal tissue structure. Traditional MRI techniques use a sequence of magnetic field gradients and radio-frequency (rf) signals to encode the position and composition of the molecules within a tissue volume. Typically, these excitation sequences are used to scan the tissue volume in a series of slices through direct sampling of the two-dimensional Fourier domain, or *k-space*, of each slice. An inverse Fourier transform is then used to reconstruct images of the tissue composition [1]. However, under appropriate conditions, the MRI acquisition process can be described by a linear system model [2]. This model allows the image acquisition to be described by a set of excitation and reconstruction vectors, and provides a mechanism for *low-order* MRI acquisition.

Low order MRI acquisition techniques use signal processing theory to enhance the performance of magnetic resonance imaging (MRI) systems. MRI is a very flexible technology. Depending on the application some image acquisitions take less than a second, while others take tens of minutes. In each case, there are tradeoffs between spatial resolution, temporal resolution, the signal to noise ratio (SNR), volume coverage, and imaging artifacts. Most past MRI research has been spent improving the imaging hardware. Low order techniques build upon this work and provide a "software tool set" to improve the MRI hardware performance.

The goal of our low order acquisition work is to decrease the dynamic sequence acquisition time without degrading other qualities (spatial resolution, SNR) of the acquisition data. Recent advances in low order image acquisition techniques include the use of wavelets [3, 4], Fourier keyhole methods [5], efficient region of interest acquisitions [6] including multiple region MRI [7], and dynamic imaging temporal resolution improvement [8].

The method we use to achieve efficient low order acquisitions follows techniques similar to system identification using adaptive filters. The primary difference is that we have complete control over the input to the system. In a signal processing context, our goal is to use a minimal number of inputs to optimally estimate the underlying MRI image, optimally choose the next input to use, and do so in an adaptive fashion to track changes in the system. In the sections that follow, we detail a solution to the first task and propose a solution for the other two.

2. THE LINEAR SYSTEM MODEL

The MR image acquisition process can be described by a linear response model if the image acquisition uses rf encoding and a low flip angle excitation [2]. For a

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given image A, the outputs Y from a set of inputs X can be described by the matrix product

$$Y = AX.$$

Each of the r columns in X and Y represent one acquisition experiment, analogous to sampling one line of k-space. Each column of X is thus an *input vector* for the system. To reconstruct the image, a set of r reconstruction vectors L are applied via $\widehat{A} = YL^H = AXL^H$. As originally presented in [2], A, X, and Y were frequency domain representations of the image and input and output vectors. Appropriate use of the inverse Fourier transform allows for a spatial/temporal interpretation of these quantities. Thus for this paper, the elements of A represent image pixel intensities.

3. DYNAMIC MRI

A promising new application for the linear system model described above is dynamic MRI. Here the objective is to provide high temporal and spatial resolution of a single slice through a series of scans, allowing one to monitor changes in the tissue structure. Applications of interest include imaging of cardiac cycles and monitoring of thermal therapy and surgical procedures [9]. For these problems, relevant changes in tissue structure occur in a small region of interest (ROI) within the scanned image plane. Current dynamic imaging methods however are based on Fourier or SVD basis sets which cover the entire image plane and are not easily tailored to the acquisition and reconstruction of local image information.

To reduce the dynamic sequence acquisition time, we take advantage of the local nature of the desired information and make use of the linear system model in [2] to determine a suitable image acquisition vector set. In an adaptive estimation context, one may train a model on a given set of images. From this model, one may then choose a set of input vectors to acquire a new set of images. This brings to mind the following three questions. How does one update the image estimate? How does one choose the “best” set of input vectors? And can one track the system, i.e., update both the image estimate and the “best” input vector set concurrently? We present a solution to the image update question in this paper. We also give a solution for the “best” input vector set for rectangular ROIs. Finally we propose a solution for the best input vector set for an arbitrary ROI problem, the details of which will be presented at the workshop.

4. AN ADAPTIVE ESTIMATION FRAMEWORK

This section details our adaptive modeling framework and proposes a new method to determine an appropriate dynamic MRI image estimation vector set for rectangular ROIs. Before proceeding, we make the following comment. In this paper we approach only rectangular ROIs. The area selected by a rectangular ROI can be regarded as simply a smaller image. Thus the notation A_n in the sections that follow refer to the rectangular sub-image of interest at time n. The formulation introduced in Section 4.2 easily allows the introduction of an arbitrary ROI [10], to which we will extend our results using the optimization problem discussed in Section 5.

4.1. Adaptive image estimation for MRI

The idea of low order techniques for the acquisition of dynamic MRI sequences was first proposed in the mid 1990’s. Zientara, et. al., proposed using a set of input vectors related to a subspace identified from a full image acquired at the beginning of the sequence [11]. Others have proposed determining the acquisition input set a priori from a large aggregate set of similar images [12, 13]. In each of these cases one set of orthogonal input vectors X is chosen to acquire the dynamic sequence and a low-order estimate of the image is formed via the matrix-vector product $\widehat{A}_n = Y_n X^H$. The estimation error at time n for this non-adaptive model is

$$\mathcal{E}_n^{(1)} = \|A_n - A_n X X^H\|_F^2 = \|A_n(I - XX^H)\|_F^2 \quad (1)$$

Our approach is to focus on estimating the change in the image rather than the image itself. For most practical dynamic MRI applications, changes in the image are relatively slow compared to the rate at which input vectors can be applied. Our adaptive approach relies on the fact that we expect only small, gradual changes in the image. In fact, we choose to approximate the image dynamics as being piece-wise constant over the interval required for a block acquisition. Thus, our adaptive model update equation is

$$\widehat{A}_n = \widehat{A}_{n-1} + \widehat{dA}. \quad (2)$$

Assuming that an input set X is given, we wish to estimate the change in the image, $\widehat{dA}$. We find the estimate through minimization of the cost function

$$\underset{\widehat{dA}}{\operatorname{argmin}} \left\| Y_n - (\widehat{A}_{n-1} + \widehat{dA})X \right\|_F^2, \quad (3)$$

where the matrices $\widehat{A}_n$ and $\widehat{dA}$ are of size $M \times N$, X and Y_n are of size $N \times r$ with $N > r$, and for an arbitrary matrix B the Frobenius norm is $\|B\|_F^2 = \sum_{i,j} b_{ij}^2$.

The solution to this minimization problem is non-unique. One solution is

$$\widehat{dA} = (Y_n - \widehat{A}_{n-1}X)X^H(XX^H)^\dagger$$

where † indicates a pseudo-inverse. Previous works [11–13] use an orthogonal input set X, and we continue that assumption in this paper. This simplifies the above equation to

$$\widehat{dA} = (Y_n - \widehat{A}_{n-1}X)X^H. \tag{4}$$

For this adaptive estimate method, the estimate error at time n is

$$\begin{aligned} \mathcal{E}_n^{(2)} &= \|A_n - \widehat{A}_n\|_F^2 \\ &= \|A_n - (\widehat{A}_{n-1} + (A_nX - \widehat{A}_{n-1}X)X^H)\|_F^2 \\ \mathcal{E}_n^{(2)} &= \|(A_n - \widehat{A}_{n-1})(I - XX^H)\|_F^2. \end{aligned}$$

This error term contains two contributions. One is from the difference between the image at time n and the previous estimate, $(A_n - \widehat{A}_{n-1})$. The second is from those modes of the image that were not probed. These modes are identified by $(I - XX^H)$. In effect, subsequent acquisitions are projections onto the subspace described by XX^H. When the estimate is formed, these projections will cancel with the complimentary subspace $(I - XX^H)$. For example, the estimate at $n-1$ is built from the estimate at $n-2$.

$$\begin{aligned} \widehat{A}_{n-1} &= \widehat{A}_{n-2} + (A_{n-1}X - \widehat{A}_{n-2}X)X^H \\ &= A_{n-1}XX^H + \widehat{A}_{n-2}(I - XX^H) \end{aligned}$$

Using this expression in $\mathcal{E}_n^{(2)}$, we find that the information in the intermediate image estimate $\widehat{A}_{n-1}$ will cancel, resulting in

$$\mathcal{E}_n^{(2)} = \|(A_n - \widehat{A}_{n-2})(I - XX^H)\|_F^2.$$

The cancellation effect will propagate all the way back to the first given estimate, which can be the first full image from which the input vectors were derived. Thus, at time n, the error of the adaptive model is given by

$$\mathcal{E}_n^{(2)} = \|(A_n - A_0)(I - XX^H)\|_F^2. \tag{5}$$

Figure 1 illustrates the improvement gained by this adaptive estimate framework for a simulated motion sequence. The original image, A_0, is shown in Figure 1(a). A second image, A_1, was generated by moving the triangle on the lower left two pixels down and two pixels to the left. The input vectors were selected from the 16 right singular vectors associated with the largest singular values of the first image. To compare the estimation error, we calculate the relative error as $\mathrm{re}(\mathcal{E}_n^{(k)}) = \mathcal{E}_n^{(k)} \;/\; \|A_n\|_F^2$.

For the non-adaptive estimate, $\widehat{A}_1 = A_1XX^H$, $\mathrm{re}(\mathcal{E}_1^{(1)}) = 0.024698$, and the absolute error image, $|A_1 - \widehat{A}_1|$, is shown in Figure 1(b). For the adaptive estimate, $\widehat{A}_1 = A_1XX^H + A_0(I - XX^H)$, $\mathrm{re}(\mathcal{E}_1^{(2)}) = 0.017784$, and the absolute error image, $|A_1 - \widehat{A}_1|$, is shown in Figure 1(c). The decrease in error between $\mathcal{E}^{(1)}$ and $\mathcal{E}^{(2)}$ is confirmed by visual comparison of the two error images. As seen in Figure 1, there is significantly less error in the image estimate formed by the adaptive method.

4.2. Determining an input vector set for rectangular regions

There is some debate in the low-order MRI acquisition community as to the "best" choice of vectors for acquiring a dynamic sequence. As mentioned previously, one proposal [11, 14] is to use the SVD of the first full image in the sequence to determine X. Other proposals involve developing a library of typical images from which to derive X, [12,13]. Based on the estimation error given in (5), we propose a third possibility: finding X from a sequence of image differences. This section examines the results of such a choice for a rectangular ROI.

If a series of $k+1$ full images are available, the generalization of (5) is

$$\sum_{i=0}^{k} \left\|(A_{i+1} - A_i)(I - XX^H)\right\|_F^2. \tag{6}$$

The matrix X that minimizes this equation can be found through the right singular vectors of the the matrix

$$\begin{bmatrix} (A_1 - A_0) \\ (A_2 - A_1) \\ \vdots \\ (A_k - A_{k-1}) \end{bmatrix}. \tag{7}$$

This formulation allows X to reflect the trajectory of the image changes and works to our advantage.

Utilizing real MRI data in a simulated contrast agent flow sequence, we compare here the results between vectors chosen using (7) with vectors chosen from a single image, as per Zientara, et. al., in [11, 14]. Figure 2 shows the first image in the sequence and the 75×75 pixel square ROI centered on the knee portion of the image.

At the start of the sequence, 3 full-order images were collected. One set of input vectors, denoted X_{diff},

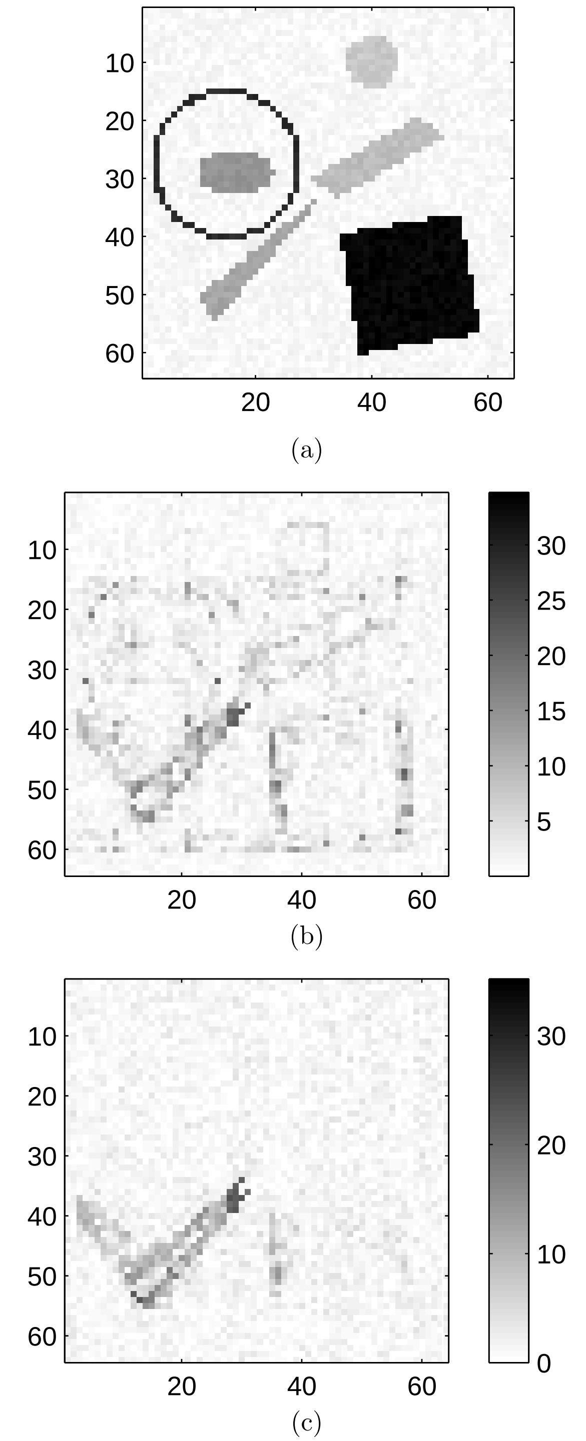


Figure 1: Comparison of image estimate error differences between a previous method and the proposed adaptive method. (a) Original Image. (b) Absolute error in estimate formed by AXX^H. (c) Absolute error in adaptive method estimate.

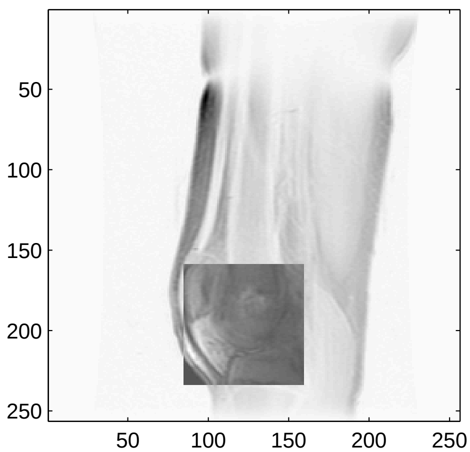


Figure 2: First image of contrast agent flow sequence. Square ROI is indicated with an inverse pixel map.

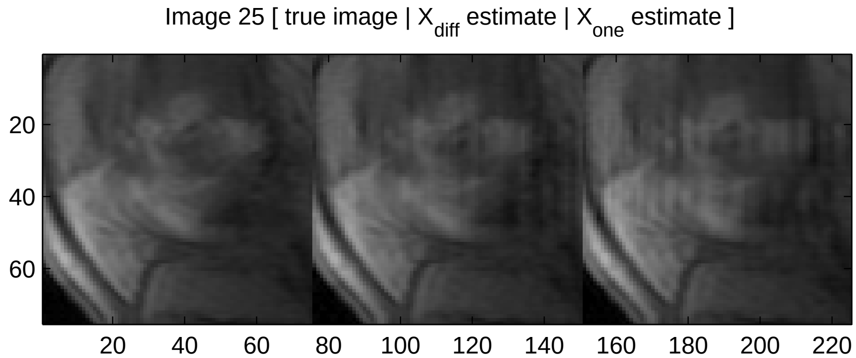
was chosen from the SVD of the stacked first order difference matrices as given in (7). A second set of inputs, denoted X_{one}, was determined from the SVD of the last full-order image in the sequence, i.e., the third image. Both sets were $r = 25$ columns wide, or 1/3 the full column width of the ROI. Each input set was then used in the adaptive estimate update equations (2) and (4) to estimate the changes in the image sequence. To compare the estimates across the entire sequence, we show the relative error, $\text{re}(A_n, \widehat{A}_n) = \|A_n - \widehat{A}_n\|_F^2 \;/\; \|A_n\|_F^2$.

Figure 3 shows both the image estimate and the absolute estimation error for each input vector choice for the final image in the sequence. Figure 4 shows the average error per pixel across the sequence for each choice of input vectors. As seen in the figures, the vectors determined from the difference matrices provide lower estimation error than vectors chosen from just a single image. Simulated motion change sequences show a similar improvement in performance.

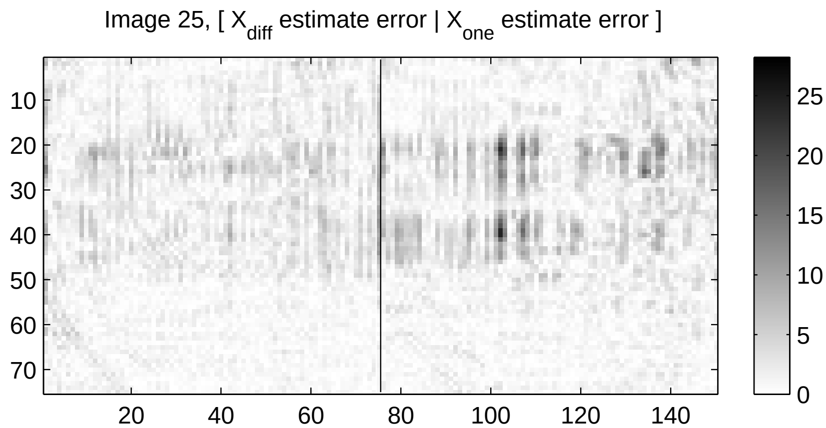
5. DISCUSSION

We have shown here that the proposed adaptive framework provides substantial improvement to the SVD method proposed by Zientara, et. al., in [11, 14]. We have also shown that input vectors chosen from the first order differences between images in the sequence gives lower estimation error than vectors chosen from just one image.

However, for any image sequence it would be advantageous to adapt both X and $\widehat{A}$ as the image progresses. Furthermore, we wish to extend the results given here to non-rectangular ROIs. For this reason we propose the following problem to recursively identify a set of



(a)



(b)

Figure 3: (a) Last image and (b) absolute error, $|A_n - \widehat{A}_n|$, in last image of simulated contrast flow sequence to compare image acquisition vector sets X_{diff} and X_{one}.

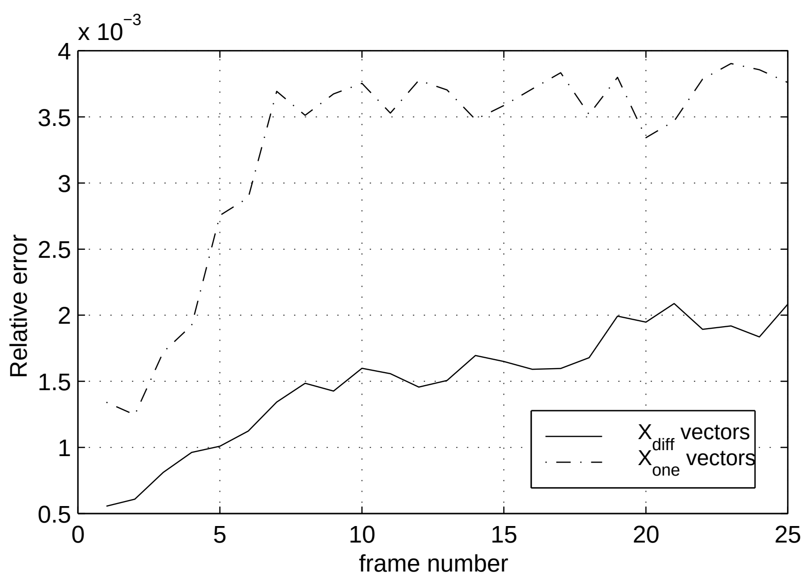


Figure 4: Comparison of relative estimation error, $\text{re}(A_n, \widehat{A}_n)$, for the simulated contrast flow sequence. '—' = Error in sequence acquired by X_{diff} vectors; '$- \cdot -$' = Error in sequence acquired by X_{one} vectors.

vectors and update the image estimate for a dynamic MRI sequence.

For non-rectangular ROIs, the SVD does not give optimal input vectors [10]. Furthermore, maintaining orthogonal columns in X gives significant advantage to the adaptive framework given in this paper. Thus we propose an optimization problem that minimizes an error function similar to (6) which incorporates an arbitrary ROI and an orthogonality constraint on X. Our approach utilizes concepts put forth in recent papers [15–17] that apply differential geometry to adaptive filtering and subspace tracking algorithms. The common idea is that requiring the solution matrix X to have orthogonal columns results in a parameter space of subspaces — i.e. a manifold. Optimizing functions over a manifold parameter space can result in algorithms with reduced complexity and strengthened robustness.

To solve the problem of jointly tracking the input vectors as well as the image itself, we will apply adaptive methods based on extended Kalman filtering concepts. We will present the details and solution of this problem at the workshop.

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