

CALCULATORS IN THE CLASSROOM: A LOOK TO THE FUTURE

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Computer symbolic algebra software on hand-held computers like the Texas Instruments TI-92 will likely become as popular as scientific calculators are today. Many paper and pencil computation methods learned today should become obsolete necessitating many changes in the mathematics curriculum of the future. The mathematics curriculum of the future can focus more on problem solving, applications concepts, and understanding.

INTRODUCTION

Electronic calculators are now over 25 years old. In the beginning they were simple "four function" devices that did only basic arithmetic such as the Texas Instruments "DataMath" costing \$120 US in 1972. They were soon followed by "electronic slide rules," scientific calculators, that did sophisticated transcendental computations with 8 to 12 digit accuracy. The first scientific calculator was the HP-35 introduced in 1972 (it cost \$395 US). Scientific calculators are now very inexpensive (\$10 to 20 US) and have significantly changed the mathematics curriculum taught in most countries. For example, we no longer spend time teaching paper and pencil methods to compute values of transcendental functions. More time could be spent on applications and conceptual understanding as scientific calculator use became widespread.

Ten years ago calculators took a giant evolutionary step and added new functionality found only on large desktop PC computers. These were the so-called graphing calculators, first invented by Casio in 1985. Graphing calculators started a revolution in the teaching and learning of mathematics. Inexpensive graphing calculators were really computers with built-in graphing software. Graphing calculators could be viewed as *computers available to all students* because of their low cost, ease of use, and portability (Demana and Waits, 1992).

Before graphing calculators, teachers had to rely exclusively on expensive computer laboratories (usually in a separate computer laboratory) to deliver computer enhanced visualization in mathematics teaching and learning. Only a few elite schools could provide such an experience to students on a regular basis. The significance of these small, inexpensive, hand-held devices should not be underestimated. Graphing calculators

now provide millions of students significant experience enhancing their mathematics learning experience with computer visualization. Teachers are now able to present mathematical ideas, concepts, and applications in both symbolic as well as numerical and graphical representations. Powerful new approaches to learning mathematics have been made possible by graphing calculators. It is now well established that a richer mathematics curriculum is possible when all students have access to graphing calculators.

Graphing calculators do have powerful built-in numerical and graphical software. However, they lack three very significant software applications for enhancing mathematics that are commonly available on expensive desktop computers: computer symbolic algebra (CSA), computer interactive geometry, and spreadsheets. Of particular significance for mathematics curriculum reform is student use of CSA. A practical (inexpensive) device for delivering powerful CSA for mathematics learning was needed. The next great leap in the evolution of hand-held calculators was provided by Texas Instruments in 1995.

A LOOK TO THE FUTURE

The Texas Instruments TI-92, introduced in late 1995, is a relatively inexpensive (2X the cost of a graphing calculator but 25X more powerful!) hand-held computer with built-in computer symbolic algebra system (using powerful DERIVE algorithms) and computer interactive geometry (an almost complete version of CABRI II). It is the first of a no doubt new generation of super graphing calculators. Other calculator manufacturers will surely follow with similar products. These new inexpensive and easy to use hand-held *personal* student computer tools contain very powerful and versatile computer software and now really represent "a computer for *all* mathematics students." These new tools and their successors will very significantly change the mathematics curriculum of the future. We will move from a focus on manipulative skill to a focus on understanding, concepts, and problem solving.

IMPLEMENTING THE NCTM STANDARDS

A major concern to us today is that the following underlying assumptions about computer technology of the well know *Curriculum and Evaluation Standards for School mathematics* (NCTM, 1989, p.124) are still nearly *impossible* to implement in typical high schools.

A computer will be available at all times in every classroom for demonstration purposes, and all students will have access to computers for individual and group work.

One reason is the high cost of computer laboratories, their upkeep, and related training issues. Another reason is that the secondary *Standards* technology strands require students to use computer software that is far more sophisticated than what can be delivered by even the most modern graphing calculators. Dependence only on desk top computers and expensive software housed in computer laboratories is still a major barrier to implementing serious curriculum reform in mathematics.

Many teachers have simply opted to *avoid* using computer symbolic algebra and computer interactive geometry in their high school mathematics classes because it was simply not practical or possible to do so. Now it is both possible and practical! For example, a set of TI-92's is an inexpensive and portable computer laboratory. Implementing the important computer technology strands of the NCTM grades 9-12 *Standards for all* students is finally feasible.

THE TWILIGHT OF TRADITIONAL PAPER AND PENCIL ALGEBRAIC SKILLS

In the past, teaching of traditional (paper and pencil) algebra symbolic skills in school mathematics *was* necessary because they were the *only* procedures available for algebraic manipulation necessary to "solve" problems. Today this simply is not the case. CSA algorithms now do algebraic manipulations (much) faster and with far better accuracy than possible with the "traditional" skills.

The cover story of the March 26, 1996 USA TODAY newspaper reported on the National Education Summit attended by 44 corporate Chief Executive Officers (CEO) invited by the USA's governors. One CEO, Richard Notebaert of Ameritech, said, "We're trying to hire. We're having difficulty finding people. We're spending hundreds of millions of dollars to put high technology in schools. But it's all wasted if teachers and students aren't able to take advantage."

We believe what is needed now is a secondary mathematics curriculum that takes advantage of computer technology to assist students in becoming powerful and thoughtful "problem solvers."

A NEW CHALLENGE

Mathematics teachers now have a new challenge. Our community can *no longer ignore* how student use of computer symbolic algebra and computer interactive geometry impacts the mathematics curriculum. This new generation of hand-held student computer tools will soon be as popular as graphing calculators are today. We *must* deal with the fact that computer

symbolic algebra and computer interactive geometry are better —far better— tools than paper and pencil for doing many of the "manipulations" associated with mathematics.

These new tools can also be used to *better* illustrate important concepts and applications of mathematics. We must redefine "basic skills" to include those paper and pencil manipulative tasks necessary to understand algebra as a language of representation. Some traditional paper and pencil skill will continue to be necessary for mathematical activities as will traditional mental skills. However, we *must* also agree to stop spending large portions of our time teaching obsolete paper and pencil algebra and calculus manipulations. These obsolete paper and pencil skills *must* be identified. That is our challenge for the future.

Here are two examples. Consider every one's favorite algebra topic of factoring algebraic expressions. Factoring *is still* very important. After all, the *Fundamental Theorem of Algebra* is a *factoring* theorem. This theorem is central to good mathematical understanding, as are the very important connections among factors, x–intercepts of the graph of functions, zeros of functions, and the behavior of functions. Figure 1 shows the result of apply-

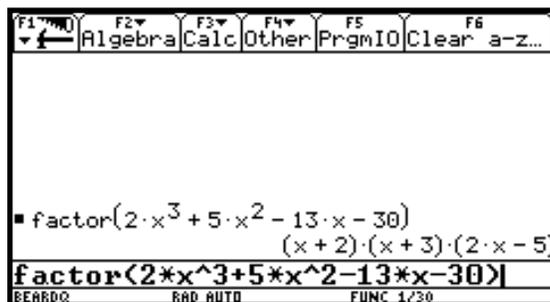


Figure 1. A CAS factoring example

ing the simple **Factor**(CAS command to the polynomial expression. The factors (by inspection) give us the solutions to the equation "expression" = 0 and tell us a great deal more information about the behavior of the polynomial function in non-factored form. Another example from calculus is "partial fractions" as an integration technique. Figure 2 shows the result of applying the **Expand**(CAS command to a simple rational function $g(x)$. Notice it is now very easy to integrate $g(x)$ (find an antiderivative) mentally from basic integration facts! The result of applying the **Integrate** CAS command *before* the CAS partial fraction decomposition is provided in Figure 3 (just to check our "mental integration" skills!).

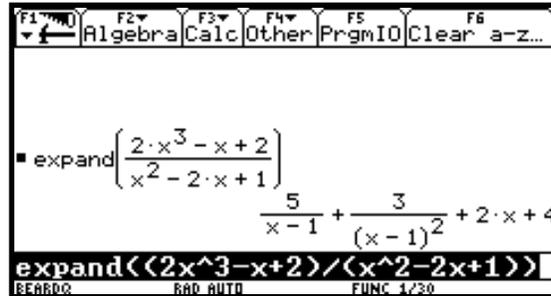


Figure 2. A CAS partial fraction decomposition

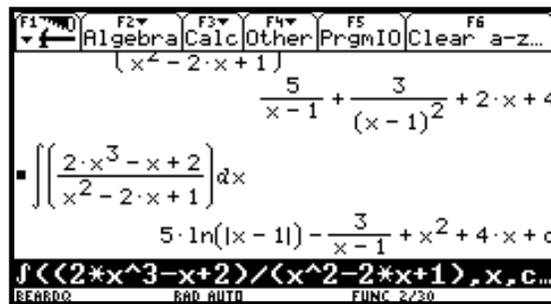


Figure 3. An indefinite integral CAS computation

We believe teachers should definitely continue to teach the concepts of factoring and partial fraction decomposition and what they mean (and why they are useful). However, the *tools* we will use "to factor" and "to find partial fraction decompositions" will move from paper and pencil (history) to student use of CAS. Teachers *should* teach the old standard and comfortable topics but should spend far *less time* with paper and pencil methods (manipulations) and far *more time* with CAS tools. Our thrust should be not to delete traditional topics but to reduce the time spent and change the tools used for these topics.

We also *must* have textbooks and tests that reflect use of these new technology tools and thus represent the *new* curriculum. We *must* as a society find appropriate ways to support and fund teachers who need in-service. Teachers will find they need to become increasingly more technology literate and become curriculum reformers as well.

A MATHEMATICS IMAGE PROBLEM

The mathematics community must do a better job of addressing the national "mathematics image problem." The public often associates "doing

mathematics" with only the mental and paper and pencil arithmetic and algebraic computations and manipulations they learned in school. We need to convincingly communicate to the general public that "doing mathematics" in the 21st century means much more than "doing" the mathematics of the past. School mathematics in the future will be far more technology enhanced, richer, interesting, and applicable than in the past. Business and industry want employees today who can think, read and understand problem situations, work cooperatively in groups, understand and use technology, and communicate effectively with others. The appropriate use of technology in mathematics teaching and learning helps build these important skills in students.

SUMMARY

Calculators with built-in *graphing* software for enhancing mathematics teaching and learning are now over ten years old. Casio invented and marketed the first graphing calculator in 1985 and started a revolution in delivering powerful and useful *computer graphing* to millions of mathematics students. Inexpensive graphing calculators have certainly fulfilled our dream of making it possible for *all* mathematics students to use computer visualization on a regular basis for both in-class and out-of-class activities—a dream that never could have been realized with desk-top PC's in computer labs.

Our world of mathematics teaching and learning will never be the same. We believe that Texas Instruments has fired the first shot in the next revolution of hand-held computer technology designed for use in school mathematics. They have produced the first inexpensive, hand-held computer symbolic algebra and computer interactive geometry tool designed for mathematics and science students. There will surely be similar products from other calculator companies in the future.

The NCTM *Standards* (NCTM, 1989) speak well to the content topics and methods that are needed in a modern mathematics curriculum for all students. Today there is a new tool that makes possible and practical the vision of the rich computer technology enhanced mathematics curriculum for *all* students represented by the NCTM *Standards*. By facilitating the reduction of time spent on paper and pencil methods, this tool will provide the additional classroom time for the new mathematical activities recommended in the *Standards*.

We now need to be more specific and explicit about a controversial issue. We can no longer spend mathematics classroom time doing everything we did in the past paper and pencil era *and* adding on the many topics and methods our students need for the technological intensive future they face. Let's all get busy today and agree on what paper and pencil computa-

tion and manipulation algorithms can better be done by computers —our students are waiting for us!

REFERENCES

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