FSLE AND FSDF JOINT DETECTORS FOR LONG CODE DS-CDMA

Luc Vandendorpe, F. Horlin, T. Sartenaer

Université catholique de Louvain, Communications and Remote Sensing Lab
Place du Levant 2, B-1348 Louvain-la-Neuve, Belgium
e-mail:{vandendorpe,horlin,sartenaer}@tele.ucl.ac.be

ABSTRACT

We investigate detection structures for a DS-CDMA multiuser uplink using codes with a period of several symbols, named long codes. We first propose a multirate description of the uplink. Then based on this formalism we revisit a number of popular receiver structures : rake receivers, minimum mean square error linear and decision feedback fractionally spaced detectors.

1. INTRODUCTION

Direct sequence (DS) code division multiple access (CDMA) has drawn the attention of the scientific community for a while. Recently this technique has also been selected by a number of standardisation bodies, for instance in second and third generation mobile cellular systems. In its basic principle each user is allocated a code and can communicate at any time on any frequency. It is now widely admitted that the receiver structures which consider the other-users-interference as additional background noise have a performance which degrades rapidly with the number of users. Besides they are highly sensitive to the near-far effect. Therefore a lot of attention has been paid lately to improved detection structures really taking into account the presence of other users and the dispersive nature of the channels. One can cite the decorrelating detector [1], the optimum detector [2], the MMSE (minimum mean square error) detector [3], decision-feedback structures [4], parallel or serial interference cancellation structures, and more recently, turbo algorithms [5].

However most of the research has been devoted to DS-CDMA systems with short codes, that is to say codes which have a length equal to the symbol period. Many standards utilize aperiodic or longer period sequences instead. In WCDMA (Wideband CDMA) [6] a long scrambling code truncated to the 10 msec frame length is used. It means that successive symbols of a given user are spread by different code segments, and a certain code segment only repeats after, say, P symbols or PN_c chips if the spreading factor is denoted by N_c. If short codes are used, the code used for all symbols of a user is the same. It may happen that the amount of interference experienced by this code is high and it will stay so. When long codes are used, the interference experienced by successive symbols changes, and one may expect that in average all users will experience similar levels of interference. Only a few contributions address topics in the specific context of long code DS-CDMA. In [7] the authors investigate a blind channel estimation method for long code multiuser CDMA. They extend correlation matching techniques to the case where the code change rapidly from bit to bit. In [8] the authors revisit linear parallel interference cancellation (PIC) for long code scenarios. They propose a weighted linear PIC system for which the weights are computed to provide the best average performance. In [9] the authors consider the downlink of long code DS-CDMA system. They investigate the performance of a chip rate zero forcing receiver followed by a correlator. In [10] the author compares the BEP (bit error probability) in the uplink of an asynchronous and multipath DS-CDMA system, for both short and long codes. Besides three receiver structures are compared : conventional, MMSE and interference cancellation for both coded and uncoded systems. MMSE reception is considered for short code systems only.

In the present paper we investigate the uplink of a DS-CDMA system using periodic long codes. We first propose a multirate description of such a system. As stated above each symbol is spread by a code segment which changes from symbol to symbol. This multirate description maps the signal of each user with periodically repeated spreading onto several parallel streams with constant spreading. In order words it translates the serial signal of each user into parallel streams which can now be described by means of linear operations only. Based on this formalism we revisit nowadays popular advanced receivers like the minimum mean square error (MMSE) linear (LE) or decision-feedback (DF) fractionally spaced (FS) joint detectors (JD) for long codes. We also comment on the matched filter or rake receiver which is the first step performed by the FS detectors.

2. TRANSMISSION SCHEME

We assume an uplink with K users each having a user-specific long code sequence of period PN_c chips where N_c is the spreading factor (the number of chips per symbol period) and P is the code period measured in symbol periods. We denote by $I_k(n)$ the symbols of user k and T the symbol period. Besides $I_{k,m_2}(m_1) = I_k(m_1P{+}m_2)$. The digital chip-rate signal produced by each user can be written as

$$x_k(n) = \sum_{m_1=-\infty}^{\infty} \sum_{m_2=0}^{P-1} I_{k,m_2}(m_1)\, a_{k,m_2}(n{-}m_1PN_c{-}m_2N_c). \quad (1)$$

The P code segments $a_{k,m_2}(n)$ have length N_c chips. One period of a user-specific long code can be obtained from the concatenation of these P code segments. $a_{k,m_2}(n)$. The signal received from the kth user after propagation over the user-specific channel whose lowpass equivalent impulse response is denoted by $c_k(t)$ is given by

$$r_{a,k}(t) = \sum_{m_1=-\infty}^{\infty} \sum_{m_2=0}^{P-1} I_{k,m_2}(m_1)\, h_{k,m_2}(t-m_1PN_cT_c) \quad (2)$$

where

$$h_{k,m_2}(t) = \sum_{n=0}^{N_c-1} a_{k,m_2}(n)\, h_k(t - nT_c - m_2 N_c T_c) \quad (3)$$

is the composite impulse response resulting from the cascade of the m_2th code segment and the impulse response $h_k(t)$. $h_k(t)$ is the composite channel corresponding to $p(t)$, the square root raised cosine chip shaping filter with roll-off α and the user-specific channel $c_k(t)$. The advantage of this representation by means of P parallel streams, is that the code used in each stream is now time-invariant. The signal received at the basestation is obviously

$$r_a(t) = \sum_{k=0}^{K-1} r_{a,k}(t) + n_a(t) \quad (4)$$

where $n_a(t)$ is the complex envelope of an additive white gaussian noise with one-sided power spectrum N_0.

3. FRACTIONAL SAMPLING

The received signal can be sampled at a rate $2/T_c$ without any loss of information. Let us define $2PN_c$ polyphase components $r_{k_2}(k_1)$ as follows:

$$\begin{aligned} r_{k_2}(k_1) &= r_a[(2k_1 PN_c + k_2)T_c/2] \\ &= \sum_{k=0}^{K-1} \sum_{m_1=-\infty}^{\infty} \sum_{m_2=0}^{P-1} I_{k,m_2}(m_1)\, h_{k,m_2,k_2}(k_1 - m_1) \\ &+ \; n_{k_2}(k_1) \end{aligned} \quad (5)$$

with $h_{k,m_2,k_2}(k_1 - m_1) = h_{k,m_2}[(k_1 - m_1)PN_cT_c + k_2T_c/2)$. Denoting by $\underline{R}(z)$ the vector of the $2PN_c$ z-transformed polyphase components of the received signal, we have

$$\underline{R}(z) = \underline{\underline{H}}(z)\, \underline{I}(z) + \underline{N}(z). \quad (6)$$

where $\underline{I}(z)$ is of size KP, $\underline{\underline{H}}(z)$ of size $2PN_c \times KP$ and $\underline{N}(z)$ is of size KP and is the vector of the $2PN_c$ z-transformed polyphase components of the noise.

4. MMSE FSLE JD

An FSLE JD device tries to build estimates of the symbols as follows:

$$\underline{\hat{I}}(z) = \underline{\underline{C}}(z)\, \underline{R}(z). \quad (7)$$

where $\underline{\underline{C}}(z)$ is a bank of $(KP \times 2PN_c)$ filters. If we want to design this IIR (infinite impulse response) linear joint detector for an MMSE criterion we can use the orthogonality principle. Let us define the vector of z-transformed estimation errors:

$$\underline{\epsilon}(z) = \underline{I}(z) - \underline{\hat{I}}(z). \quad (8)$$

The orthogonality principle requires $\underline{\underline{S}}_{\underline{\epsilon}\underline{R}}(z) = \underline{\underline{0}}$ where $\underline{\underline{S}}_{XY}$ stands for the crossspectrum beween $\underline{X}(z)$ and $\underline{Y}(z)$. From the orthogonality principle and the matrix inversion lemma, we get

$$\underline{\underline{C}}(z) = \sigma_N^{-2} \left[\sigma_I^{-2}\, \underline{\underline{E}}_{KP} + \sigma_N^{-2}\, \underline{\underline{H}}^H(1/z^*) \underline{\underline{H}}(z)\right]^{-1} \underline{\underline{H}}^H(1/z^*) \quad (9)$$

where the matrix inversion lemma has been used to get the last equation. ^{H} is used for transposition and conjugation. $\underline{\underline{E}}_{KP}$ denotes an identity matrix of order KP. Besides we have assumed white noise with variance after presampling filter given by σ_N^2. Equation 9 shows that the first operation performed by the equalizer is to apply a matched filter. This result is well-known [11] but it is its interpretation which is interesting. The matched filter receiver is made of a bank of KP branches, each one matched to a particular code segment of a particular user. In fact, the P parallel matched filters associated with a particular user are nothing but a parallel description (with P time invariant branches) of a single periodically time varying matched filter. The outputs of these matched filters are downsampled to the symbol rate $(1/PT)$. All these operations are achieved by $\underline{\underline{H}}^H(1/z^*)$. Then the symbol MIMO (multi-input/multi-output) equalization is applied. It has a MIMO transfer function given by the inverse of the key power spectrum:

$$\underline{\underline{S}}_c(z) = \left[\sigma_I^{-2}\, \underline{\underline{E}}_{KP} + \sigma_N^{-2}\, \underline{\underline{H}}^H(1/z^*) \underline{\underline{H}}(z)\right]. \quad (10)$$

This MIMO equalizer cares about multiple access interference but also about the possible interference between the different code segments of each user. In this MIMO filter, P outputs are associated with each user. Each one of these outputs is obtained by processing the PK matched filter outputs by means a specific set of KP filters. This agains appears to be a parallel version of a time varying process. For a given user the P sets of KP time invariant filters are in fact equivalent to a single set of KP time varying filters, with period P.

About the estimation errors after optimal linear joint detection we have

$$\underline{\underline{S}}_{\underline{\epsilon}\underline{\epsilon}}(z) = \left[\sigma_I^{-2}\, \underline{\underline{E}}_{KP} + \sigma_N^{-2}\, \underline{\underline{H}}^H(1/z^*) \underline{\underline{H}}(z)\right]^{-1} = \underline{\underline{S}}_c^{-1}(z) \quad (11)$$

The estimation errors are provided by the diagonal elements of the order 0 matrix in the expansion of $\underline{\underline{S}}_{\underline{\epsilon}\underline{\epsilon}}(z)$ as a z polynomial.

5. MMSE FSDF JD

The DF joint detector is made of a $(KP \times 2PN_c)$ bank of forward filters, denoted by $\underline{\underline{C}}(z)$ and of a causal feedback bank of $KP \times KP$ filters, denoted by $\underline{\underline{B}}(z) - \underline{\underline{E}}_{KP}$, and builds an estimate $\underline{\hat{I}}(z)$ in the following way:

$$\underline{\hat{I}}(z) = \underline{\underline{C}}(z)\, \underline{R}(z) - \left[\underline{\underline{B}}(z) - \underline{\underline{E}}_{KP}\right] \underline{I}(z). \quad (12)$$

The feedback section processes previous decisions. However we assume that they are correct and hence use the original symbols. The filter bank $\underline{\underline{B}}(z)$ is stable, causal and monic ('1's on the main diagonal of the order 0 matrix in the z expansion).

If this IIR equalizer with a causal IIR feedback section is designed for an MMSE criterion, we can again use the orthogonality principle. It comes

$$\underline{\underline{C}}(z) = \underline{\underline{B}}(z)\, \underline{\underline{S}}_{\underline{I}\underline{R}}(z)\, \underline{\underline{S}}_{\underline{R}\underline{R}}^{-1}(z) \quad (13)$$

With this result it appears that about the power spectrum of the estimation error, we have (using again the matrix inversion lemma)

$$\underline{\underline{S}}_{\epsilon\epsilon} = \underline{\underline{B}}(z) \left[\sigma_I^{-2} \underline{\underline{E}}_{KP} + \sigma_N^{-2} \underline{\underline{H}}^H(1/z^*)\underline{\underline{H}}(z)\right]^{-1} \underline{\underline{B}}^H(1/z^*). \tag{14}$$

Considering the key power spectrum, it can be factored in the following way [12]:

$$\underline{\underline{S}}_c(z) = \underline{\underline{D}}^H(1/z^*)\,\underline{\underline{\Lambda}}\,\underline{\underline{D}}(z), \tag{15}$$

where matrix $\underline{\underline{\Lambda}}$ is diagonal and $\underline{\underline{D}}(z)$ is a causal, monic and stable matrix transfer function with causal and stable inverse. The geometrical mean of powers of estimation errors will be minimized if we select $\underline{\underline{B}}(z) = \underline{\underline{D}}(z)$ [13, 11]. It such a case, $\underline{\underline{S}}_{\epsilon\epsilon}(z) = \underline{\underline{\Lambda}}^{-1}$. As the power spectrum $\underline{\underline{S}}_{\epsilon\epsilon}(z)$ is a pure diagonal matrix, it appears that the estimation noise is white. The geometrical mean of powers of estimation errors that is what we are interested in can also be computed by using the fact that ([12]):

$$\log_2 \det \underline{\underline{\Lambda}} = \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} \log_2 \det[\underline{\underline{S}}_c(e^{j\Omega})]\mathrm{d}\Omega\right]. \tag{16}$$

About the forward filters we also have

$$\underline{\underline{C}}(z) = \sigma_N^{-2}\,\underline{\underline{\Lambda}}^{-1}\left[\underline{\underline{D}}^H(1/z^*)\right]^{-1}\underline{\underline{H}}^H(1/z^*). \tag{17}$$

This filter also appears to be made first of a bank of matched filters. Then it is cascaded with an anticausal bank. The feedback section $\underline{\underline{B}}(z) - \underline{\underline{E}}_{KP}$ is also a parallel representation of a period P time varying feedback process. The triangular nature of the order 0 feedback matrix also means that each time a new decision is available it is used for the next estimate to be computed.

6. COMPUTATIONAL RESULTS

Computations have been made for a system with $K = 5$ active users, BPSK (binary phase shift keying) modulation, complex spreading with short Hadamard codes of length $N_c = 16$, scrambling with Gold codes of length $P \times N_c = 32 \times 16 = 512$, a half root Nyquist chip shaping filter $p(t)$ with roll-off 0.22. Power control is applied in order to get the same averaged (over P symbols) $E_b/N_0 = 15$dB for each user. Static multipath channels with $L = 6$ paths are used for each user, with random delays and amplitudes. A typical value of the delay between the first path and the last path is $100T_c$.

Figure 1 shows the matched filter bound (MFB) and the SNRs (signal-to-noise ratios) at the output of the matched filter as a function of the symbol under consideration when long codes are used. The 32 SNRs associated with the first user are first given, then the 32 ones of the 2nd user, etc up to 160 as we have 5 users with period $P = 32$. It turns out that for a certain average E_b/N_0 the different symbols of a certain users may have quite different MFBs. Besides it also appears that the performance at the output of the matched filter bank is rather poor in the scenario considered here. Figure 2 shows the performance after FSLE JD and FSDF JD (assuming perfect decisions). The performance of these two detectors is quite close, and much above that of the matched filter. It also turns out as already mentioned that not all symbols have the same SNR. For the FSLE and FSDF JD, in average one can say that all users experience a similar SNR. It is interesting to

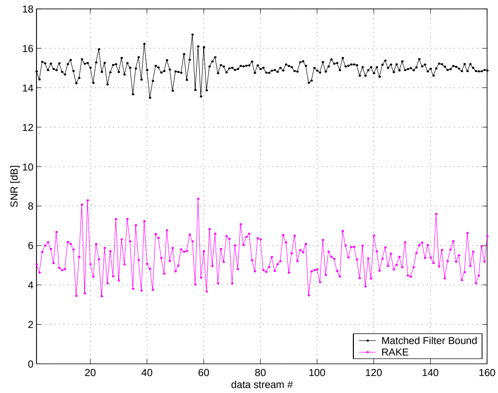

Figure 1: Matched filter bound and the SNRs at the output of the rake receiver (or perfect matched filter) as a function of the symbol under consideration when long codes are used, $K = 5$, $L = 6$, $P = 32$, $N_c = 16$

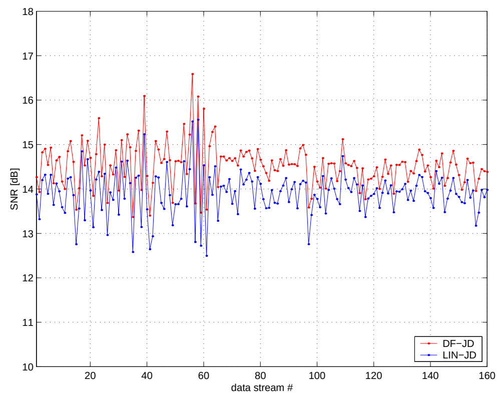

Figure 2: Performance after FSLE JD and FSDF JD as a function of the symbol under consideration when long codes are used, $K = 5$, $L = 6$, $P = 32$, $N_c = 16$

compare these results with those obtained without the long code, that is to say Hadamard codes only, for the same scenario. As all symbols of a particular user now behave in a similar way, we have 5 SNRs (as we have 5 users). Figure 3 reports the matched filter bound (MFB) and the SNRs at the output of the matched filter as a function of the symbol under consideration when short codes are used. Figure 4 reports the performance after FSLE or FSDF JD. It appears that for the choice of codes made here some users may experience very bad transmission conditions.

7. CONCLUSIONS

We have considered the uplink of a long code DS-CDMA system. In order to describe the signals by means of time invariant operations only, or render the input cyclostationary, a multirate description of each user-specific signal has been proposed. It can be seen as a parallel or vector description of a periodically time-varying process. On the basis of this formalism, the structures of MMSE FSLE and FSDF JD have been investigated. It appeared that each

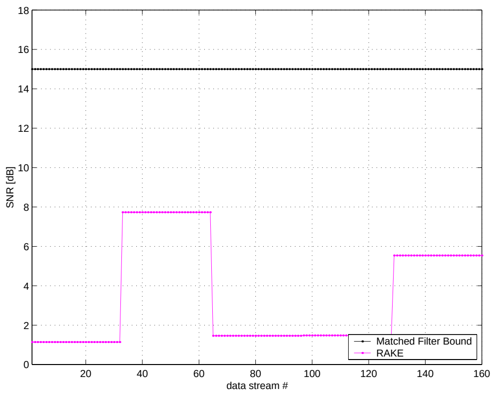

Figure 3: Matched filter bound and the SNRs at the output of the rake receiver (or perfect matched filter) as a function of the symbol under consideration when short codes only are used, $K = 5$, $L = 6$, $N_c = 16$

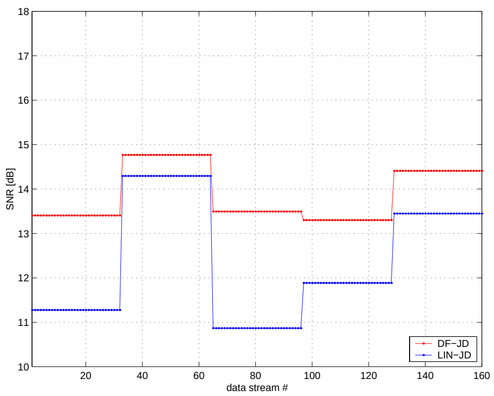

Figure 4: Performance after FSLE JD and FSDF JD as a function of the symbol under consideration when short codes only are used, $K = 5$, $L = 6$, $N_c = 16$

user requires as many sets of filters as there are different code segments in the periodic long code. For the FSLE JD the filter is made of a bank of matched filters (as many as users, K, multiplied by code segments, P) followed by a low rate MIMO detector. This parallel representation of the receivers with KP outputs appears to be equivalent to K time varying receivers, with period P. As for the DF receiver these comments also apply to the forward filter which is however different. The feedback filter also appears to be a parallel representation of a time-varying process.As already pointed out in [10] all symbols of a certain user no longer have the same signal to noise ratio.

8. REFERENCES

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