New Approximation Results for the Multiprocessor Open and Flow Shop Scheduling Problem

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Abstract

We investigate the multiprocessor multi-stage open shop and flow shop scheduling problem. In both problems, there are s stages each consisting of a number m_i of parallel identical machines for $1 \leq i \leq s$. Each job consists of s operations with one operation for each stage. The goal is to find a non-preemptive schedule that minimizes the makespan. We propose polynomial time approximation schemes for the multiprocessor open shop and flow shop scheduling problem when the number of stages s is constant and the numbers of machines m_i are non-constant.

Problem Definition. A flow shop (or open shop) is a multi-stage production process with the property that all jobs have to pass through the stages. For flow shops the order in which the jobs pass through the stages is the same, whereas for open shops the order is immaterial. There are n jobs J_j, with $j = 1, \ldots, n$, where each job J_j consists of s operations $O_{1j}, \ldots, O_{sj}$. The operation O_{ij}, with $i = 1, \ldots, s$, has to be processed at stage i of the production process, and p_{ij} is the processing time or length of operation O_{ij}.

In the classical open and flow shop problem, there is only one machine available for each stage. In the multiprocessor open and flow shop problem, for every stage i there are m_i identical machines available that can process operations in parallel. Since more than n machines on a stage are not necessary, we may assume that $m_i \leq n$. At any time step, every job is processed by at most one machine and every machine executes at most one job. We assume that preemption is not allowed, i.e. once an operation is started, it must be completed without interruption. The goal is to find a schedule that minimizes the makespan C_{max}, that is the maximum completion time among all jobs. The minimum makespan among all schedules is denoted by C^*_{max}.

Following the three-field notation scheme [10], the makespan minimization problem in a classical open and flow shop with s stages is denoted by $Os||C_{max}$ and $Fs||C_{max}$ (or $O||C_{max}$ and $F||C_{max}$ depending on whether the number s of stages is constant or not), respectively. The makespan minimization in a multiprocessor s-stage open and flow shop is denoted by $Os(P)||C_{max}$ and $Fs(P)||C_{max}$ (or $O(P)||C_{max}$ and $F(P)||C_{max}$), respectively.

Complexity Results. Gonzales and Sahni [5] proved that $Os||C_{max}$ is NP-hard in the weak sense, and Williamson et al. [15] showed that $O||C_{max}$ is NP-hard in the strong sense. On the other hand, Garey et al. [4] showed that $F3||C_{max}$ is strongly NP-hard, and Hoogeveen et al. [8] proved that $F2(P2, P1)||C_{max}$ (with two stages, two machines on the first stage and one machine on the second stage) and $F2(P1, P2)||C_{max}$ are already strongly NP-hard. If we have only one stage $s = 1$, then we have the classical strongly NP-hard scheduling problem $P||C_{max}$ of independent jobs on identical machines [3].

Approximability Results. If the number s of stages is part of the input, Williamson et al. [15] proved that the existence of an approximation algorithm with worst case ratio $< 5/4$ for the problem $O||C_{max}$ or $F||C_{max}$ would imply $P = NP$. On the positive side, polynomial time approximation schemes (PTAS) have been found for $Fs||C_{max}$, $Os||C_{max}$ and the more general problem $Js|op \leq \mu|C_{max}$ in [6, 13, 9]. All three PTAS's can be generalized to the case where the number of stages and number of machines per stage are all constant. Chen and Strusevich [2] have developed an approximation algorithm for $O(P)||C_{max}$ (the multiprocessor open shop problem) with worst case ratio $2 + \epsilon$. For $O2(P)||C_{max}$, they have derived a worst case ratio of $2 - 2/m^2$ where $m = \max(m_1, m_2) \geq 2$. Schuurman and Woeginger [11] have found an approximation algorithm for $O(P)||C_{max}$ with improved worst case ratio 2. Furthermore, a $(3/2 + \epsilon)$ - approximation algorithm for the problem $O2(P)||C_{max}$ (with two stages) is given in [11]. The existence of an approximations scheme for $Os(P)||C_{max}$ with constant $s \geq 2$ number of stages and arbitrary number of machines per stage was posed as an open problem by Schuurman and Woeginger [11]. Several approximation algorithms have been studied for the two - stage multiprocessor flow shop problem, see e.g. in [1, 12, 14]. The best result, a polynomial time approximation scheme for $F2(P)||C_{max}$ is given in [12]. Determining the approximability behaviour of $Fs(P)||C_{max}$ with constant $s \geq 3$ number of stages and arbitrary number of machines per stage was posed as an open question in a paper by Hall [6].

New Results. In this paper, we propose polynomial time approximation schemes for both the multiprocessor open shop and flow shop scheduling problem when the number of stages s is constant and the numbers of machines m_i on stage i, $1 \leq i \leq s$ are part of the input. For open shops, this improves even for two stages the best previous known result of $3/2 + \epsilon$ in [11]. Furthermore, we answer the open question by Schuurman and Woeginger [11] for $Os(P)||C_{max}$ and the open question by Hall [6] for $Fs(P)||C_{max}$. Notice that we can not expect a fully polynomial time approximation scheme (since both problems are strongly NP-hard even for a constant number of stages), unless P=NP. In our approach, we use dynamic programming combined with several ideas from Hall [6], Hochbaum and Shmoys [7], Schuurman and Woeginger [11, 12] and Sevastianov and Woeginger [13].

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