

FINITE SETS AND SYMMETRIC SIMPLICIAL SETS

MARCO GRANDIS

ABSTRACT. The category of finite cardinals (or, equivalently, of finite sets) is the symmetric analogue of the category of finite ordinals, and the ground category of a relevant category of presheaves, the *augmented symmetric simplicial sets*. We prove here that this ground category has characterisations similar to the classical ones for the category of finite ordinals, by the existence of a universal symmetric monoid, or by generators and relations. The latter provides a definition of symmetric simplicial sets by faces, degeneracies and transpositions, under suitable relations.

Introduction

The category Δ^{\sim} of finite ordinals (and monotone mappings) is the basis of the presheaf category \mathbf{Smp}^{\sim} of augmented simplicial sets. It has well known characterisations, as:

- (a) *the free strict monoidal category with an assigned internal monoid;*
- (b) *the subcategory of \mathbf{Set} generated by finite ordinals, their faces and degeneracies;*
- (c) *the category generated by faces and degeneracies, under the cosimplicial relations.*

The last characterisation is currently used in the usual description of an augmented simplicial set as a sequence of sets with faces and degeneracies, subject to the (dual) simplicial relations. The restriction of this characterisation (c) to the category Δ of *positive* finite ordinals plays the same role for *ordinary* (non augmented) simplicial sets (while (a) cannot be so restricted).

Here, in Theorems 4.1 and 4.2, we give similar characterisations for the "symmetric analogue", the category $!\Delta^{\sim}$ of *finite cardinals*, with the same objects $n \geq 0$ and all mappings, equivalent to the (large) category of finite sets. $!\Delta^{\sim}$ is thus:

- (d) *the free strict monoidal category with an assigned symmetric monoid;*
- (e) *the subcategory of \mathbf{Set} generated by faces, degeneracies and main transpositions;*
- (f) *the category generated by faces, degeneracies and main transpositions, under the symmetric cosimplicial relations (Section 3).*

Again, the last characterisation gives a presentation of the non-augmented symmetric simplicial site $!\Delta$, and provides a definition of symmetric simplicial sets by faces, degeneracies and transpositions, under the dual relations.

To motivate the interest of such characterisations, let us recall that *symmetric simplicial sets*, i.e. the presheaves $X: !\Delta^{op} \rightarrow \mathbf{Set}$ on finite positive cardinals, have been studied

Received by the editors 2001 February 19.

Transmitted by Walter Tholen. Published on 2001 April 4. References [3], [5] updated 2002 July 13.
2000 Mathematics Subject Classification: 18G30, 55U10, 18D10, 20F05.

Key words and phrases: Simplicial sets, monoidal categories, generators and relations.

© Marco Grandis, 2001. Permission to copy for private use granted.

in [4, 5], where a combinatorial homotopy theory has been introduced for their category $\mathbf{!Smp} = \mathbf{Set}^{\Delta^{op}}$, extending a previous theory for simplicial complexes [3].

As a crucial advantage of the extension, we have a *fundamental n -groupoid* functor $\Pi_n: \mathbf{!Smp} \rightarrow n\text{-}\mathbf{Gpd}$ ($n \leq \omega$) left adjoint to a *symmetric nerve* M_n , which yields a strong, simple version of the Seifert-van Kampen theorem: Π_n preserves all colimits. Analogously, a notion of (non-reversible) *directed homotopy* has been developed in the ordinary simplicial topos \mathbf{Smp} , with applications to image analysis as in [3]; we have now a *fundamental n -category functor* $\uparrow\Pi_n$, left adjoint to a *nerve* N_n .

A classical reference on simplicial sets is May's book [11]. The characterisations of the category Δ^\sim of finite ordinals can be found in Mac Lane's text [10]. For monoidal categories, see [10] and Kelly's book [8]. The case $n = 1$ of the adjunction $\Pi_n \dashv M_n$ was already noted by Lawvere [9], and was at the origin of this research.

NOTATION. As usual, finite ordinals and finite cardinals coincide, and are constructed as $0 = \emptyset$, $n = (n - 1) \cup \{n - 1\} = \{0, 1, \dots, n - 1\}$. The term "graph" stands for *oriented graph*.

1. Reviewing the simplicial site

The category Δ^\sim of finite ordinals has a rich structure (cf. [10], VII.5, where Δ^\sim, Δ are written as Δ, Δ^+ , respectively); it is reviewed here as a leading frame for our symmetric analogue.

To begin with, Δ^\sim is a strict monoidal category, with respect to the ordinal sum $m + n$ (non-symmetric). The object 1 is an internal monoid

$$\begin{aligned} \partial : 0 \rightarrow 1 \leftarrow 2 : e \\ e(\partial + 1) = \text{id} = e(1 + \partial), \quad e(e + 1) = e(1 + e) \end{aligned} \tag{1}$$

with unit (or face) ∂ and multiplication (or degeneracy) e . Then, the terminal mapping $e^{(k)}: k \rightarrow 1$ appears to be an iterated multiplication, with

$$e^{(0)} = \partial, \quad e^{(1)} = \text{id}, \quad e^{(2)} = e, \quad e^{(3)} = e(e + 1) = e(1 + e), \dots$$

and each monotone mapping $f: m \rightarrow n$ can be uniquely decomposed as a sum $f = e^{(m_0)} + \dots + e^{(m_{n-1})}$ of iterated multiplications, where $m_i = \#(f^{-1}\{i\})$, and $m = m_0 + \dots + m_{n-1}$.

The usual (co)faces and (co)degeneracies can be constructed with the structural maps ∂, e and the monoidal structure (for $0 \leq i \leq n$)

$$\partial_i^n = i + \partial + (n - i): n \rightarrow n + 1, \quad e_i^n = i + e + (n - i): n + 2 \rightarrow n + 1 \tag{2}$$

(the injective monotone map which omits i and the surjective monotone map which repeats i , respectively); the *cosimplicial relations* follow easily from the previous formulae:

$$\begin{aligned} \partial_i \partial_j &= \partial_{j+1} \partial_i \quad (i \leq j), & e_j e_i &= e_i e_{j+1} \quad (i \leq j), \\ e_j \partial_i &= \partial_i e_{j-1}, \text{ or } 1, \text{ or } \partial_{i-1} e_j & (i < j \text{ or } i = j, j + 1 \text{ or } i > j + 1). \end{aligned} \tag{3}$$

A monotone mapping $f: m \rightarrow n$ has a canonical factorisation

$$f = \partial_{j_1} \cdot \partial_{j_2} \cdot \dots \cdot e_{i_2} \cdot e_{i_1} \quad (m - 1 > i_1 > i_2 > \dots \geq 0; \quad n > j_1 > j_2 > \dots \geq 0) \quad (4)$$

by faces and degeneracies; every composite of faces and degeneracies can be put in canonical form, using the cosimplicial relations as rewriting rules (from the left).

Taking advantage of all this, the category Δ^\sim of finite ordinals is characterised as:

- (a) the free strict monoidal category with an assigned internal monoid, 1;
- (b) the subcategory of **Set** generated by finite ordinals, their faces and degeneracies;
- (c) the category generated by the graph (2), subject to the cosimplicial relations (3).

2. Symmetric monoids

The category $!\Delta^\sim$ of finite cardinals (equivalent to the category of finite sets) has a strict monoidal structure $m + n$, the (categorical) sum of cardinals, with a canonical symmetry (provided by the sum)

$$s: m + n \rightarrow n + m, \quad (5)$$

$$s(i) = n + i \quad (0 \leq i < m), \quad s(m + j) = j \quad (0 \leq j < n)$$

which is not strict (note that the identity $m + n = n + m$ is not natural). Now, $(1; \partial, e)$ is a commutative monoid within this enriched structure, satisfying the obvious axioms

$$e(\partial + 1) = \text{id} = e(1 + \partial), \quad e(e + 1) = e(1 + e), \quad es = e. \quad (6)$$

However, we want to be able to deal with "commutative" (or *symmetric*) monoids, within a mere monoidal category without symmetry (which is necessary for symmetric monads, cf. Section 6); this can be done by transferring the symmetry to the monoid itself. The object 1 is now viewed as an internal symmetric monoid, with a *unit* (or face) ∂ , *multiplication* (or degeneracy) e and *symmetry* s ,

$$\partial : 0 \rightarrow 1 \leftarrow 2 : e \quad s : 2 \rightarrow 2 \quad (s(t) = 1 - t), \quad (7)$$

satisfying the axioms below (containing a Yang-Baxter condition on s , see [7] and references therein)

$$\begin{aligned} e(\partial + 1) &= \text{id} = e(1 + \partial), & e(e + 1) &= e(1 + e), \\ ss &= 1, & (s + 1)(1 + s)(s + 1) &= (1 + s)(s + 1)(1 + s), \\ s(\partial + 1) &= 1 + \partial, & es = e, & \quad s(1 + e) = (e + 1)(1 + s)(s + 1). \end{aligned} \quad (8)$$

(In the previous case the four new identities hold automatically, by the coherence theorem of symmetric monoidal categories and by naturality of s .)

3. The symmetric site

After higher faces and degeneracies, we can also construct in $!\Delta^\sim$ the main transpositions s_i (the permutation which exchanges $i, i + 1$, for $0 \leq i \leq n$)

$$s_i = s_i^n = i + s + (n - i): n + 2 \rightarrow n + 2 \tag{9}$$

subject to the *Moore relations*:

$$s_i \cdot s_i = 1, \quad s_i \cdot s_j \cdot s_i = s_j \cdot s_i \cdot s_j \ (i = j - 1), \quad s_i \cdot s_j = s_j \cdot s_i \ (i < j - 1). \tag{10}$$

This is precisely the usual *Moore presentation* of the symmetric group S_{n+2} , the group of automorphisms of the set $n + 2$: generators $s_i = (i, i + 1)$, subject to the relations (10); see Coxeter-Moser [2], 6.2; or Johnson [6], Section 5, Thm. 3. (S_{n+2} also admits systems of two generators, e.g. the cyclic permutation $(0, 1, \dots, n + 1)$ and $s_0 = (0, 1)$; but then, the relations are complicated, cf. [2].)

Now, *faces, degeneracies and main transpositions form a system of generators for Δ^\sim* : an arbitrary mapping $f: m \rightarrow n$ can be factorised as

$$f = h \cdot \rho, \quad h = f_0 + \dots + f_{n-1}, \tag{11}$$

$$(f_j = e^{(m_j)}: m_j \rightarrow 1, \quad m_j = \#(f^{-1}\{j\}))$$

where $\rho: m \rightarrow m$ is a permutation and h is monotone; the latter is uniquely determined by f , as above, while ρ is not unique, generally: $h\rho = h\sigma$ iff $h\rho\sigma^{-1} = h$, iff $\rho\sigma^{-1}$ can be decomposed in a sum of permutations $\sigma_0 + \dots + \sigma_{n-1}$, coherently with the set-decomposition $m = m_0 + \dots + m_{n-1}$. (However, ρ is uniquely determined if we ask that ρ^{-1} be strictly monotone on each interval of the preceding decomposition of m ; then, depending on conventions, ρ and ρ^{-1} are respectively called an (m_0, \dots, m_{n-1}) -*shuffle* and an (m_0, \dots, m_{n-1}) -*deal*, or vice versa.)

Our generators satisfy the *symmetric cosimplicial relations*, consisting:

- of the usual *cosimplicial relations* for faces and degeneracies (3),
- of the *Moore relations* for transpositions (10),
- of the following *mixed relations*

$$\begin{aligned} s_i \partial_j &= \partial_j s_i, & s_i e_j &= e_j s_i & (i < j - 1), \\ s_i \partial_i &= \partial_{i+1}, & s_i e_i &= e_{i+1} s_i s_{i+1}, \\ s_i \partial_j &= \partial_j s_{i-1}, & s_i e_j &= e_j s_{i+1} & (i > j), \end{aligned} \tag{12}$$

$$e_i s_i = e_i, \tag{13}$$

which again follow easily from the structural properties (8).

It follows easily that $s_i \partial_{i+1} = \partial_i$ and $s_i e_{i+1} = e_i s_{i+1} s_i$, so that the previous relations (12) can be viewed as rewriting rules for $s_i \partial_j: n + 1 \rightarrow n + 2$ and $s_i e_j: n + 3 \rightarrow n + 2$ ($i \leq n; j \leq n + 1$), which allow one to transfer permutations to the right of monotone maps.

In general, given a strict monoidal category $(\mathbf{A}, +, 0)$ and an internal symmetric monoid $(a; \partial, e, s)$, the "multiples" $a + \dots + a$ are linked by a system of maps (for $0 \leq i \leq n$)

$$\begin{aligned} \partial_i^n &= ia + \partial + (n - i)a: na \rightarrow (n + 1)a, \\ e_i^n &= ia + e + (n - i)a: (n + 2)a \rightarrow (n + 1)a, \\ s_i^n &= ia + s + (n - i)a: (n + 2)a \rightarrow (n + 2)a, \end{aligned} \tag{14}$$

which satisfies the symmetric cosimplicial relations. As in Section 1, we write $e^{(n)} : na \rightarrow a$ the n -ary multiplication, inductively defined as

$$e^{(0)} = \partial, \quad e^{(1)} = \text{id}, \quad e^{(n+1)} = e(e^{(n)} + 1) = e(1 + e^{(n)}); \tag{15}$$

one can easily deduce from (13), by induction, that $e^{(n)} \cdot s_i^{n-2} = e^{(n)}$.

4. Main results

4.1. THEOREM. (The internal symmetric monoid) $!\Delta^\sim$ can be characterised as:
 (d') the free strict monoidal category with an assigned symmetric monoid, 1.

PROOF. Let a strict monoidal category $(\mathbf{A}, +, 0)$ be given, together with an internal symmetric monoid $(a; \partial, e, s)$; we have to show that there is a unique strictly monoidal functor $F: !\Delta^\sim \rightarrow \mathbf{A}$ sending 1 to a and preserving the structure. We already know that the "multiples" $a + \dots + a$ form a symmetric cosimplicial object (14). From Section 1, there is a unique strictly monoidal functor $F: \Delta^\sim \rightarrow \mathbf{A}$ sending 1 to a and preserving unit and multiplication; it operates in the obvious way on generators

$$F(n) = na, \quad F(\partial_i^n) = \partial_i^n, \quad F(e_i^n) = e_i^n. \tag{16}$$

Consider now the group S_{n+2} of automorphisms of $n + 2$ in $!\Delta^\sim$; on the main transpositions $s_i^n = i + s + (n - i): n + 2 \rightarrow n + 2$ we must have

$$F(s_i^n) = ia + s + (n - i)a = s_i^n: (n + 2)a \rightarrow (n + 2)a; \tag{17}$$

on the other hand, since this setting is consistent with the Moore relations (10), we have defined a sequence of group-homomorphisms $F: S_{n+2} \rightarrow \text{Aut}((n + 2)a)$, and extended the functor F to all bijections of $!\Delta^\sim$.

Take now an arbitrary mapping $f = h\rho: m \rightarrow n$, factorised as above (11): h is monotone and ρ is a permutation; we must set $Ff = Fh \cdot F\rho$; to show that the definition is correct, it is sufficient to verify that $Fh = Fh \cdot Fs_i$, for each main transposition s_i "acting within a summand" of $m_0 + \dots + m_{n-1}$; taking for instance $0 \leq i < m_0 - 1$, we have (also by the identity $e^{(n)} \cdot s_i^{n-2} = e^{(n)}$, at the end of Section 3)

$$Fh \cdot Fs_i = (Ff_0 + \dots + Ff_{n-1}) \cdot Fs_i = (Ff_0 \cdot Fs_i) + \dots + Ff_{n-1} = Fh. \tag{18}$$

Last, we must prove that the extended mapping F preserves composition; let us begin showing that, for each monotone mapping $h: m \rightarrow n$ and each permutation $\sigma: n \rightarrow n$

$$\begin{array}{ccc}
 m & \xrightarrow{\tau} & m \\
 h \downarrow & & \downarrow k \\
 n & \xrightarrow{\sigma} & n
 \end{array} \tag{19}$$

one can find a permutation $\tau: m \rightarrow m$ and a monotone $k: m \rightarrow n$ such that the square above commutes, as well as its F -image in \mathbf{A} . One can assume that $\sigma = s_i$; let $h = h_u \dots h_1$ be the canonical factorisation of a monotone map (4); applying the rewriting rules deriving from the mixed relations (12), we obtain a factorisation $\sigma h = k\tau$, with a monotone map $k = k_v \dots k_1$ (canonical factorisation (4)) and a permutation $\tau = \tau_w \dots \tau_1$ (product of main transpositions). The same relations hold in \mathbf{A} (for its $\partial_i^n, e_i^n, s_i^n$) and F preserves the composition within monotone maps and within bijections, whence

$$F\sigma \cdot Fh = F\sigma \cdot Fh_u \cdot \dots \cdot Fh_1 = Fk_v \cdot \dots \cdot Fk_1 \cdot F\tau_w \cdot \dots \cdot F\tau_1 = Fk \cdot F\tau. \tag{20}$$

Now, the functorial property for F follows easily: if the mappings $f = h\rho: m \rightarrow n$ and $f' = h'\sigma: n \rightarrow p$ are factorised as above, in (11), we rewrite $\sigma h = k \cdot \tau$ as in diagram (19), and

$$Ff' \cdot Ff = Fh' \cdot F\sigma \cdot Fh \cdot F\rho = Fh' \cdot Fk \cdot F\tau \cdot F\rho = F(h'k) \cdot F(\tau\rho) = F(f'f). \tag{21}$$

(For the last equality, note that $(h'k) \cdot (\tau\rho) = h'\sigma \cdot h\rho = f'f$ is an admissible factorisation of $f'f$, i.e. a permutation followed by a monotone map.) ■

4.2. THEOREM. (Presentation). *The category $!\Delta^\sim$ can also be characterised as:*

(b') *The subcategory of **Set** generated by faces ($\partial_i: n \rightarrow n + 1$), degeneracies ($e_i: n + 2 \rightarrow n + 1$) and main transpositions ($s_i: n + 2 \rightarrow n + 2$), where $0 \leq i \leq n$.*

(c') *The category generated by faces, degeneracies and main transpositions, under the symmetric cosimplicial relations (Section 3).*

PROOF. (b') is already known and (c') follows easily from the previous characterisation (Theorem 4.1). Let $!\Delta^\sim$ be defined by the presentation above. It is strictly monoidal: define the sum-functor in the obvious way ($\partial_i + q = \partial_i, p + \partial_i = \partial_{p+i}$, etc.) and check the consistency with relations. Then take a strict monoidal category $(\mathbf{A}, +, 0)$ with an internal symmetric monoid $(a; \partial, e, s)$; a strict monoidal functor $F: !\Delta^\sim \rightarrow \mathbf{A}$ sending 1 to a and preserving the structure is uniquely determined on generators, as in (16), (17): $F(n) = na, F(\partial_i^n) = ia + \partial + (n - i)a$, etc. Conversely, defining F in this way is obviously consistent with relations, since all of them can be deduced from the axioms (8) and the monoidal structure. ■

5. Symmetric simplicial sets

Of course, the (non-augmented) symmetric simplicial site $!\Delta$ can be presented as the category generated by faces, degeneracies and main transpositions between positive cardinals, under the restricted relations.

Therefore, a symmetric simplicial set $X:!\Delta^{op} \rightarrow \mathbf{Set}$ can be assigned by the corresponding data $(X_n, \partial_i^n, e_i^n, s_i^n)$, where we write $X_n = X[n] = X(n+1)$, as usual; faces, degeneracies and main transpositions (for $0 \leq i \leq n$)

$$\partial_i^n: X_n \rightarrow X_{n-1}, \quad e_i^n: X_n \rightarrow X_{n+1}, \quad s_i^n: X_{n+1} \rightarrow X_{n+1} \tag{22}$$

are to satisfy the symmetric simplicial relations (dual to the ones considered in Section 3). Equivalently, one can assign a simplicial set $(X_n, \partial_i^n, e_i^n)$ and a right action of each symmetric group S_{n+1} on the component X_n ($x\rho = \rho^*(x)$), coherently with faces and degeneracies (i.e., the latter have to satisfy the dual *mixed relations* with the main transpositions, cf. (12), (13)).

The usual embedding of Δ in \mathbf{Top} extends easily to $!\Delta$, forming a symmetric cosimplicial object with the same components, the standard topological simplices $!\Delta_n = \Delta_n$, and extended actions λ^* (for all mappings $\lambda: m \rightarrow n$)

$$(!\Delta_n, \lambda^*): !\Delta \rightarrow \mathbf{Top}, \quad \lambda^*((t_i)_{i=0,\dots,n}) = (\sum_{\lambda i=j} t_i)_{j=0,\dots,m}. \tag{23}$$

This model of $!\Delta$ in \mathbf{Top} gives rise to the functor $!S_*$ of *symmetric singular simplices*

$$!S_*: \mathbf{Top} \rightarrow \mathbf{!Smp}, \quad !S_n(X) = \mathbf{Top}(!\Delta_n, X) \tag{24}$$

where the transposition $s_i^n: !S_{n+1}(X) \rightarrow !S_{n+1}(X)$ amounts to a reflection of simplices, with respect to the symmetry hyperplane of the i -th, $(i+1)$ -th vertices of Δ_n .

Its left adjoint is the (symmetric) geometric realisation functor $\mathbf{!Smp} \rightarrow \mathbf{Top}$: the realisation of the symmetric simplicial set X is the coend $\int^{[n]} X_n \cdot !\Delta_n$ (of the inner functor $!\Delta^{op} \times !\Delta \rightarrow \mathbf{Top}$). By Yoneda, the realisation of $!\Delta[n]$ is $!\Delta_n = \Delta_n$.

6. Symmetric comonads

A comonad (K, ∂, e) in the category \mathbf{A} is a comonoid in the category $\mathbf{End}(\mathbf{A})$ of endomorphisms of \mathbf{A} , with the strict monoidal structure of composition

$$\begin{aligned} \partial: K \rightarrow 1, & & e: K \rightarrow K^2, \\ \partial K \cdot e = \text{id}K = K\partial \cdot e & & eK \cdot e = Ke \cdot e; \end{aligned} \tag{25}$$

it generates an augmented simplicial object in $\mathbf{End}(\mathbf{A})$, and - by evaluation - a functor $K_*: \mathbf{A} \rightarrow \mathbf{Smp}\tilde{(\mathbf{A})}$ with values in the category of augmented simplicial objects on \mathbf{A} (cf. [1])

$$\begin{aligned} K_*(X) = ((K_{n+1}(X)), (\partial_i^n), (e_i^n)) & \quad (n \geq -1; 0 \leq i \leq n), \\ \partial_i^n = K^{n-i}\partial K^i: K^{n+1} \rightarrow K^n, & \quad e_i^n = K^{n-i}eK^i: K^{n+1} \rightarrow K^{n+2}. \end{aligned} \tag{26}$$

For a category \mathbf{C} , the category of augmented simplicial objects $Smp\tilde{\mathbf{C}}$ has a well known comonad, given by the shift (or decalage) \mathbf{K}

$$\begin{aligned} \mathbf{K}: Smp\tilde{\mathbf{C}} &\rightarrow Smp\tilde{\mathbf{C}}, & \mathbf{K}X &= ((X_{n+1}), (\partial_{i+1}^{n+1}), (e_{i+1}^{n+1}))_{n \geq -1}, \\ \partial: \mathbf{K}X &\rightarrow X, & \partial &= \partial_0^{n+1}: X_{n+1} \rightarrow X_n, \\ e: \mathbf{K}X &\rightarrow \mathbf{K}^2 X, & e &= e_0^{n+1}: X_{n+1} \rightarrow X_{n+2} \end{aligned} \tag{27}$$

with counit ∂ and comultiplication e consisting of the discarded faces and degeneracies.

In fact, $Smp\tilde{\mathbf{C}}$ is the cofree category-with-comonad on \mathbf{C} , with respect to the forgetful functor $|-|$ from categories with a comonad to categories. The counit-component $|Smp\tilde{\mathbf{C}}| \rightarrow \mathbf{C}$ sends the augmented simplicial object X to X_{-1} , while the unit-component $\mathbf{A} \rightarrow Smp\tilde{(|\mathbf{A}|)}$ is the functor K_* considered above.

Similarly, a *symmetric comonad* (K, ∂, e, s) will be a symmetric comonoid in $\text{End}(\mathbf{A})$: with respect to the previous structure, in (25), we have to add a *symmetry* $s: K^2 \rightarrow K^2$, satisfying

$$\begin{aligned} ss &= 1, & sK \cdot Ks \cdot sK &= Ks \cdot sK \cdot Ks, \\ \partial K \cdot s &= K\partial, & se &= e, & eK \cdot s &= Ks \cdot sK \cdot Ke. \end{aligned} \tag{28}$$

By the characterisation theorems of Section 4, it generates an augmented symmetric simplicial object in $\text{End}(\mathbf{A})$ and a functor $K_*: \mathbf{A} \rightarrow !Smp\tilde{\mathbf{A}}$. Again, $!Smp\tilde{\mathbf{C}}$ is the cofree category with symmetric comonad over \mathbf{C} .

References

- [1] M. Barr and J. Beck, Homology and standard constructions, in: Seminar on triples and categorical homology theory, Lecture Notes in Math. 80, Springer 1969, 245-335.
- [2] H.S.M. Coxeter and W.O.J. Moser, Generators and relations for discrete groups, Springer, Berlin 1957.
- [3] M. Grandis, An intrinsic homotopy theory for simplicial complexes, with applications to image analysis, Appl. Categ. Structures 10 (2002), 99-155.
- [4] M. Grandis, Simplicial toposes and combinatorial homotopy, Dip. Mat. Univ. Genova, Preprint 400 (1999).
- [5] M. Grandis, Higher fundamental functors for simplicial sets, Cahiers Topologie Geom. Differentielle Categ XLII (2001), 101-136.
- [6] D.L. Johnson, Topics in the theory of presentation of groups, Cambridge Univ. Press, Cambridge 1980.
- [7] A. Joyal and R. Street, Braided tensor categories, Adv. Math. 102 (1993), 20-78.
- [8] G.M. Kelly, Basic concepts of enriched category theory, Cambridge University Press, 1982.

- [9] F.W. Lawvere, Toposes generated by codiscrete objects, in combinatorial topology and functional analysis, Notes for Colloquium lectures given at North Ryde, NSW, Aus (1988), at Madison, Wisconsin, USA (1989), and at Seminario Matematico e Fisico, Milano, Italy (1992).
- [10] S. Mac Lane, *Categories for the working mathematician*, Springer 1971.
- [11] J.P. May, *Simplicial objects in algebraic topology*, Van Nostrand 1967.

Dipartimento di Matematica, Università di Genova
Via Dodecaneso 35, 16146-Genova, Italy
Email: `grandis@dima.unige.it`

This article may be accessed via WWW at <http://www.tac.mta.ca/tac/> or by anonymous ftp at <ftp://ftp.tac.mta.ca/pub/tac/html/volumes/8/n8/n8.dvi>, [ps](ftp://ftp.tac.mta.ca/pub/tac/html/volumes/8/n8/n8.ps)

THEORY AND APPLICATIONS OF CATEGORIES (ISSN 1201-561X) will disseminate articles that significantly advance the study of categorical algebra or methods, or that make significant new contributions to mathematical science using categorical methods. The scope of the journal includes: all areas of pure category theory, including higher dimensional categories; applications of category theory to algebra, geometry and topology and other areas of mathematics; applications of category theory to computer science, physics and other mathematical sciences; contributions to scientific knowledge that make use of categorical methods.

Articles appearing in the journal have been carefully and critically refereed under the responsibility of members of the Editorial Board. Only papers judged to be both significant and excellent are accepted for publication.

The method of distribution of the journal is via the Internet tools WWW/ftp. The journal is archived electronically and in printed paper format.

SUBSCRIPTION INFORMATION. Individual subscribers receive (by e-mail) abstracts of articles as they are published. Full text of published articles is available in .dvi, Postscript and PDF. Details will be e-mailed to new subscribers. To subscribe, send e-mail to `tac@mta.ca` including a full name and postal address. For institutional subscription, send enquiries to the Managing Editor, Robert Rosebrugh, `rrosebrugh@mta.ca`.

INFORMATION FOR AUTHORS. The typesetting language of the journal is \TeX , and \LaTeX is the preferred flavour. \TeX source of articles for publication should be submitted by e-mail directly to an appropriate Editor. They are listed below. Please obtain detailed information on submission format and style files from the journal's WWW server at <http://www.tac.mta.ca/tac/>. You may also write to `tac@mta.ca` to receive details by e-mail.

EDITORIAL BOARD.

John Baez, University of California, Riverside: `baez@math.ucr.edu`

Michael Barr, McGill University: `barr@barrs.org`, *Associate Managing Editor*

Lawrence Breen, Université Paris 13: `breen@math.univ-paris13.fr`

Ronald Brown, University of North Wales: `r.brown@bangor.ac.uk`

Jean-Luc Brylinski, Pennsylvania State University: `jlb@math.psu.edu`

Aurelio Carboni, Università dell'Insubria: `aurelio.carboni@uninsubria.it`

P. T. Johnstone, University of Cambridge: `ptj@dpms.cam.ac.uk`

G. Max Kelly, University of Sydney: `maxk@maths.usyd.edu.au`

Anders Kock, University of Aarhus: `kock@imf.au.dk`

F. William Lawvere, State University of New York at Buffalo: `wlawvere@acsu.buffalo.edu`

Jean-Louis Loday, Université de Strasbourg: `loday@math.u-strasbg.fr`

Ieke Moerdijk, University of Utrecht: `moerdijk@math.uu.nl`

Susan Niefield, Union College: `niefiels@union.edu`

Robert Paré, Dalhousie University: `pare@mathstat.dal.ca`

Andrew Pitts, University of Cambridge: `Andrew.Pitts@cl.cam.ac.uk`

Robert Rosebrugh, Mount Allison University: `rrosebrugh@mta.ca`, *Managing Editor*

Jiri Rosicky, Masaryk University: `rosicky@math.muni.cz`

James Stasheff, University of North Carolina: `jds@math.unc.edu`

Ross Street, Macquarie University: `street@math.mq.edu.au`

Walter Tholen, York University: `tholen@mathstat.yorku.ca`

Myles Tierney, Rutgers University: `tierney@math.rutgers.edu`

Robert F. C. Walters, University of Insubria: `walters@fis.unico.it`

R. J. Wood, Dalhousie University: `rjwood@mathstat.dal.ca`