Performance of MC-CDMA Systems with Frequency Offsets Over a Correlated Rayleigh Fading Channel

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***Abstract*-In this contribution we analyzed and compared the bit error rate performance for MC-CDMA (multicarrier code-division multiple access) systems using IFFT/FFT (inverse fast Fourier transform/fast Fourier transform) and conventional correlation receiver over a frequency selective Rayleigh fading channel. Carrier frequency offset is considered in the analysis with either equal gain combining or maximum ratio combining. It is shown that the system using IFFT/FFT is more efficient to implement than the conventional one, with only slightly performance loss.**

I. INTRODUCTION

MC-CDMA is a multicarrier transmission scheme which combines two radio access techniques, namely OFDM (orthogonal frequency division multiplexing) [1] and CDMA (code-division multiple access) [2]. The main advantages of using MC-CDMA are the efficient frequency diversity and high bandwidth efficiency. In addition to this, an MC-CDMA system solves the difficult inter-symbol interference (ISI) problem encountered with high data rates across multipath channels by dividing the available bandwidth into many low rate orthogonal channels. An interesting aspect of MC-CDMA is that its modulation and demodulation can be implemented easily by means of IFFT and FFT operators [3]. However, a major drawback of an MC-CDMA system is its sensitivity to synchronization errors between the transmitter and receiver. In an MC-CDMA link, the subcarriers are perfectly orthogonal only if transmitter and receiver use exactly the same frequencies. Inter-carrier interference (ICI) results when orthogonality among subcarriers is destroyed by frequency errors.

Recently, there are some papers investigating the effect of frequency offset on MC-CDMA systems. In [4] and [5], the impact of frequency offset on MC-CDMA systems is evaluated for the cases of downlink and uplink Gaussian channels, respectively. When downlink and uplink frequency selective fading channels are considered, the BER performance of MC-CDMA systems is addressed in [6]-[11]. Nevertheless, all of the analyses mentioned above are based on the conventional MC-CDMA system model, which uses conventional method for subcarrier modulation and demodulation. In [12], the author employed a new system model using IFFT/FFT for subcarrier modulation/demodulation, but the analysis was done for an MC-DS-CDMA system with focus on SNR performance.

In this paper, we present BER performance analysis on the effect of frequency offset errors for a downlink MC-CDMA system. IFFT/FFT for subcarrier modulation/demodulation are considered with carrier frequency offset in a correlated Rayleigh fading channel.

The rest of this paper is organized as follows: the MC-CDMA system model using IFFT/FFT and the channel model are described in Section II. The BER performance is analyzed in Section III. Section IV provides numerical results and discussions, while section V concludes the paper.

II. SYSTEM MODEL

A. MC-CDMA System Model

The transmitter and receiver of the MC-CDMA system using IFFT/FFT are shown in Fig. 1. For clarity of analysis, we assume that the spreading factor of the system is equal to the number of subcarriers N, which means that there is a total of N chips in each data bit. Assuming that user m transmits data bit d_m with a spreading code $\{c_{n,m}, n = 0, 1, \dots N\text{-}1\}$ over the zero-th symbol duration $[0, NT_c]$, according to the block diagram of the MC-CDMA transmitter shown in Fig. 1, the output of the IFFT block can be represented by

$$s_k = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} d_m c_{n,m} e^{j2\pi nk/N} \qquad k = 0, 1, \dots, N\text{-}1 \tag{1}$$

where N is the number of subcarriers, M is the number of users. Assuming that the digital to analog converter (DAC) is ideal and does not introduce any distortion, the transmitted lowpass equivalent downlink MC-CDMA signal can be written as

$$s(t) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} d_m c_{n,m} e^{j2\pi \frac{n}{NT_c} t} \qquad 0 \le t \le NT_c \tag{2}$$

where $\dfrac{n}{NT_c}$ is the lowpass frequency of the nth subcarrier.

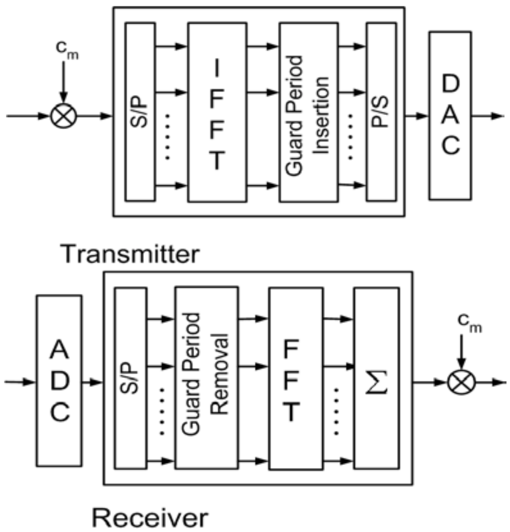

Figure 1. Block diagram of the MC-CDMA transmitter and receiver using IFFT/FFT.

B. Multipath Channel Model

We consider a frequency selective multipath slow Rayleigh fading channel where the fading characteristic is constant over at least one symbol duration. In open literatures the frequency selective multipath Rayleigh fading channel is commonly modeled by the finite length tapped delay line [13]. The equivalent complex channel gain h_n for the nth subcarrier can be represented by the complex multipath gains in the following equation

$$h_n=\sum_{l=0}^{L-1}\alpha_l e^{j\Psi_l}e^{-j\omega_n(lT_c)} \quad (3)$$

where L is the number of multipath, $\{\alpha_l,\ l = 0, 1, \ldots L\text{-}1\}$ are the multipath envelops which are independent Rayleigh random variables (r.v.'s), $\{\Psi_l,\ l = 0, 1, \ldots, L\text{-}1\}$ are the multipath phases which are independent, identically distributed (i.i.d.) r.v.'s uniform in $(0, 2\pi)$, and ω_n is the angular frequency of the nth subcarrier. Equation (3) can be further rewritten as $h_n=\beta_n e^{j\Phi_n}$ where β_n and Φ_n are the magnitude and phase of the channel gain for the nth subcarrier, respectively. As can be seen, although the channel is frequency selective, the narrowband signal transmitted over each subcarrier experiences frequency nonselective Rayleigh fading. Channel gains for subcarriers are identically distributed zero-mean complex Gaussian r.v.'s and are mutually correlated. The variance of h_n is denoted as σ^2. The maximum delay due to multipath is $(L\text{-}1)T_c$, which is referred to as channel delay spread T_d. Defining Ω_l as the second moment of $\{\alpha_l\}$ (i.e., $\Omega_l = E[(\alpha_l)^2]$), we assume a exponential multipath intensity profile (MIP) distribution given by $\Omega_l=\Omega_0 e^{-\delta l}$, $\delta \geq 0$. Where the parameter δ reflects the rate at which this decay occurs.

III. PERFORMANCE ANALYSIS

After passing though the multipath channel, the received ISI-free part of the zero-th symbol takes the form

$$r(t)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} d_m c_{n,m}\beta_n e^{j\Phi_n} e^{j2\pi(\frac{n}{NT_c}+\delta f)t}+\eta(t) \quad (4)$$

where $\eta(t)$ is the lowpass equivalent of the additive white Gaussian noise (AWGN) with zero mean and unilateral power spectral density N_0, and δf is the frequency offset. After sampling, the signal sample at $t = kT_c$ is given by

$$r(kT_c)=\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} d_m c_{n,m}\beta_n e^{j\Phi_n} e^{j2\pi nk/N} e^{j2\pi\delta f kT_c}+\eta(kT_c) \quad (5)$$

Assuming that user p is the reference user and the frequency offset is less than Δf, the pth user's ith chip obtained after FFT operation is found to be

$$\begin{aligned}\hat{d}_{i,p} &= \frac{1}{N}\sum_{k=0}^{N-1} r(kT_c)e^{-j2\pi ki/N} \\ &= \frac{1}{N}P\cdot d_p c_{i,p}\beta_i e^{j\Phi_i}+\frac{1}{N}\sum_{\substack{m=0\\ m\neq p}}^{M-1}P\cdot d_p c_{i,m}\beta_i e^{j\Phi_i} \\ &\quad +\frac{1}{N}\sum_{m=0}^{M-1}\sum_{\substack{n=0\\ n\neq i}}^{N-1}Q(n,i)\cdot d_m c_{n,m}\beta_n e^{j\Phi_n}+\eta_i'\end{aligned} \quad (6)$$

where $P=\dfrac{e^{j2\pi\frac{\delta f}{\Delta f}}-1}{e^{j2\pi\frac{\delta f}{N\cdot\Delta f}}-1}$, $Q(n,i)=\dfrac{e^{j2\pi(n-i+\frac{\delta f}{\Delta f})}-1}{e^{j2\pi(n-i+\frac{\delta f}{\Delta f})/N}-1}$. It is easy to show that after FFT, the resultant noise component η_i' is still Gaussian.

Normally, there are two ways of combining the chips of the same data bit: equal gain combining (EGC) and maximum ratio combining (MRC). Both will be studied in the following.

A. Equal Gain Combining

With EGC, knowledge of $\{\Phi_i,\ i = 0, 1, \ldots, N\text{-}1\}$ is assumed known at the receiver. After despreading with EGC, the signal obtained can be written as

$$\hat{d}_p = \sum_{i=0}^{N-1} \hat{d}_p \cdot c_{i,p} e^{-j\Phi_i} \quad (7)$$

By taking the real part of $\hat{d}_p$, we can obtain the decision variable U_p as

$$U_p = \sum_{i=0}^{N-1} P' \cdot d_p \beta_i + \sum_{i=0}^{N-1} \sum_{\substack{m=0 \\ m\neq p}}^{M-1} P' \cdot d_m c_{i,m} c_{i,p} \beta_i + \sum_{i=0}^{N-1} \sum_{m=0}^{M-1} \sum_{\substack{n=0 \\ n\neq i}}^{N-1} Q'(n,i) \cdot d_m c_{n,m} c_{i,p} \beta_n + \mathrm{Re}(\sum_{i=0}^{N-1} \eta_i' \cdot c_{i,p} e^{-j\Phi_i}) \quad (8)$$

where

$$P' = \frac{\cos[(1-\frac{1}{N}) \cdot \pi \cdot \frac{\delta f}{\Delta f}] \sin(\pi \frac{\delta f}{\Delta f})}{N \cdot \sin(\pi \frac{\delta f}{N \cdot \Delta f})}, \quad (9)$$

$$Q'(n,i) = \frac{\cos[\pi(n-i+\frac{\delta f}{\Delta f})(1-\frac{1}{N}) + (\Phi_n - \Phi_i)] \sin[\pi(n-i+\frac{\delta f}{\Delta f})]}{N \cdot \sin[\pi(n-i+\frac{\delta f}{\Delta f})/N]} \quad (10)$$

The decision variable U_p can be divided into five parts:

$$U_p = D_p + \xi + ICI_1 + ICI_2 + MAI \quad (11)$$

The term D_p is the desired output

$$D_p = \sum_{i=0}^{N-1} P' \cdot d_p \beta_i \quad (12)$$

The term $\xi = \mathrm{Re}(\sum_{i=0}^{N-1} \eta_i' \cdot c_{i,p} e^{-j\Phi_i})$ is the interference term due to Gaussian noise. It is easy to see that ξ is Gaussian with zero mean and variance $N_0 N$. The two terms of ICI: ICI_1 is the interference from same user, due to different subcarriers

$$ICI_1 = \sum_{i=0}^{N-1} \sum_{\substack{n=0 \\ n\neq i}}^{N-1} Q'(n,i) \cdot d_p c_{n,p} c_{i,p} \beta_n \quad (13)$$

while ICI_2 is the interference from other users, due to different subcarriers

$$ICI_2 = \sum_{i=0}^{N-1} \sum_{\substack{m=0 \\ m\neq p}}^{M-1} \sum_{\substack{n=0 \\ n\neq i}}^{N-1} Q'(n,i) \cdot d_m c_{n,m} c_{i,p} \beta_n \quad (14)$$

MAI is the interference from other users, due to same subcarriers

$$MAI = \sum_{i=0}^{N-1} \sum_{\substack{m=0 \\ m\neq p}}^{M-1} P' \cdot d_m c_{i,m} c_{i,p} \beta_i \quad (15)$$

It is easy to see that the mean and variance of the signal will depend on the correlation conditions of the subcarrier channel gains. Following our channel model, all the subcarrier channel gains are mutually correlated. However, many previous researches employed the common assumption of independent subcarrier channel gains. In order to compare the results of this paper with previous research under the same conditions, we consider two cases in the sequel.

Case 1: $\{h_n, n = 0, 1, \ldots, N\text{-}1\}$ are mutually independent

In this case, the interference terms ICI_1, and ICI_2 and *MAI* are Gaussian with zero mean and variances. Assuming a "one" is transmitted, then the mean and variance of U_p are given, respectively, by

$$E(U_p) = P' \cdot \sum_{i=0}^{N-1} E(\beta_i) \quad (16)$$

and

$$Var(U_p) = P'^2 \cdot \sum_{i=0}^{N-1} Var(\beta_i) + N_0 N + \sum_{i=0}^{N-1} \sum_{\substack{n=0 \\ n\neq i}}^{N-1} Var[Q'(n,i)\beta_n] + (M-1) \cdot \sum_{i=0}^{N-1} \sum_{\substack{n=0, \\ n\neq i}}^{N-1} Var[Q'(n,i)\beta_n] + (M-1) \cdot P'^2 \cdot \sum_{i=0}^{N-1} Var[\beta_i] \quad (17)$$

The probability of error or average bit error rate (BER) is given by:

$$P(e) = \frac{1}{2} erfc(\frac{E(U_p)}{\sqrt{2Var(U_p)}}) \quad (18)$$

Case 2: $\{h_n, n = 0, 1, \ldots, N\text{-}1\}$ are mutually correlated

In this case U_p is conditional Gaussian conditioned on $\{\beta_n\}$ and $\{\Phi_n\}$. Hence the mean and variance of U_p conditioned on $\{\beta_n\}$ and $\{\Phi_n\}$ is given by

$$E(U_p \mid \{\beta_n\}, \{\Phi_n\}) = P' \cdot \sum_{i=0}^{N-1} \beta_i \quad (19)$$

and

$$Var(U_p \mid \{\beta_n\}, \{\Phi_n\}) = N_0 N + \sum_{i=0}^{N-1} \sum_{\substack{n=0 \\ n\neq i}}^{N-1} [Q'(n,i)]^2 \beta_n^{\ 2} + (M-1) \cdot \sum_{i=0}^{N-1} \sum_{\substack{n=0, \\ n\neq i}}^{N-1} [Q'(n,i)]^2 \beta^2{}_n + (M-1) \cdot P'^2 \cdot \sum_{i=0}^{N-1} \beta_i^2 \quad (20)$$

respectively. The probability of error conditioned on $\{\beta_n\}$ and $\{\Phi_n\}$ is simply given by

$$P[e \mid \{\beta_n\}, \{\Phi_n\}] = \frac{1}{2} erfc(\frac{E(U_p \mid \{\beta_n\}, \{\Phi_n\})}{\sqrt{2Var(U_p \mid \{\beta_n\}, \{\Phi_n\})}}) \quad (21)$$

and the BER is obtained by averaging (21) over $\{\beta_0, \beta_1, ..., \beta_{N-1}\}$ and $\{\Phi_0, \Phi_1, ..., \Phi_{N-1}\}$.

B. Maximum Ratio Combining

For MRC, the complex channel gain of each subcarrier must be continuously estimated, which may not be feasible in practice. However, MRC gives a lower bound of the system BER. With MRC, equation (7) can be rewritten as

$$\hat{d}_p = \sum_{i=0}^{N-1} \hat{d}'_p c_{i,p} \beta_i e^{-j\Phi_i} \tag{22}$$

and the system BER with MRC can be evaluated by following a similar procedure as that described for EGC for both cases.

IV. NUMERICAL RESULT

The effect of frequency offset on BER performance is studied by numerical evaluation of (21). We set $N = 64$ and $L = 6$. We assume that the power of each multipath is 4dB less than that of its neighboring early path, thus δ is approximately 0.9210 (-*log*(1*10^-0.4)).

Fig. 2 shows the BER curves of the MC-CDMA system with 64 subcarriers and 10 users experiencing various carrier frequency offsets using EGC combining. From the figure we can see that as the degree of frequency offsets increases, so does the performance deterioration. This is because the frequency offset will cause the attenuation of the desired signal and the enhancement of the interference signal.

Performance comparison between MRC and EGC for the MC-CDMA system with 64 subcarriers and 10 users experiencing 4% of carrier frequency offset is shown in Fig. 3. It is obvious that MRC performs better than EGC.

In Fig. 4, we show the system performance under big frequency offsets compared to the frequency spacing between adjacent subcarriers. From the figure we can see that when the carrier frequency offset reaches 40%, the performance loss is significant, a BER of 10^{-3} is no longer possible. Hence the system cannot allow big frequency offsets. It can also be concluded that the frequency offset must be kept below 10% to prevent significant performance loss.

In order to observe the effects of correlation between subcarrier channel gains on the BER performance, we plotted the BERs of independent and correlated subcarrier channel gains in Fig. 5. It's easy to see that the system performance is better when the channel gains are mutually independent.

The performance comparison of the MC-CDMA model using IFFT/FFT with that of the conventional model is also done in this contribution. We use [9] as the reference paper,

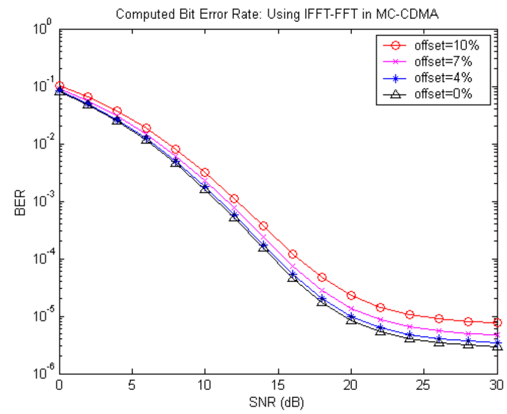

Figure 2. BERs of an MC-CDMA system with 64 subcarriers and 10 users experiencing 0%, 4%, 7%, 10% carrier frequency offset in percentage of Δf. (EGC)

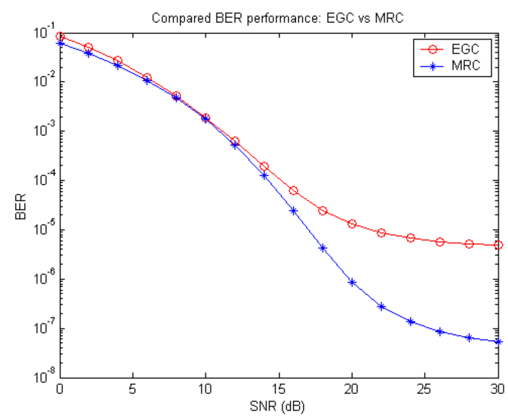

Figure 3. BER performance comparison: MRC vs. EGC, with 64 subcarriers and 10 users.

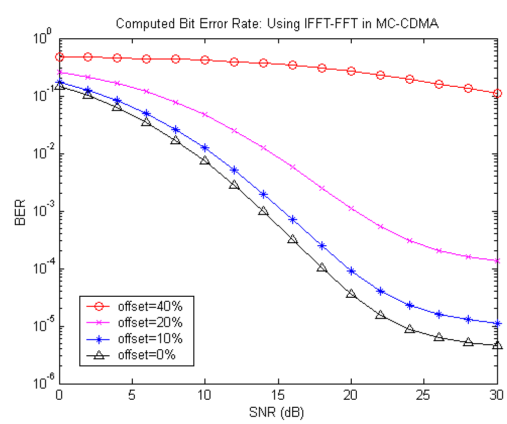

Figure 4. BERs of an MC-CDMA with 64 subcarriers and 10 users experiencing carrier frequency offsets of 0%, 10%, 20%, 40%, respectively. (EGC)

following his result, we did the simulation under the condition of equal gain combining and independent number is 64 and 10 respectively. Fig. 6 shows the performance comparison between the conventional method and the IFFT/FFT method, with frequency offsets 4% and 10%, respectively. From the simulation result we can see that when the frequency offset is small, the performance of the two systems is almost the same. With the increase of frequency offset, the IFFT/FFT method degrades faster than the conventional method. For frequency offset below 10%, the degradation between the IFFT/FFT method and the conventional method is small and negligible, but IFFT/FFT will be more efficient and easier to implement than the conventional method.

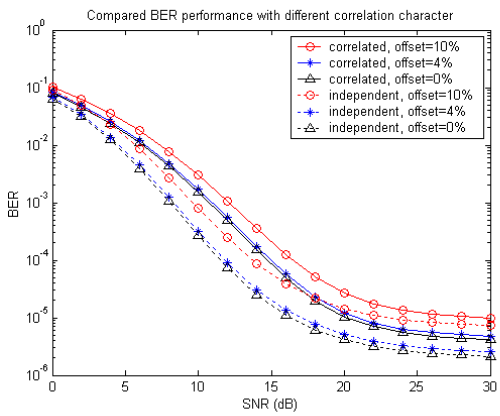

Figure 5. Comparison of BER performance with different correlation situations.

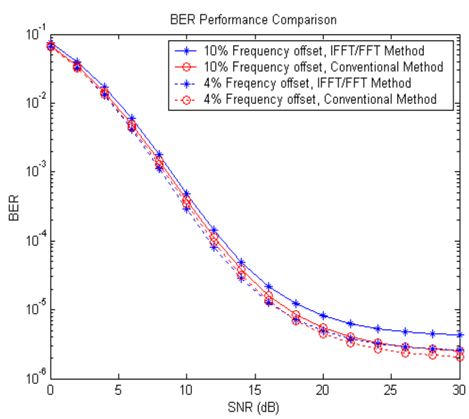

Figure 6. Comparison of BER performance with conventional MC-CDMA system and proposed MC-CDMA system, with frequency offsets 4% and 10%, respectively. (EGC)

V. CONCLUSION

The BER performance of a downlink MC-CDMA system using FFT/IFFT has been presented. The analysis has been done for both EGC and MRC. Numerical results indicate that frequency offset and channel correlation have adverse effects on the performance of the MC-CDMA system. A big frequency offset as compared to the frequency spacing between adjacent subcarriers will cause severe BER performance loss. The comparison between EGC and MRC indicates that MRC performs better. The comparison between the conventional system model and the new system model shows that we can implement MC-CDMA more efficiently and easily using IFFT/FFT, with the cost of slightly more performance degradation.

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