SCATTERING FUNCTION AND TIME-FREQUENCY SIGNAL PROCESSING

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ABSTRACT

The estimation of the scattering function in time-frequency selective fading mobile environment is considered. The scattering function explicitly reveals the time-frequency selective behavior of the fading channel under the well-known WSSUS assumption. We propose two classes of estimators based on a time-frequency framework that generalize the existing estimators while giving an extra freedom according to different criteria wanted to be achieved in the estimation of the scattering function. Instead of using Woodward ambiguity function or symmetric ambiguity function, we use the generalized ambiguity function which comes from the general class of quadratic time-frequency distributions.

1. INTRODUCTION

One of the major effects to wideband transmission in mobile radio communications due to multipath propagation is the time and frequency dispersion as the results of time-delays over the multipaths and Doppler-shifts from random motion of scatterers. This effect is known as time-frequency selective fading [1, 2].

In practice, this type of channels is often modeled as a random linear time-varying filters. Its second order statistics is completely characterized by its scattering function under the wide-sense stationary Gaussian process with uncorrelated scattering (WSSUS) assumption [3]. The scattering function explicitly reveals the time-frequency selective behavior of the fading channel. The importance of the scattering function is emphasized by the extensive literature [4, 5, 6, 7, 8, 9, 10, 11] (and references therein).

The estimation of the scattering function of the random linear time-varying is considered. A common approach is to relate the scattering function with the symmetric ambiguity function [4] or Woodward ambiguity ambiguity function [8, 9] of the input signal. However, a classical problem faced in this approach is the division of zero. In order to solve it, thresholding method and its derivatives have been approached (review of this can be found in [8]).

We propose two classes of estimators based on a time-frequency framework that generalize the existing estimators while giving an extra freedom according to different criteria wanted to be achieved in the estimation of the scattering function. Instead of using Woodward ambiguity function or symmetric ambiguity function, we use the *generalized* ambiguity function which comes from a general class of quadratic time-frequency distributions [12].

2. WSSUS CHANNEL

A complex baseband received signal, $r(t)$, through a wireless mobile communication channel can be modeled[1] as follows [3]

$$\begin{aligned} r(t) &= x(t) + \varepsilon(t), \qquad \text{with } 0 \le t \le T \\ &= \int h(t,\tau) s(t-\tau)\, d\tau + \varepsilon(t) \qquad (1) \\ &= \iint \mathcal{U}(\nu,\tau) s(t-\tau)\, e^{j2\pi\nu t}\, d\tau\, d\nu + \varepsilon(t) \qquad (2) \end{aligned}$$

where $h(t,\tau)$ is the channel impulse response representing the linear time-varying behavior; $s(t)$ is the complex baseband transmitted signal; T is the symbol duration; $\varepsilon(t)$ is the additive white Gaussian noise with zero mean and variance σ_ε^2; τ and ν denote the time-delay and Doppler shift variables; and $\mathcal{U}(\nu,\tau)$, the Fourier transform of $h(t,\tau)$ from t to ν, is called Delay-Doppler Spread function of the LTV channel. By applying the Fourier transform among the variables t, f, τ and ν, we can define several system functions [2, 3] with their relationship shown on Fig. 1.

The Delay-Doppler Spread function is often modeled as a wide-sense stationary Gaussian process with uncorrelated scattering (WSSUS) [2, 3] whose second-order statistics can be represented by[2]

$$\mathcal{E}\left\{\mathcal{U}(\nu,\tau)\cdot\mathcal{U}^*(\nu',\tau')\right\} = P_{\mathcal{U}}(\nu',\tau')\cdot\delta(\nu-\nu')\delta(\tau-\tau') \qquad (3)$$

[1]In practice, the double integral is bounded by the ranges of delays and Doppler-shifts, however, without loss of generality, we the full range $(-\infty,\infty)$ and drop them for short notation.

[2]$\mathcal{E}\{\cdot\}$ denotes the expected value operator.

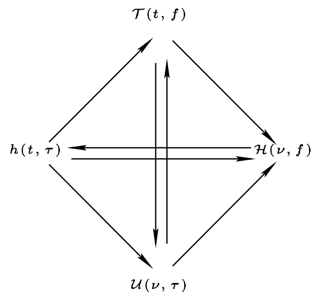

Figure 1: Relationship among Bello system functions. $\mathcal{T}(t,f)$ and $\mathcal{H}(\nu,f)$ are the Time-Variant System and Output Doppler Spread functions, respectively.

where $P_{\mathcal{U}}(\nu,\tau)$ is the delay-Doppler spectrum and referred to as the *scattering function* of the channel. It follows that the WSSUS channel may be represented as a collection of non-scintillating uncorrelated scatterers which cause both delays and Doppler shifts. The statistics of the WSSUS channel is shown on Fig. 2 [13].

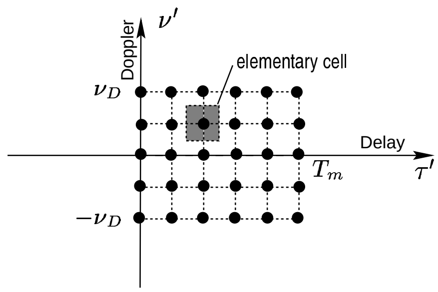

Figure 2: Second order statistics of frequency-selective fast fading channel with WSSUS assumption. (T_m and ν_D are the maximum time-delay and Doppler-shift for a particular mobile environment).

3. A BRIEF ON TIME-FREQUENCY

Time-frequency signal processing is a natural extension of both the time domain and frequency domain processing, that involves representing signals in 2-D space which reveals more information of the signal. Such a representation is intended to provide a distribution of signal energy versus both time and frequency simultaneously. For this reason, the representation is commonly called a time-frequency distribution (TFD) [12].

A general class of quadratic TFDs is defined as [12]

$$\rho_z(t,f) \triangleq \iiint e^{j2\pi\nu(u-t)}\, g(\nu,\tau) \times z(u+\frac{\tau}{2})z^*(u-\frac{\tau}{2})\, e^{-j2\pi f\tau}\, d\nu\, du\, d\tau \quad (4)$$

where $g(\nu,\tau)$ is a two dimensional function in the ambiguity domain, (ν,τ), and is called the *kernel*. The kernel determines the distribution and its properties. We can obtain and study the distributions with certain desired properties by properly constraining the kernel.

Knowing the Wigner-Ville distribution [12] of an analytic signal $z(t)$

$$W(t,f) = \int K_z(t,\tau)\, e^{-j2\pi f\tau}\, d\tau$$

where $K_z(t,\tau) = z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})$, one can arrive to the Fourier relationship shown in Fig. 3 where $A_z(\nu,\tau)$ is the symmetric ambiguity function.

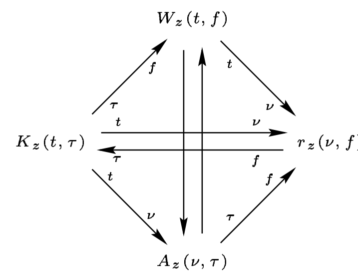

Figure 3: Quadratic representations corresponding to WVD [12]. (Note the similarity compared to Fig. 1).

Moreover, with the use of general quadratic time-frequency class in (4), we have another relationship illustrated in Fig. 4 where the $\mathcal{A}_z(\nu,\tau)$ is the *generalized* ambiguity function.

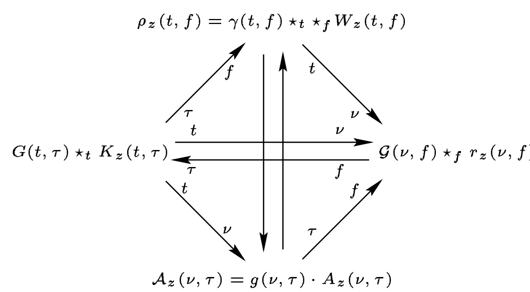

Figure 4: Dual domains of signal quadratic representations [12].

4. CHANNEL SECOND ORDER STATISTICS

Given the WSSUS assumption to the channel, it is important to examine the relationship of the transmitted and received signals through their second order statistics. In this paper, we consider the noise-free case of the channel. The correlation function of the received signal given in (1) can be defined as

$$R_{xx}(t,\Delta t) \triangleq \mathcal{E}\left\{x(t+\frac{\Delta t}{2})\cdot x^*(t-\frac{\Delta t}{2})\right\}$$

Taking Fourier transform from t to Δf on both sides of the above, we then have (see Appendix)

$$\mathcal{E}\left\{A_x(\Delta f,\Delta t)\right\} = A_s(\Delta f,\Delta t)\cdot R_{\mathcal{T}}(\Delta f,\Delta t) \quad (5)$$

where $R_{\mathcal{T}}(\Delta f, \Delta t)$ is the double Fourier transform[3] of the scattering function $P_{\mathcal{U}}(\nu', \tau')$. Equation (5) represents the second-order statistic relationship in the ambiguity domain based on WSSUS assumption. By applying inverse double Fourier transform on both sides of (5), we have another representation of the second-order statistic relationship in the time-frequency domain (W is the Wigner-Ville distribution, see Fig. 3)

$$\mathcal{E}\left\{W_x(\nu', \tau')\right\} = W_s(\nu', \tau') \star_{\nu'} \star_{\tau'} P_{\mathcal{U}}(\nu', \tau') \qquad (6)$$

One often renames the variables (Δf, Δt, ν' and τ') by (ν, τ, t and f), respectively [4]. We adopt this for the ease of visualization in the time-frequency context, as a result, (5) and (6) can be rewritten as

$$\mathcal{E}\left\{A_x(\nu, \tau)\right\} = A_s(\nu, \tau) \cdot R_{\mathcal{T}}(\nu, \tau) \qquad (7)$$

$$\mathcal{E}\left\{W_x(t, f)\right\} = W_s(t, f) \star_t \star_f P_{\mathcal{U}}(t, f) \qquad (8)$$

By multiplying both sides of (7) with an *arbitrary* kernel $g(\nu, \tau)$, and taking the inverse double Fourier transform of the result, we derive to two general equations representing the second-order relationship in terms of the generalized ambiguity function $\mathcal{A}$ and the general quadratic time-frequency distribution ρ (see Fig. 4), as

$$\mathcal{E}\left\{\mathcal{A}_x(\nu, \tau)\right\} = \mathcal{A}_s(\nu, \tau) \cdot R_{\mathcal{T}}(\nu, \tau) \qquad (9)$$

$$\mathcal{E}\left\{\rho_x(t, f)\right\} = \rho_s(t, f) \star_t \star_f P_{\mathcal{U}}(t, f) \qquad (10)$$

It should be noted that the result given in [6, 7] are the special cases (expressed in terms of spectrogram and Wigner-Ville distribution) of the general case presented in (10).

5. SCATTERING FUNCTION ESTIMATORS

Since the kernel $g(\nu, \tau)$, implicitly included in (9) and (10), is arbitrary, two general classes of estimators for the scattering functions are proposed: *deconvolution* and *direct-implementation*.

5.1. Class of deconvolution estimators

The class of deconvolution estimators is defined based on the division of (9) by the generalized ambiguity function of the input signal

$$\hat{P}_{\mathcal{U}}^{(g,1)}(t, f) \triangleq \mathcal{F}^{-1}_{\nu \to t}\left\{\mathcal{F}_{\tau \to f}\left\{\frac{\mathcal{E}\left\{\mathcal{A}_x(\nu, \tau)\right\}}{\mathcal{A}_s(\nu, \tau)}\right\}\right\} \qquad (11)$$

Similar to the approach in [8], zero-division problem in (11) is encountered. A classical solution is to threshold the symmetric ambiguity function $A_s(\nu, \tau)$, or the Woodward ambiguity function (when matched filtering is pre-applied at the output signal), at the points equal to zero. Mathematically, it replaces $A_s(\nu, \tau)$ by $A_{s'}(\nu, \tau) = A_s(\nu, \tau) + \lambda C(\nu, \tau)$.

This replacement creates a problem in which the signal $s'(t)$ may not exist. One could also use an interrogating signal (as in the case of image processing [8]) with some well-behaved characteristics in order to achieve better estimation of $P_{\mathcal{U}}(t, f)$. However, in communications, interrogating (pilot) signals are not encouraged since the communication becomes more expensive with extra maneuver of the pilot signals.

On the other hand, by using of the kernel $g(\nu, \tau)$, in turns, making use of the well-defined generalized ambiguity function, the problems of the nonexistence of $s'(t)$ and using interrogating signals are avoided. Also the kernel can be used to smooth $A_s(\nu, \tau)$ in the sense that it represents the entire energy within an elementary cell (see Fig. 2) to a value at the center of the cell and the estimates of the scattering function need only to be evaluated at these centers of the cells. This helps partially minimize the zero division problem. Thresholding, not for zero division problem, can be applied after smoothing in order to discard the cells that have negligible energy. As a result, computational efficiency of post-processing in each cell for different purposes (e.g. detection [13]) can be significantly improved.

5.2. Class of direct-implementation estimators

One can choose the kernel $g(\nu, \tau)$ so that the distribution $\rho_s(t, f)$ in (10) is impulse-like (we would ideally wish to have a delta representation in the time-frequency plane, this, however, does not exist due to the constraint of minimum time-frequency bandwidth according to Heisenberg's uncertainty principle), the left-hand side of (10), then, approximates to $P_{\mathcal{U}}(t, f)$. Thus, we define another class, namely direct-implementation, of estimators

$$\hat{P}_{\mathcal{U}}^{(g,2)}(t, f) \triangleq \mathcal{E}\left\{\rho_x(t, f)\right\} \qquad (12)$$

An example of this class can be obtained by choosing kernel such that the time-frequency distribution $\rho_s(t, f)$ is approximated to that of Hermite functions known for having well-localized time-frequency representation [14].

6. CONCLUSION

We have proposed two classes of estimators for the scattering function of the time-frequency dispersive fading mobile channels with the use of the generalized ambiguity function familiarized in the context of time-frequency signal processing. The degree of freedom introduced by the arbitrary kernel $g(\nu, \tau)$ results in different estimators. This avoids or overcomes the problems encountered in the existing estimators. The selection of optimum criteria for the estimators and, in turns, the performance of the estimators, depends on the selection of the kernel with specific property. Detailed analysis of this is beyond the scope of the paper.

[3] Definition of double Fourier transform:

$$\Psi(\alpha, \beta) = \mathcal{F}_{x \to \alpha}\left\{\mathcal{F}^{-1}_{y \to \beta}\left\{\Phi(x, y)\right\}\right\}$$
$$= \iint \Phi(x, y)\, exp\left\{-j2\pi\alpha x + j2\pi\beta y\right\}\, dx\, dy$$

7. REFERENCES

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Appendix

Proof of equation (5)

The correlation function of the received signal given in (1) is expressed as

$$R_{xx}(t,\Delta t) \triangleq \mathcal{E}\left\{x(t+\frac{\Delta t}{2})\cdot x^*(t-\frac{\Delta t}{2})\right\}$$

$$= \mathcal{E}\Bigg\{\left(\iint \mathcal{U}(\nu,\tau)s(t+\frac{\Delta t}{2}-\tau)\,e^{j2\pi\nu(t+\Delta t/2)}\,d\tau\,d\nu\right)$$

$$\times\left(\iint \mathcal{U}^*(\nu',\tau')s^*(t-\frac{\Delta t}{2}-\tau')\right.$$

$$\left.e^{-j2\pi\nu'(t-\Delta t/2)}\,d\tau'\,d\nu'\right)\Bigg\}$$

$$= \iiiint \mathcal{E}\left\{\mathcal{U}(\nu,\tau)\mathcal{U}^*(\nu',\tau')\right\}s(t+\frac{\Delta t}{2}-\tau)$$

$$\times s^*(t-\frac{\Delta t}{2}-\tau')\cdot e^{j2\pi(\nu(t+\Delta t/2)-\nu'(t-\Delta t/2))}\,d\tau\,d\nu\,d\tau'\,d\nu'$$

Using (3), and integrating over ν and τ, one can obtains

$$R_{xx}(t,\Delta t) = \iint P_{\mathcal{U}}(\nu',\tau')$$

$$\times s(t+\frac{\Delta t}{2}-\tau')s^*(t-\frac{\Delta t}{2}-\tau')\,e^{j2\pi\nu'\Delta t}\,d\tau'\,d\nu'$$

Taking Fourier transform from t to Δf on both sides of the above we then have[4]

$$\mathcal{E}\left\{A_x(\Delta f,\Delta t)\right\} =$$

$$= \iiint P_{\mathcal{U}}(\nu',\tau')s(t+\frac{\Delta t}{2}-\tau')s^*(t-\frac{\Delta t}{2}-\tau')$$

$$\times\, e^{j2\pi\nu'\Delta t}\,e^{-j2\pi t\Delta f}\,d\tau'\,d\nu'\,dt$$

$$= \iiint P_{\mathcal{U}}(\nu',\tau')K_s(t-\tau',\Delta t)e^{j2\pi\nu'\Delta t}e^{-j2\pi t\Delta f}\,d\tau'\,d\nu'\,dt$$

$$= \iint P_{\mathcal{U}}(\nu',\tau')\left(\int K_s(t-\tau',\Delta t)e^{-j2\pi t\Delta f}\,dt\right)$$

$$e^{j2\pi\nu'\Delta t}\,d\tau'\,d\nu'$$

$$= \iint P_{\mathcal{U}}(\nu',\tau')\cdot A_s(\Delta f,\Delta t)e^{-j2\pi\tau'\Delta f}e^{j2\pi\nu'\Delta t}\,d\tau'\,d\nu'$$

$$= A_s(\Delta f,\Delta t)\cdot\mathcal{F}_{\tau'\to\Delta f}\left\{\mathcal{F}^{-1}_{\nu'\to\Delta t}\left\{P_{\mathcal{U}}(\nu',\tau')\right\}\right\}$$

where $A_x(\cdot)$ and $K_s(\cdot)$ are defined in Fig. 3 for different signals ($x(t)$ and $s(t)$ instead of $z(t)$) and different variables. Note that, the notations t, f, ν and τ used in Fourier transforms, see Fig. 3, are only for familiar convention in signal processing point of view, one can use any others as long as they satisfy the definition of Fourier transform). It should be noted that the result (5) appears also in [4] without proof.

[4]Fourier transform operator and expected value operator are interchangeable under some specific conditions.

$$\mathcal{F}_{t\to\Delta f}\left\{R_{xx}(t,\Delta t)\right\} =$$

$$\mathcal{E}\left\{\mathcal{F}_{t\to\Delta f}\left\{x(t+\frac{\Delta t}{2})\cdot x^*(t-\frac{\Delta t}{2})\right\}\right\} = \mathcal{E}\left\{A_x(\Delta f,\Delta t)\right\}$$