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ABSTRACT: To determine the design water levels along the major rivers in the Netherlands, two-dimensional flow models, based on the software package WAQUA, are applied. Calibration of these mathematical models is, in general, a time consuming process. This process can be automated by function minimisation with the simplex algorithm as proposed by Nelder and Mead (1965). In the present paper, this methodology and the results of an application to the Dutch IJssel river are described. To prevent unrealistic values for the parameters, constraints have been implemented. The results are encouraging.

1 Introduction

In a low-lying country, such as the Netherlands, protection against high water levels during (extreme) river floods is of great importance. The dike heights are based on the water levels for the design discharge. This discharge is based on a frequency analysis of the (annual) maximum discharges at the upstream Dutch border and has an average return period of 1250 years. The maximum discharge during the recent flood waves of 1993 and 1995 has an influence on the frequency analysis, resulting in a larger value of the design discharge. To cope with such trends in the discharge and possible other trends that may affect the water levels in the rivers, such as changes in land use, river geometry and bed level changes due to erosion or sedimentation, the law prescribes a regular update of the design water levels. The local dike height will be based on the design water levels. For the next update, the river flow models and the methodology for calibration are reviewed.

For the assessment of the water levels during a flood wave in the Dutch main rivers, the flow conditions in the river are simulated with a mathematical model. Although depth and width averaged flow simulations (one-dimensional flow models) can also be used to predict the water levels in open channels, two-dimensional depth-averaged flow simulations (2DH-models), based on the software package WAQUA (Rijkswaterstaat, 1992) are preferable. Depth-averaged modelling allows a more precise representation of river characteristics, resulting in a more accurate prediction of water levels than would be possible with 1D-models. Apart from more accurate results, a 2DH-model also generates information of local flow conditions, which can be used to evaluate the effects of works to be executed in the major bed. Compensating measures can be defined to mitigate possible negative effects.

Assuming that the river geometry has been thoroughly checked and that the measured hydraulic conditions in the river have been validated, the model can be calibrated by reducing the differences between measured and calculated flow conditions. Usually these differences are reduced by expert judgement with regard to the effect of various specified hydraulic roughness components, such as grass, woodland, shrubs and reed, on the flow conditions, followed by a recalculation of the flow conditions with adapted hydraulic roughness values. In general, this is a trial and error approach that depends mostly on the skills of the expert, while the process is time consuming. Hence, there is a need for a systematic approach. In the present paper a more systematic approach is developed on the basis of a function minimisation using the simplex method as proposed by Nelder and Mead (1965).

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2 Methodology

For the purpose of calibrating two-dimensional water-flow models, the roughness parameters must be determined as such that the measured and calculated water levels coincide as much as possible. This can be achieved by minimising the sum of the squared differences between measurements and calculations. A difficulty in using this method of the least squares is that one WAQUA simulation requires almost half an hour computing time (on a workstation). Therefore, the minimisation procedure to be chosen has to be fast, robust, and simple. Furthermore, computing the derivatives of the object function is not feasible in this situation.

A well-known minimisation algorithm using only function value information is the simplex algorithm of Nelder and Mead (1965). When the number of parameters is small (i.e. smaller than about 15), this direct search method is often competitive with more complex algorithms that require many algebraic calculations. Another advantage of the Nelder-Mead simplex algorithm is the small number of calculations that is needed to initialise the algorithm (this number equals the number of parameters *n* plus one). More advanced global optimisation techniques, such as neural networks, simulated annealing, and genetic algorithms, may require even one thousand iterations for finding the optimum (Solomatine, 1995). An efficient implementation of Dennis and Woods (1987) has been used to program the Nelder-Mead simplex method.

In mathematical terms, the object function that represents the squares of differences between measurements and calculations can be written as

$$f(\mathbf{x}) = \sum_{j=1}^{m} \sum_{k=1}^{p} w_{jk} [h_{jk} - j_{jk}(\mathbf{x})]^2 , \quad (1)$$

$$\mathbf{x} = (x_1, \ldots, x_n) = $$
$$= (g_{\text{grass}}, g_{\text{woodland}}, g_{\text{shrubs}}, a_1, \ldots, a_r, b_1, \ldots, b_r),$$

where *j* stands for the index of the river discharge Q_j, *k* for the index of the measuring location, w_{jk} for the weight attached to discharge *j* and measuring location *k*, h_{jk} and j_{jk} for the measured and calculated water level at measuring location *k* given discharge *j*, respectively, and the vector **x** for the *n* multiplication coefficients that are introduced in Sections 3-4.

The combination $\mathbf{x}^*$ of multiplication coefficients is optimal when it minimises the object function, i.e. when

$$f(\mathbf{x}^*) = \min_{\mathbf{x} \in R^n} f(\mathbf{x}) .$$

This unconstrained minimisation problem can be solved using the Nelder-Mead simplex method. At each iteration *n*+1 points are used as the vertices of an *n*-dimensional simplex. In R^2, for example, three points determine a triangle. First, on basis of the currently-used parametric vector, the other *n* vertices of the simplex are created by enlarging one co-ordinate, e.g. with 5%, and keeping the other *n*-1 co-ordinates fixed (this is done for all *n* co-ordinates). Next, trial steps are generated by the four operations of reflection, expansion, contraction, and shrinkage. A reflected vertex is computed by reflecting the worst vertex (having the largest object function value) through the centroid of the *n* remaining vertices. Roughly, there are now three possibilities:

1. If the reflected vertex has a function value which is less than the largest function value of the *n* remaining vertices and greater than or equal to the smallest function value of the *n* remaining vertices, then the reflected vertex is accepted and it is added to the set of the *n* remaining vertices.
2. If the reflected vertex has a function value which is less than the smallest function value of the *n* remaining vertices, then the trial step has produced a good point and an expansion vertex is computed. The expansion vertex is accepted (and the simplex is expanded) if its function value is less than the smallest function value, otherwise the reflected vertex is accepted.
3. If the reflected vertex has a function value which is greater than or equal to the largest function value of the *n* remaining vertices, then the simplex is contracted or shrunk.

The iteration process stops when the simplex is small. The simplex then contains the

minimum or is very close to it. The number of evaluations of the object function is approximately proportional to the power law $(n+1)^{2.11}$. For details, see Nelder and Mead (1965) or Dennis and Woods (1987).
To prevent the roughness parameters to take unrealistic values, lower and upper bounds for the parameters can be introduced by applying the transformation

$$\tilde{x}_i(x_i) = \tan\left(\left[-\frac{1}{2}+\frac{x_i-l_i}{u_i-l_i}\right]p\right), \quad i=1,\ldots,n,$$

where l_i and u_i are the lower and upper bound of x_i, respectively, $i = 1,\ldots,n$. Using this transformation, the constrained minimisation problem

$$\min_{\mathbf{x}\in R^n} f(\mathbf{x})$$
$$\text{sub } l_i \le x_i \le u_i, \quad i=1,\ldots,n$$

can now easily be transformed into the unconstrained minimisation problem

$$\min_{\tilde{\mathbf{x}}\in R^n} f(\mathbf{x}(\tilde{\mathbf{x}})),$$

which can be solved using the simplex algorithm.

3 Application

To test the function minimisation with the simplex method of Nelder and Mead the IJssel river (one of the Rhine branches in the Netherlands) was selected. The method is applied to a section between Zutphen and Zwolle (see
Figure 1).

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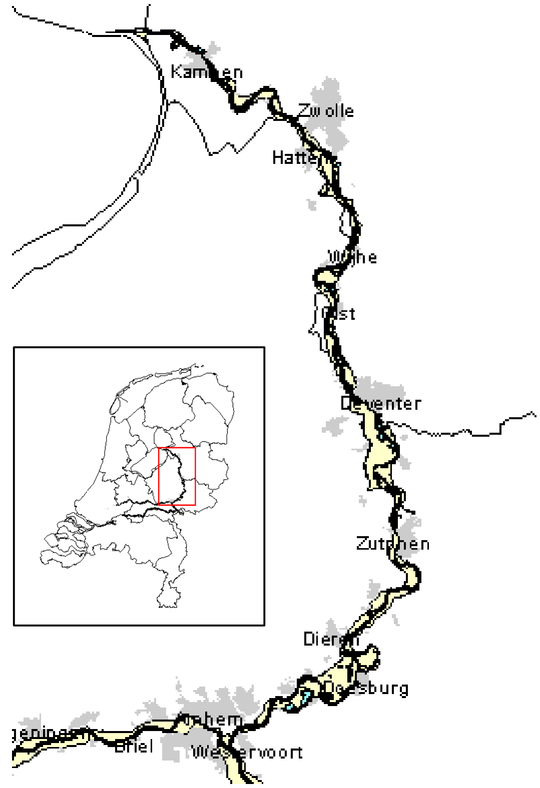

Figure 1: The IJssel river between Zutphen and Zwolle, the Netherlands

This river section consists of a minor bed, with a more or less uniform flow width of approx. 90 m, and a major bed, with a width (storage and flow width) varying from 350 m to 3100 m. The minor bed is an open channel with non-cohesive bed material. Different bed forms occur in the minor bed. The flood plain is mostly open with different kinds of grass and patches of reed, hedges, shrubs and woodland. For some locations in the flood plain, plans were developed for Nature Restoration, such as the Duursche Waarden on the right bank, opposite Veessen (see

Figure 2).
At those locations there is more reed and woodland in the flood plain. Often, herds of

some kind graze in these areas to create an area with specific biological and ecological characteristics (ecotope). In the mathematical model, the flow over grass and over reed, and through shrubs and woodland is represented by a (local) roughness value.

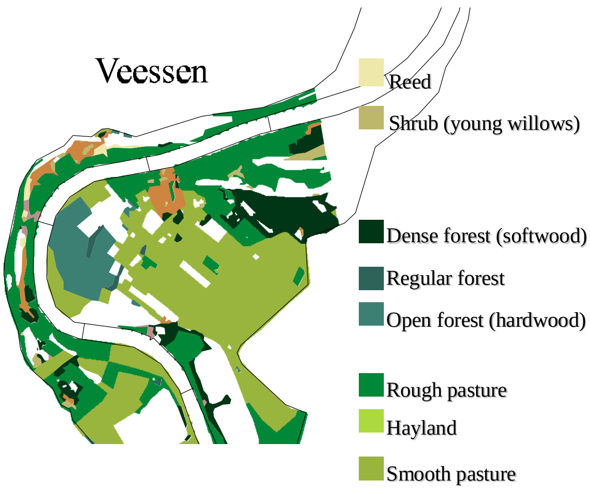

Figure 2: Types of vegetation in the Duursche Waarden (between Olst and Wijhe).

Some of the relations for the hydraulic roughness, such as the roughness due to grass, can be represented by the roughness formulation as proposed by Nikuradse:

$$C = 18\log\left(\frac{12h}{k_s}\right)$$

with C = Chézy value, h = water depth and k_s = roughness according to Nikuradse. Others, such as the flow through woodland, require a more complicated relation between roughness and the actual flow velocity and/or water depth, see Klopstra et al, (1997):

$$C_t = \frac{1}{\sqrt{\frac{1}{C_b^2} + \frac{C_d \sum A_i}{2gA_s L} R}}$$

with A_i = flow area of vegetation, A_s = flow area, C_b = Chézy value due to bed friction alone, C_d = drag coefficient of vegetation, C_t = total Chézy value, g = acceleration of gravity, L = length of vegetation in flow direction and R = hydraulic radius.

In the mathematical model, approximately 20 types of vegetation are distinguished with their own influences on the hydraulic roughness. It is, however, not practical to optimise all vegetation types in the calibration of the mathematical model using the simplex algorithm. The vegetation types with only a limited area of appearance in the river section are excluded from the parameter optimisation.

A further reduction of the number of parameters is obtained by assuming a fixed ratio between different classes of the same vegetation type, such as short grass, normal grass and long grass. The hydraulic roughness is represented by a Nikuradse type of roughness, with $k_{short\ grass} = k_{normal\ grass}/2.5$ and $k_{long\ grass} = 5\ k_{normal\ grass}$. The function minimisation results in an optimal value for normal grass, but due to the roughness ratio between normal grass and short and long grass, the hydraulic roughness of these vegetation types are also influenced. A similar approach has been applied for woodland and shrubs. For the representation of the hydraulic roughness of bed forms in the minor bed, the roughness predictor according to Van Rijn (1985) and Ogink (1986), simplified by Van Velzen (1998), has been used:

$$k_s = a\ h^{0.7}(1 - \exp\{-b\ h^{-0.3}\})$$

with a,b = coefficients, h = water depth and k_s = minor bed roughness (Nikuradse). In this formulation the influence of the characteristic grain size (D_{50}) on the roughness of the minor bed is incorporated in the coefficients a and b. Due to the variation of the grain size in longitudinal direction both coefficients are not constant and are also parameters in the function minimisation. For all the types of roughness a multiplication coefficient is defined, which is varied in the function minimisation to obtain a minimum value for the object function. To reduce the number of parameters related to the minor bed, a fixed ratio was assumed for the values of the coefficient *a* in the various sections and similarly the coefficient *b*.

The simplex algorithm can be applied to a complete hydrograph. The object function is the sum of the squared differences between measured and calculated water levels at a regular time interval during the hydrograph for a given number of measuring locations. To reduce the required computing time for the function minimisation, the hydrograph is (temporarily) represented by m = 5 steady state discharges (Q_1 = 750 m^3/s, Q_2 = 1050 m^3/s, Q_3 =1350 m^3/s, Q_4 = 1600 m^3/s and Q_5 = 1850 m^3/s). In the river section that is modelled, the water levels are measured in p = 4 stations (Zutphen, Deventer, Olst and Wijhe) upstream of the outflow boundary location in the mathematical model. The measured water levels for the locations mentioned above were obtained from stage-discharge relations (see Figure 3).

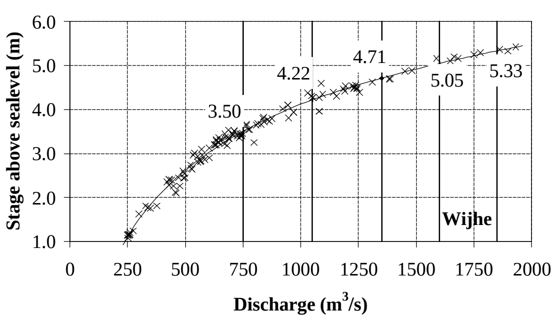

Figure 3: Stage-discharge relation for measuring location Wijhe.

The mathematical model is, amongst others, used for the prediction of water levels at extreme flood conditions with an average return period of 1250 years. It is therefore important to reduce the uncertainty in the predicted water levels at high discharges. This can be achieved by weighing the differences between measured and calculated water levels. The water level differences were given weight j for river discharge Q_j, i.e. $w_{jk} = j$ for $j = 1,...,m$ and $k = 1,...,p$. Note that all measuring locations are given equal weight.

The minimised value $\mathbf{x}^*$ of the object function (1) gives an indication of the average inaccuracy of the calculated water levels in comparison to the measured water levels according to:

$$\sqrt{\frac{f(\mathbf{x}^*)}{\sum_{j=1}^{m}\sum_{k=1}^{p} w_{jk}}} = \sqrt{\frac{f(\mathbf{x}^*)}{p\cdot[m(m+1)/2]}} \,. \qquad (3)$$

4 Results

Computations have been carried out with a different number of parameters in the function minimisation. In the first series only $n = 5$ roughness parameters were used (grass, *a* and *b* constant in longitudinal direction, woodland and shrubs) without constraints in the value of the roughness parameters, according to Equation (1) as long as they are positive and the model calculations are stable.

However, the roughness parameters tend to take unrealistic values. To prevent unrealistic values, a lower and an upper bound is specified for each roughness parameter and the calculations were repeated. The development of the object function (2) during the function minimisation is given in Figure 4. The corresponding iteration process of the multiplication factors for the various types of roughness is shown in
Figure 5 up to and including
Figure 9.

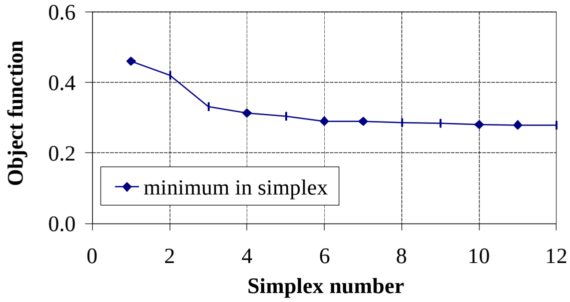

Figure 4: Decrease of object function each time a new minimum is found in the simplex (5 parameters).

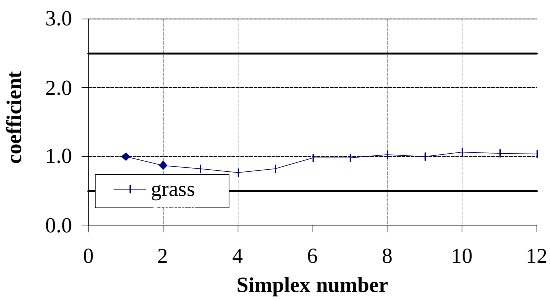

Figure 5: Multiplication coefficient for grass.

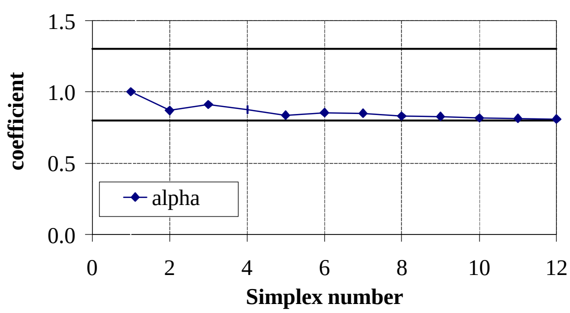

Figure 6: Multiplication coefficient for *a* of minor bed roughness.

From Figure 4 it follows that the simplex algorithm approaches the minimum value asymptotically. This also holds for the multiplication coefficients of the various types of roughness. Apart from the multiplication coefficient for grass, the resulting values of the multiplication coefficients approach either the lower bound or the upper bound. There is a simple explanation for this. Since most of the flood plain is covered with grass, slight adjustments in this type of roughness result in noticeable changes in the calculated water levels, improving the object function. The other types of vegetation cover only relatively small areas. Therefore, for a small change in water level, large changes in roughness are required to improve the object function and the resulting multiplication coefficient approaches the lower or upper bound.

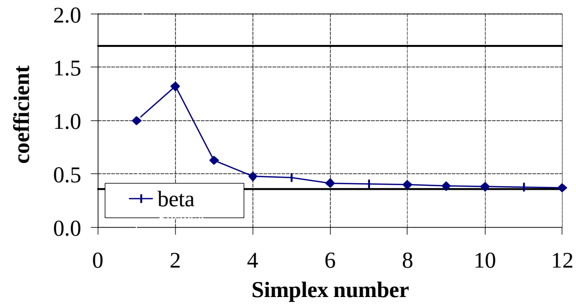

Figure 7: Multiplication coefficient for *b* of minor bed roughness.

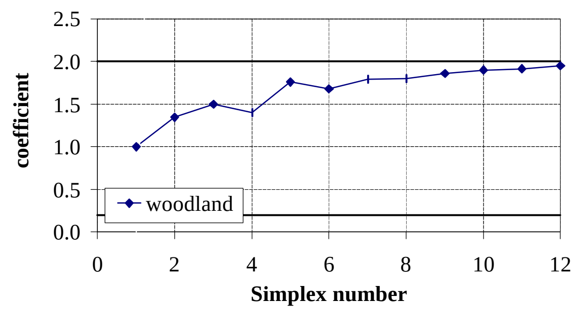

Figure 8: Multiplication coefficient for woodland.

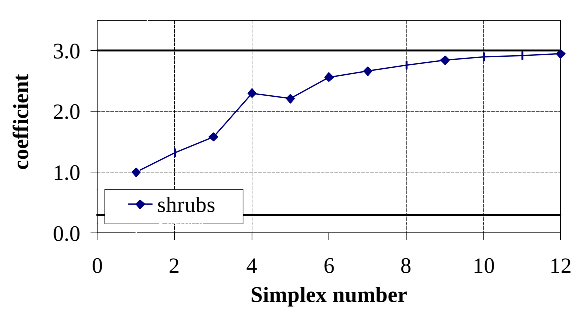

Figure 9: Multiplication coefficient for shrubs.

In the function minimisation so far, the roughness for the minor bed is assumed to be constant in longitudinal direction. In reality, however, the minor bed roughness varies in longitudinal direction. The variation in longitudinal direction was taken into account by increasing the number of variables in the simplex algorithm. In the flow direction $r = 6$ sections were distinguished, each section having its own minor bed roughness. Because each section adds two more parameters to the simplex assessment, the total number of parameters in the simplex was increased to $n = 15$. Due to the larger number of parameters, more simulations with the mathematical model were required to minimise the object function. To reduce the time required for computations, the number of discharge levels was reduced from $m = 5$ to $m = 2$ levels ($Q_1 = 1650$ m^3/s and $Q_2 = 1800$ m^3/s). Due to the reduced number of discharge levels, the object function has a smaller starting value. The iteration process of the object function minimisation is shown in

Figure 10. According to Equation (3), the average inaccuracy in the computed water levels is approximately 0.04 m.

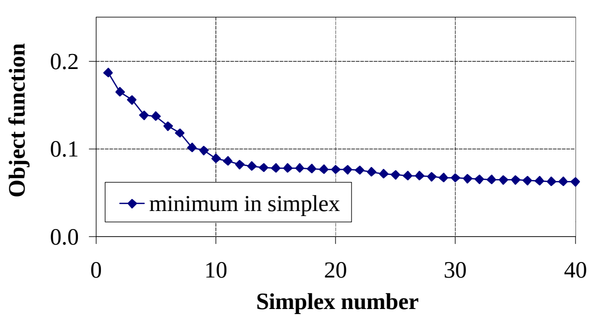

Figure 10: Decrease of object function each time a new minimum is found for the simplex (15 parameters).

The optimised values of the multiplication coefficients g_{grass}, $g_{woodland}$, g_{shrubs}, a_1, ..., a_r, b_1, ..., b_r are given in Table 1, where $r = 6$.

Table 1: Multiplication coefficients for the parameters in the simplex determination.

Multiplication Coëfficiënt for:	Lower bound	Value	Upper bound
γ_{grass}	0.50	0.982	2.50
$\gamma_{woodland}$	0.20	0.227	2.00
γ_{shrubs}	0.30	0.369	3.00
a_1, section 1	0.80	1.298	1.30
a_2, section 2	0.80	1.300	1.30
a_3, section 3	0.80	0.801	1.30
a_4, section 4	0.80	0.805	1.30
a_5, section 5	0.80	0.802	1.30
a_6, section 6	0.80	1.299	1.30
b_1, section 1	0.36	1.693	1.70
b_2, section 2	0.36	1.692	1.70
b_3, section 3	0.36	1.692	1.70
b_4, section 4	0.36	0.365	1.70
b_5, section 5	0.36	0.366	1.70
b_6, section 6	0.36	0.369	1.70

5 Conclusions

The present study demonstrates that the differences between measured and calculated water levels can be minimised by applying the simplex algorithm of Nelder and Mead. With prescribed constraints based on reasonable physical bounds for the parameters, the calibration process is comparable to the process that would have been used otherwise, but with much less human effort. With or without constraints, the simplex algorithm converges to a minimum value for the object function. The final value of the object function is an indication of the average inaccuracy of the calculated water levels compared to the measured water levels. Application of the simplex algorithm can be compared to a reverse solution of the momentum and continuity equation. Instead of solving the water levels for given discharge and roughness, the simplex algorithm solves the roughness for given discharge and water levels. The larger the area of a particular roughness, the more the roughness value influences the water levels, and the more suitable such a parameter will be in the function minimisation. The optimised parameters also depend on the measured water levels. Small changes in water levels may have a significant effect on the value of one or more parameters, especially if the roughness area is small in comparison to the total area between successive measuring points.

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