# COMPARISON OF DIGITAL MULTI-CARRIER WITH DIRECT SEQUENCE SPREAD SPECTRUM IN THE PRESENCE OF MULTIPATH

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**ABSTRACT**

We compare single user digital Multi-Carrier Spread Spectrum modulation with Direct Sequence Spread Spectrum in the presence of frequency-selective multipath fading. We derive closed-form expressions for the bit error probability and show that MC-SS is more robust to multipath fading than is DS-SS.

## 1. INTRODUCTION

The increasing interest in and applications of direct sequence spread spectrum (DS-SS) technology stem from its robustness to fading, its anti-interference capability, and the potential for (even uncoordinated) multiple access. With a wide bandwidth and thus a short chip period, multiple paths can be resolved with DS-SS transmissions and a RAKE receiver can be used to mitigate fading and improve system performance [6].

An alternative approach to combat frequency-selective multipath is multicarrier modulation. Multi-Carrier Spread-Spectrum (MC-SS) [7] and the corresponding multiple access scheme: Multicarrier (MC) CDMA [10] has gained increasing popularity in recent years. By exploiting multiple carriers and a narrow band DS waveform on each subcarrier, it has been shown that multicarrier DS CDMA outperforms single carrier CDMA for wideband transmissions in the presence of narrow band interference [4].

Although most existing MC approaches rely on analog carrier modulations, digital implementations through FFTs are also available [1]. Thanks to the rapid development of digital devices and digital signal processing (DSP) technologies, the Digital to Analog (D/A) and Analog to Digital (A/D) converters are being pushed closer to the transceiver's end. Starting from a discrete-time equivalent model, we investigate the performance of digital MC-SS and compare it with DS-SS. The main contributions of this paper are the novel results on performance analysis of digital MC-SS in the presence of multipath. Further results on the performance analysis of digital MC-SS in the presence of narrow band interference (NBI) and the presence of both NBI and multipath may be found in [11].

## 2. UNIFYING TRANSCEIVER MODELS

The diagram in the upper part of Fig. 1 describes the discrete-time baseband equivalent model of an MC-SS system. The length-$N$ symbol periodic digital spreading code $\mathbf{c}_{mc} := [c_{mc}(0), \ldots, c_{mc}(N-1)]^T$ spreads the $i$th information symbol $s(i)$. The resulting sequence $\mathbf{c}_{mc}s(i)$ is then IFFT processed to obtain the $N \times 1$ vector $\mathbf{F}_N^{\mathcal{H}}\mathbf{c}_{mc}s(i)$, where $\mathbf{F}_N$ is the $N \times N$ FFT matrix with $(m,n)$ entry $(1/\sqrt{N})e^{-j2\pi mn/N}$ and $^{\mathcal{H}}$ denotes Hermitian transpose.

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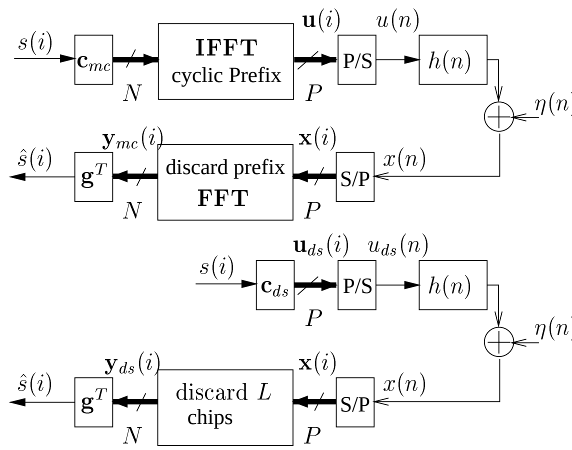


**Fig. 1**. Equivalent model: MC-SS (upper) and DS-SS (lower)

To avoid channel-induced inter symbol/block interference (ISI/IBI), we replicate the last $P-N$ entries (Cyclic Prefix (CP)) of the vector $\mathbf{F}_N^{\mathcal{H}}\mathbf{c}_{mc}s(i)$ at the front to form the $P \times 1$ transmitted block $\mathbf{u}(i)$, as in conventional OFDM systems, e.g., [1]. The received signal, after conversion to baseband and receive filtering, is sampled at the chip rate, to yield

$$x(n) = \sum_{l=0}^{L} h(l)u(n-l) + \eta(n), \qquad (1)$$

where $h(\ell)$ is the overall channel (transmit and receive filters, and propagation channel), $\eta(n)$ is the filtered additive Gaussian noise (AGN), and $L$ is the maximum order of the FIR channel. To avoid ISI, the CP length should be larger than the channel order: $P-N \geq L$. To avoid bandwidth overexpansion, we choose the smallest block length $P = N + L$ here.

To convert (1) from a serial to a convenient matrix-vector form, we define the $P \times 1$ vector: $\mathbf{x}(i) := [x(iP), x(iP+1), \ldots, x(iP+P-1)]^T$ (likewise for $\boldsymbol{\eta}(i)$), and the $P \times P$ Toeplitz channel matrices $\mathbf{H}_0, \mathbf{H}_1$ with $(k,l)$th entries $h(k-l)$ and $h(k-l+P)$, respectively. Since $h(l) = 0, \forall l \notin [0, L]$, and $P = N + L$, we can write (1) as:

$$\mathbf{x}(i) = \mathbf{H}_0\mathbf{u}(i) + \mathbf{H}_1\mathbf{u}(i-1) + \boldsymbol{\eta}(i), \qquad (2)$$

where the second term represents IBI.

At the receiver, the CP is removed by dropping the first $P-N$ elements of $\mathbf{x}(i)$, thus eliminating IBI. After the CP removal and FFT processing, we have

$$\mathbf{y}_{mc}(i) = \mathbf{F}_N\tilde{\mathbf{H}}\mathbf{F}_N^{\mathcal{H}}\mathbf{c}_{mc}s(i) + \mathbf{F}_N\bar{\mathbf{w}}(i), \qquad (3)$$

where $\tilde{\mathbf{H}}$ is the resulting channel matrix and $\bar{\mathbf{w}}(i)$ is the $N \times 1$ truncated noise vector: $\bar{\mathbf{w}}(i) := \mathbf{R}_{cp}\boldsymbol{\eta}(i)$, where $\mathbf{R}_{cp} :=$

$[\mathbf{0}_{N\times(P-N)}, \mathbf{I}_N]$ is the CP-removing matrix. Matrix $\tilde{\mathbf{H}}$ is an $N \times N$ circulant matrix with $(k,l)$th entry given by $h((k-l)\text{mod}N)$. Because (I)FFTs diagonalize circulant matrices, the circulant matrix $\tilde{\mathbf{H}}$ can be decomposed as $\tilde{\mathbf{H}} = \mathbf{F}_N^{\mathcal{H}}\mathbf{D}(\tilde{\mathbf{h}})\mathbf{F}_N$, where $\tilde{\mathbf{h}} := [H(\exp(0)), H(\exp(j2\pi/N)), \ldots, H(\exp(j2\pi(N-1)/N))]^T$ whose entries are the channel frequency response $H(z) := \sum_{l=0}^{L} h(l)z^{-l}$ evaluated at the subcarriers $z_k = \exp(j2\pi k/N)$, and $\mathbf{D}(\tilde{\mathbf{h}}) := \text{diag}(\tilde{\mathbf{h}})$ denotes a diagonal matrix with the $(i,i)$th entry being the $i$th element of the vector $\tilde{\mathbf{h}}$; see [9] for more details. Therefore, we can rewrite (3) as:

$$\mathbf{y}_{mc}(i) = \mathbf{D}(\tilde{\mathbf{h}})\mathbf{c}_{mc}s(i) + \mathbf{F}_N\bar{\mathbf{w}}(i)\,. \tag{4}$$

With $\mathbf{D}(\mathbf{c}_{mc}) := \text{diag}(\mathbf{c}_{mc})$, we verify that $\mathbf{D}(\tilde{\mathbf{h}})\mathbf{c}_{mc} = \mathbf{D}(\mathbf{c}_{mc})\tilde{\mathbf{h}}$. Define $\mathbf{h} := [h(0), \cdots, h(L)]^T$ and $\mathbf{V}$ as the $N \times (L+1)$ Vandermonde matrix formed by the first $L+1$ columns of $\sqrt{N}\mathbf{F}_N$; thus, $\tilde{\mathbf{h}} = \mathbf{V}\mathbf{h}$ represents a scaled FFT operation in matrix form. We then can rewrite (4) as:

$$\mathbf{y}_{mc}(i) = \mathbf{D}(\mathbf{c}_{mc})\mathbf{V}\mathbf{h}s(i) + \mathbf{F}_N\bar{\mathbf{w}}(i). \tag{5}$$

Since the spreading sequence is binary, i.e., $\mathbf{c}_{mc}$ has entries $\pm 1$, it holds that $\mathbf{D}^{\mathcal{H}}(\mathbf{c}_{mc})\mathbf{D}(\mathbf{c}_{mc}) = \mathbf{I}_N$, and after multiplying (5) with $\mathbf{D}^{\mathcal{H}}(\mathbf{c}_{mc})$ we arrive at

$$\mathbf{D}^{\mathcal{H}}(\mathbf{c}_{mc})\mathbf{y}_{mc}(i) = \mathbf{V}\mathbf{h}s(i) + \mathbf{D}^{\mathcal{H}}(\mathbf{c}_{mc})\mathbf{F}_N\bar{\mathbf{w}}(i). \tag{6}$$

Our primary goal is to compare the ability of MC-SS and DS-SS to combat multipath fading; therefore, we now describe the discrete time baseband model of DS-SS that is depicted in the lower part of Fig. 1.

Without FFT and CP insertion at the transmitter, the transmitted block in DS-SS is $\mathbf{u}_{ds}(i) = \mathbf{c}_{ds}s(i)$, where $\mathbf{c}_{ds} := [c_{ds}(0), c_{ds}(1), \ldots, c_{ds}(P-1)]^T$ is a $P \times 1$ vector having the same block length as the MC-SS system (the upper part of Fig. 1). Replacing $\mathbf{u}(i)$ in (2) by $\mathbf{u}_{ds}(i)$, and with $\mathbf{R}_{cp}$ eliminating IBI as in (3), we arrive at:

$$\mathbf{y}_{ds}(i) = \mathbf{R}_{cp}\mathbf{H}_0\mathbf{c}_{ds}s(i) + \bar{\mathbf{w}}(i). \tag{7}$$

Because $\mathbf{H}_0\mathbf{c}_{ds}$ represents in matrix-vector form the linear convolution between $\mathbf{h}$ and $\mathbf{c}_{ds}$, we can commute $\mathbf{h}$ and $\mathbf{c}_{ds}$ to obtain $\mathbf{H}_0\mathbf{c}_{ds} = \mathbf{C}_{ds}\mathbf{h}$, with $\mathbf{C}_{ds}$ denoting a $P \times (L+1)$ Toeplitz matrix with first column $\mathbf{c}_{ds}$ and first row $[c_{ds}(0), 0, \ldots, 0]$. Let us now define the truncated $N \times 1$ code vector for DS-SS as $\bar{\mathbf{c}}_{ds} := \mathbf{R}_{cp}\mathbf{c}_{ds}$. Multiplying $\mathbf{R}_{cp}$ with $\mathbf{C}_{ds}$ yields a truncated $N \times (L+1)$ Toeplitz matrix $\bar{\mathbf{C}}_{ds}$ with first column $\bar{\mathbf{c}}_{ds}$ and first row $[c_{ds}(L), \ldots, c_{ds}(0)]$. Therefore, we can rewrite (7) as:

$$\mathbf{y}_{ds}(i) = \bar{\mathbf{C}}_{ds}\mathbf{h}s(i) + \bar{\mathbf{w}}(i). \tag{8}$$

Comparing (6) with (8), we *unify* MC-SS and DS-SS in the following equivalent model:

$$\mathbf{y}(i) = \mathbf{C}\mathbf{h}s(i) + \mathbf{w}(i) = \mathbf{c}s(i) + \mathbf{w}(i), \tag{9}$$

where $\mathbf{c} := \mathbf{C}\mathbf{h}$ denotes the equivalent signature code vector after channel convolution and receiver processing. For convenience, we list the corresponding vectors for MC-SS and DS-SS unified by (9):

$$\mathbf{c} = \mathbf{V}\mathbf{h}, \quad \mathbf{w}(i) = \mathbf{D}^{\mathcal{H}}(\mathbf{c}_{mc})\mathbf{F}_N\bar{\mathbf{w}}(i), \text{ for MC-SS}\,, \tag{10}$$

$$\mathbf{c} = \bar{\mathbf{C}}_{ds}\mathbf{h}, \quad \mathbf{w}(i) = \bar{\mathbf{w}}(i)\quad, \text{ for DS-SS}\,. \tag{11}$$

We assume the additive noise is white, i.e., $\mathbf{R}_{\bar{w}\bar{w}} := \text{E}\left\{\mathbf{w}(i)\mathbf{w}^{\mathcal{H}}(i)\right\} = \sigma_w^2\mathbf{I}_N$. Starting with the unifying model (9), the Maximum Ratio Combiner (MRC) output becomes: $\hat{s}(i) = \mathbf{c}^{\mathcal{H}}\mathbf{y}(i)$. With $\sigma_s^2 := \text{E}\left\{s(i)s^{\mathcal{H}}(i)\right\}$, the output SNR becomes: $SNR = \mathbf{c}^{\mathcal{H}}\mathbf{c}\sigma_s^2/\sigma_w^2$, where $\mathbf{c}$ is defined in (10) for MC-SS and in (11) for DS-SS.

We next analyze the system bit error rate (BER) for random multipath channels.

## 3. RANDOM MULTIPATH FADING CHANNELS

Recall that $\mathbf{c} = \mathbf{V}\mathbf{h}$ for MC-SS and $\mathbf{c} = \bar{\mathbf{C}}_{ds}\mathbf{h}$ for DS-SS. The corresponding SNRs for a given channel $\mathbf{h}$ are:

$$SNR^{(mc)} = \mathbf{h}^{\mathcal{H}}\mathbf{V}^{\mathcal{H}}\mathbf{V}\mathbf{h}\sigma_s^2/\sigma_w^2 = N\mathbf{h}^{\mathcal{H}}\mathbf{h}\sigma_s^2/\sigma_w^2, \tag{12}$$

$$SNR^{(ds)} = \mathbf{h}^{\mathcal{H}}\bar{\mathbf{C}}_{ds}^{\mathcal{H}}\bar{\mathbf{C}}_{ds}\mathbf{h}\sigma_s^2/\sigma_w^2. \tag{13}$$

Eqs. (12) and (13) clearly show that the SNR, and thus the BER, in MC-SS *do not* depend on the code choices, whereas they do so in DS-SS. In [4] it is assumed that the self-interference due to multipath is *negligible*, i.e., the shifts of the spreading code are nearly orthogonal to itself so that $\bar{\mathbf{C}}_{ds}^{\mathcal{H}}\bar{\mathbf{C}}_{ds} = N\mathbf{I}_{L+1}$. Under this assumption, we have that $SNR^{(mc)} = SNR^{(ds)}$, which indicates that MC-SS and DS-SS exhibit the same ability in resisting multipath effects, which agrees with the results in [4]. In general, the Toeplitz matrix $\bar{\mathbf{C}}_{ds}$ does not have orthogonal columns. The columns of $\bar{\mathbf{C}}_{ds}$ can be approximately orthogonal (so that self-interference is negligible) only when the code length $P$ is sufficiently large relative to the channel order $L$, and the code is well constructed. Unlike [4], where focus is placed on multiuser interference and narrow band interference but the multipath-induced self-interference is ignored, here, we explicitly consider this self-interference effect and compare the multipath resistance of DS-SS with that of MC-SS. Thanks to the FFT processing and the CP insertion at the transmitter, the FIR multipath is converted to parallel frequency-flat subchannels in MC-SS, so that the self-interference on each subcarrier is accounted for and absorbed in the fading coefficient for that subchannel. As confirmed by (12), the performance of MC-SS is independent of code choices. We next show the advantages of MC-SS over DS-SS in the randomly faded multipath channel scenario.

For random channels $\mathbf{h}$ with covariance matrix $\mathbf{R}_{hh} := \text{E}\left\{\mathbf{h}\mathbf{h}^{\mathcal{H}}\right\}$, the BER for BPSK can be expressed in terms of the output SNR as: $P_b = \text{E}_{\mathbf{h}}\left\{\mathcal{Q}\left(\sqrt{SNR}\right)\right\}$ .This expression is difficult to evaluate by averaging over the statistics of the fading amplitude random variables directly [8], since $\mathcal{Q}(x)$ is a nonlinear function of $x$. However, by using an alternative representation of $\mathcal{Q}(\cdot)$, a closed-form BER expression for independent faded channels has been obtained in [8]. Following the steps of [8], and assuming that the channel estimates at the receiver are error-free, we will first derive a general BER expression for MC-SS and DS-SS, and then compare their capabilities in resisting multipath.

We first diagonalize $\mathbf{R}_{hh}$ via its spectral decomposition:

$$\mathbf{R}_{hh} = \mathbf{U}_h\mathbf{D}_h\mathbf{U}_h^{\mathcal{H}}, \quad \mathbf{D}_h = \text{diag}(\lambda_{11}, \ldots, \lambda_{LL}), \tag{14}$$

where $\mathbf{U}_h$ is unitary and $\lambda_{ii} \geq 0$ denotes the $i$th eigenvalue of $\mathbf{R}_{hh}$. Similarly, we decompose the signature code covariance matrix $\mathbf{R}_{cc} := \mathrm{E}\left\{\mathbf{c}\mathbf{c}^{\mathcal{H}}\right\}$ as:

$$\mathbf{R}_{cc} = \mathrm{E}\{\mathbf{C}\mathbf{h}\mathbf{h}^{\mathcal{H}}\mathbf{C}^{\mathcal{H}}\} = \mathbf{C}\mathbf{R}_{hh}\mathbf{C}^{\mathcal{H}} = \mathbf{U}_c\mathbf{D}_c\mathbf{U}_c^{\mathcal{H}}, \quad (15)$$

where $\mathbf{U}_c$ is a $P \times (L+1)$ matrix with orthonormal columns and $\mathbf{D}_c$ is a diagonal matrix with entries $\bar{\lambda}_{ii}, i \in [1, L+1]$. When $\mathbf{R}_{hh}$ is diagonal and $\mathbf{C}$ has orthonormal columns, we have $\bar{\lambda}_{ii} = \lambda_{ii}, \forall i \in [1, L+1]$.

Pre-multiplying $\mathbf{y}(i)$ in (9) with $\mathbf{U}_c^{\mathcal{H}}$ yields:

$$\begin{aligned}\mathbf{y}'(i) &:= \mathbf{U}_c^{\mathcal{H}}\mathbf{y}(i) = \mathbf{U}_c^{\mathcal{H}}\mathbf{C}\mathbf{h}s(i) + \mathbf{U}_c^{\mathcal{H}}\mathbf{w}(i) \\ &:= \mathbf{h}'s(i) + \mathbf{w}'(i),\end{aligned} \quad (16)$$

where $\mathbf{h}' := \mathbf{U}_c^{\mathcal{H}}\mathbf{C}\mathbf{h}$ and $\mathbf{w}'(i) := \mathbf{U}_c^{\mathcal{H}}\mathbf{w}(i)$ denote equivalent channel and noise vectors. Because $\mathbf{R}_{h'h'} = \mathbf{U}_c^{\mathcal{H}}\mathbf{R}_{cc}\mathbf{U}_c = \mathbf{D}_c$, the entries of $\mathbf{h}'$ are uncorrelated, while $\mathbf{w}'(i)$ is still white since $\mathbf{R}_{w'w'} = \sigma_w^2\mathbf{I}_{L+1}$. The MRC symbol estimate $\hat{s}(i) = (\mathbf{h}')^{\mathcal{H}}\mathbf{y}'(i)$ equals the MMSE/MF receiver output operating on $\mathbf{y}(i)$: $\hat{s}(i) = \mathbf{c}^{\mathcal{H}}\mathbf{y}(i)$. As a result, a closed form Symbol Error Rate (SER) expression for MPSK ($M$ constellation points) signals can then be obtained by direct substitution from [8, eq. (44)]:

$$P_s(E) = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\prod_{i=1}^{L} I_i(\bar{\lambda}_{ii}\sigma_s^2/\sigma_v^2, \gamma_{PSK}, \theta)d\theta, \quad (17)$$

where $\gamma_{PSK} := \sin^2(\pi/M)$, and $I_i(x, \gamma_{PSK}, \theta)$ is the moment of the probability density function of $h_i'$ evaluated at $-\gamma_{PSK}/\sin^2(\theta)$ (see [8, eq. (24)]). For example, if $h_i'$ is Rayleigh distributed, we have

$$I_i(x, \gamma_{PSK}, \theta) = [1 + \gamma_{PSK}x\sigma_s^2/(\sigma_v^2\sin^2(\theta))]^{-1}. \quad (18)$$

The moment $I_i(x, \gamma_{PSK}, \theta)$ for other distributions such as Nakagami, and the resulting SER for different constellations (e.g., QAM) can be found in [8].

To establish the optimality of MC-SS over DS-SS, let us consider the generic model of [3]:

$$\tilde{\mathbf{y}}(i) = \tilde{\mathbf{C}}\mathbf{h}s(i) + \tilde{\mathbf{w}}(i), \quad (19)$$

where $\tilde{\mathbf{w}}(i)$ is white and $\tilde{\mathbf{C}}$ is an arbitrary $N \times (L+1)$ matrix obeying the power constraint: $\mathrm{tr}\left\{\tilde{\mathbf{C}}^{\mathcal{H}}\tilde{\mathbf{C}}\right\} = \mathcal{P}_0$, prescribed by the transmit-power budget.

Starting with the generic model (19), it is possible to choose the precoder $\tilde{\mathbf{C}}$ according to the optimality criterion specified in the following theorem:

**Theorem 1 [3]**: *If $\mathbf{h}$ and $\tilde{\mathbf{w}}(i)$ in* (19) *are uncorrelated and $\tilde{\mathbf{w}}(i)$ is white, the optimum precoding matrix $\tilde{\mathbf{C}}$ is given by: $\tilde{\mathbf{C}}_{opt} = \mathbf{\Phi}\mathbf{D}_f\mathbf{U}_h^{\mathcal{H}}$, where $\mathbf{U}_h$ is defined in* (14)*; diagonal matrix $\mathbf{D}_f$ is the optimal power loading matrix selected as in [3, eq. (17) and (18)], and $\mathbf{\Phi}$ an arbitrary $N \times (L+1)$ matrix with orthonormal columns. Optimality of $\tilde{\mathbf{C}}_{opt}$ pertains to either minimizing the error in estimating the random channel, $E\left\{\|\mathbf{h} - \hat{\mathbf{h}}\|^2\right\}$, or, maximizing the conditional mutual information $I(\bar{\mathbf{x}}, \mathbf{h}|s)$ if $\mathbf{h}$ is complex Gaussian distributed.*

If the entries of $\mathbf{h}$ are independent and identically distributed (i.i.d.), i.e., $\mathbf{R}_{hh} = \sigma_h^2\mathbf{I}$ with $\mathbf{U}_h = \mathbf{I}_{L+1}$, then the optimal power loading matrix $\mathbf{D}_f = \alpha\mathbf{I}_{L+1}$, where $\alpha^2 = \mathcal{P}_0/(L+1)$ [3]. In this case, the optimal precoder $\tilde{\mathbf{C}}_{opt} = \alpha\mathbf{\Phi}$ should have orthogonal columns. Because the Vandermonde matrix $\mathbf{V}$ has orthogonal columns while the Toeplitz matrix $\bar{\mathbf{C}}_{ds}$ does not, MC-SS is optimal in this setting and it thus outperforms DS-SS considerably.

The optimality in Theorem 1 amounts to minimizing the mean-square channel estimation error, which implies that channel estimation accuracy dictates the overall BER performance. However, for special cases, it is possible to have the power loading of Theorem 1 optimize the overall BER directly (see, e.g., [2] for differential QPSK constellations which lead to a simple closed-form BER expression).

However, when the entries of $\mathbf{h}$ are i.i.d. with Gaussian distribution and covariance matrix $\mathbf{R}_{hh} = \sigma_h^2\mathbf{I}$, we can directly establish the optimality based on the SER expression in (17). Because $\mathbf{R}_{hh} = \sigma_h^2\mathbf{I}_{L+1}$, we have $\mathbf{D}_c$ in (15) for MC-SS as: $\mathbf{D}_c^{(mc)} = N\sigma_h^2\mathbf{I}_{L+1}$. Therefore, $\mathbf{D}_c^{(mc)}$ for MC-SS has equal diagonal entries, which is not the case for DS-SS because $\bar{\mathbf{C}}_{ds}$ for DS-SS in (15) does not have orthogonal columns in general. However, the total transmitted power is the same because

$$\mathrm{tr}\left\{\mathbf{D}_c^{(ds)}\right\} = \sigma_h^2\mathrm{tr}\left\{\bar{\mathbf{C}}_{ds}^{\mathcal{H}}\bar{\mathbf{C}}_{ds}\right\} = N(L+1)\sigma_h^2 = \mathrm{tr}\left\{D_c^{(mc)}\right\}.$$

Let us denote the $i$th diagonal element of $\mathbf{D}_c^{(ds)}$ by $\bar{\lambda}_{ii}^{(ds)}$ and of $\mathbf{D}_c^{(mc)}$ by $\bar{\lambda}_{ii}^{(mc)}$. We then have $\bar{\lambda}_{ii}^{(mc)} = (\sum_{i=1}^{L+1}\bar{\lambda}_{ii}^{(ds)})/(L+1)$. Applying the inequality: $(x_1 + x_2 + \ldots + x_N) \geq N(x_1x_2\cdots x_N)^{1/N}, x_i > 0$, we obtain $(x_1x_2\cdots x_N)^{-1} \geq [(x_1 + x_2 + \ldots + x_N)/N]^{-N} \geq 0$, and after taking into account (18), we arrive at the following inequality:

$$\begin{aligned}&\prod_{i=1}^{L+1} I_i(\bar{\lambda}_{ii}^{(ds)}\sigma_s^2/\sigma_v^2, \gamma_{PSK}, \theta) \quad (20)\\ &= \prod_{i=1}^{L+1}\left[1 + \frac{\gamma_{PSK}\bar{\lambda}_{ii}^{(ds)}\sigma_s^2}{\sigma_v^2\sin^2(\theta)}\right]^{-1} \geq \left[I_i(\bar{\lambda}_{ii}^{(mc)}\frac{\sigma_s^2}{\sigma_v^2}, \gamma_{PSK}, \theta)\right]^{L+1}\end{aligned}$$

Substituting (20) back into (17), we thus obtain:

$$P_s^{(ds)}(E) \geq P_s^{(mc)}(E), \quad (21)$$

where equality is achieved when the Toeplitz matrix $\bar{\mathbf{C}}_{ds}$ for DS-SS has orthogonal columns, i.e., when self-interference is zero. Inequality (20) implies that equal power loading optimizes BER for i.i.d. Gaussian channels. By distributing its power evenly across all subbands, MC-SS provides maximum protection against random frequency-selective multipath fading in this case.

If $\mathbf{h}$ is not i.i.d., equipower loading $\mathbf{D}_f = \alpha\mathbf{I}_{L+1}$ turns out to be near optimal at high SNR [3]. The selected precoder matrix $\tilde{\mathbf{C}} = \alpha\mathbf{\Phi}\mathbf{U}_h$ has orthogonal columns, which corroborates the near-optimality of MC-SS at high SNR.

To shed further light on the performance of digital MC-SS relative to DS-SS and to study the code dependence of DS-SS, we consider the following scenarios.

We construct three channel models, assuming that the channel $\mathbf{h}$ is Gaussian distributed of order $L = 2$, Channel 1 is i.i.d. with $\mathbf{R}_{hh} = \mathrm{diag}(1, 1, 1)/3$; channel 2 has $\mathbf{R}_{hh} = \mathrm{diag}(1, 0.5, 0.1)/1.6$, i.e., the first path shows a 3dB gain over

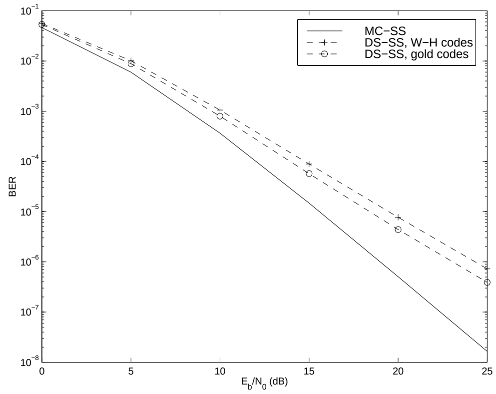


**Fig. 2**. MC-SS versus DS-SS with $P = 8$

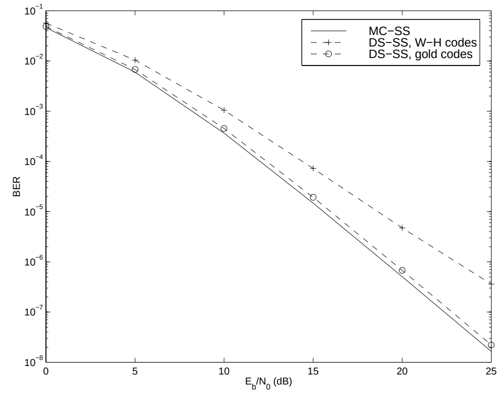


**Fig. 3**. MC-SS versus DS-SS with $P = 16$

the second and 10dB gain over the third path; and channel 3 is adopted from [2] with $\mathbf{R}_{hh} = \text{diag}(1, 0.05, 0.01)/1.06$, i.e., the first path has a 13dB gain over the second and 20dB gain over the third path.

To avoid the code dependence for DS-SS, we adopt the code-hopping scheme of [5] and average the BER over all possible code choices. It is known that W-H codes have poor autocorrelation properties. Therefore, we also employ Gold codes, which have better autocorrelation properties [6]. In Figs. 2 and 3 we compare the BER of MC-SS with the average BER of DS-SS with W-H codes of length $P = 8, 16$ and with Gold codes $P = 7, 15$, respectively. First, we see that MC-SS outperforms DS-SS with W-H codes considerably because the multipath induced self-interference of W-H codes is large. When Gold sequences are employed, we observe that the BER of DS-SS approaches that of MC-SS when the code length increases, as the self-interference becomes relatively smaller and smaller. In Fig. 2, note that MC-SS offers a 4 to 5 dB advantage over DS-SS at BER of $10^{-6}$.

With colored channels, we observe similar results as those in Figs. 2 and 3 for i.i.d channels. We compare in Fig. 4 MC-SS against DS-SS with code length 16 for both channels 2 and 3. Although MC-SS is not optimum (near optimum at high SNR) in these two channel settings, we clearly see that MC-SS outperforms DS-SS alternatives considerably, especially when the spreading codes for DS-SS are not well constructed. In a nutshell, the superiority of MC-SS over DS-SS in the presence of multipath justifies its increasing popularity.

## 4. CONCLUSIONS

We used results from [3] for the optimal coding matrix, and showed that in the case of uncorrelated and equal power paths, the optimal code leads to multi-carrier spread-spectrum (MC-SS) which may significantly outperform direct-sequence spread spectrum (DS-SS). We developed closed-form expressions for the BER performance of digital MC-SS and DS-SS schemes in the presence of frequency-selective multipaths (which destroy code orthogonality). The performance of MC-SS does not depend upon the spreading code; in contrast, the performance of DS-SS does depend upon the spreading code. In general, MC-SS outperforms DS-SS; the performance of DS-SS approaches that of MC-SS if the spreading gain is large and the codes are well chosen. In the case of colored channels (correlated paths and/or paths with unequal powers), MC-SS outperforms DS-SS, especially for short spreading lengths.

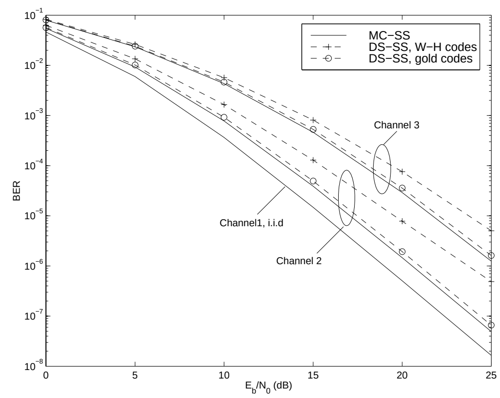


**Fig. 4**. MC-SS vs DS-SS, different channels

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