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Improving Distance Based Geographic Location Techniques in Sensor Networks

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Abstract. Supporting nodes without Global Positioning System (GPS) capability, in wireless ad hoc and sensor networks, has numerous applications in guidance and surveying systems in use today. At issue is that a procedure be available so that the subset of nodes with GPS capability succeed in supporting the maximum possible number of nodes without GPS capability and as a result enable the highest connectivity of the underlying network infrastructure. In this paper, we identify incompleteness in the standard method for computing the position of a node based on three GPS enabled neighbors, in that it may fail to support the maximum possible subset of sensors of the wireless network. We give a new complementary algorithm (the three/two neighbor algorithm) that indeed succeeds in providing a higher fraction of nodes (than the 3-Neighbour algorithm) with their position. We prove its correctness and test its performance with simulations.

1 Introduction

In wireless ad hoc systems, location determination can be an important parameter in reducing information overhead, thus simplifying the distribution of information and limiting infrastructure reliance. Location awareness has proven to be an important component in designing communication algorithms in such systems and there have been many papers making use of this paradigm (e.g., Kranakis et al. [13] Bose et al. [4], Kuhn et al. [14], and Boone et al. [3], to mention a few) thus making possible the execution of *location based* routing using only local (i.e., information in the vicinity of the node) as opposed to global knowledge on the status of the nodes. In addition, guidance and surveying systems in use today have numerous military and civilian applications. The current Global Positioning System (GPS) is satellite based and determines the position of a GPS equipped device using the radiolocation method. However, there are instances where devices may not have GPS capability either because the signal is too weak (due to obstruction) or integration is impossible. Adding to these the fact that such devices are easy to jam and there have been calls to declare the GPS a critical infrastructure.

Typically, sensors so equipped can determine their position with their GPS devices. The remaining nodes have no option but to query neighbors for their location and thus determine their position using radiolocation. There are several radiolocation methods in use (see Bahl et al. [2]) including signal strength, angle of arrival, time of arrival, and time difference of arrival. However, despite the method used at issue is that a procedure be available so that the subset of nodes with GPS capability succeed in supporting the maximum possible number of nodes without GPS capability and as a result enable the highest connectivity of the underlying network infrastructure.

Consider a set S of n sensors in the plane. Further assume that the sensors have the same reachability radius r. This paper addresses the problem of supporting nodes that do not have GPS capability within a sensor network. We assume that all the sensors have identical reachability radius r. We are interested in algorithms that will position the maximum number of sensors in the set S. The paper is organized as follows. In Section 2, we give an overview of radiolocation.

In Section 3, we show first that the usual positioning algorithm that determines the position of a node based on the presence of its three GPS enabled neighbors may fail to determine the position of the maximum number possible of sensors. We give a new algorithm that is based on using the distance one neighborhood. It may outperform the traditional algorithm in the sense that it determines correctly the position of more sensors than the usually used three neighbor algorithm. Later we explore the distance k ($k \geq 1$) neighborhood of a node and derive an algorithm that captures the maximum number of nodes of a sensor system that can compute their geographic position. In Section 4, we explore the performance of these algorithms in a random setting whereby sensors are dropped randomly and independently with the uniform distribution in the interior of a unit square. We investigate what is the impact of the size of the reachability radius r of the sensors as a function of the number of sensors so that with high probability all the sensors in the unit square determine correctly their position. Our approach to improve distance based geographic location techniques is fairly general and can be applied over an existing algorithm like the one proposed by Capkun et al. [9].

2 Overview of Radiolocation

The position of wireless nodes can be determined using one of four basic techniques, namely time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA) or signal strength.

The TOA technique is pictured in Figure 3. Node A is unaware of its position. Three position aware nodes are involved, let us say B_1, B_2, and B_3. Each position aware node B_i sends a message to A and the trip time of the signal is measured. The trip time multiplied by the propagation speed of signals (i.e. the speed of light) yields a distance d_i. The distance $d(A, B_i)$ defines a circle around node $d(A, B_i)$. The position of A is on the circumference of this circle. In a two dimensional model, the position of A is unambiguously determined as the intersection of three such circles.

Trip time measurement from each node B_i to node A requires synchronized and accurate clocks at both locations. If round-trip time is measured instead (halved to obtain trip time), then this requirement is relaxed. No clock synchronization is required and accurate clocks are required only at the B_i's.

TDOA is pictured in Figure 1. Nodes B_1, B_2, and B_3 are aware of their position while node A is not. B_1 and B_2 simultaneously send a signal. Times of arrivals t_1 and t_2 of signals from B_1 and B_2, respectively, are measured by A. Node A has the capability to recover the identity of the sender of a signal and its position. The time difference of arrival is calculated, i.e. $\delta_t = t_2 - t_1$. The time difference δ_t multiplied by the speed of light is mapped to the distance difference δ_d. The position (x_1, y_1) of B_1, position (x_2, y_2) of B_2 and δ_d define a hyperbola h with equation:

$$\sqrt{(x - x_1)^2 + (y - y_1)^2} - \sqrt{(x - x_2)^2 + (y - y_2)^2} = \delta_d \tag{1}$$

The positions of B_1 and B_2 are at the foci of the hyperbola. The position of node A is a solution of Equation 1. The geometrical properties of the hyperbolas are such that all points located on the line of h are of equal time difference δ_t and equal distance difference δ_d.

Two such hyperbolas can be defined by involving two different pairs of nodes (B_1, B_2) and (B_2, B_3) which produce two time differences of arrival δ_1 and δ_2. In a two dimensional model, the observer of the times of arrival δ_1 and δ_2, i.e. node A, is at the position corresponding to the intersection of the two hyperbolas. Note that there are cases in which the two hyperbolas intersect at two points. In these cases, a third independent measurement is required to resolve the ambiguity.

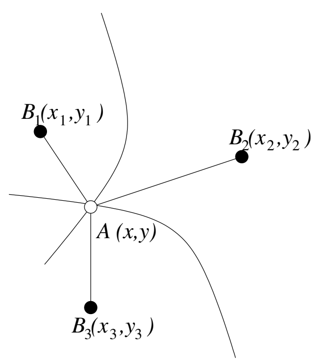

Fig. 1. The TDOA technique.

The AOA technique is pictured in Figure 2. Two position aware nodes, let us say B_1 and B_2, are required to determine the position of a node A. Nodes B_1 and B_2 have to be able to determine the direction from which a signal is coming. This can be achieved with an array antenna [10]. An imaginary line is drawn from B_1

to A and another imaginary line is drawn from B_2 to A. The angle of arrival is defined as the angle that each of these lines make with a line directed towards a common reference. The intersection of these two lines unambiguously determines the position of A. Note however, that if A, B_1, and B_2 all lie on the same straight line, another independent measurement is required to resolve the ambiguity. Accuracy of the AOA technique is largely dependent on the beamwidth of the antennas. According to Pahlavan and Krishnamurthy [15] the TOA technique is superior to the AOA technique. In CDMA cellular networks, Caffery and Stüber [8] come to similar conclusions.

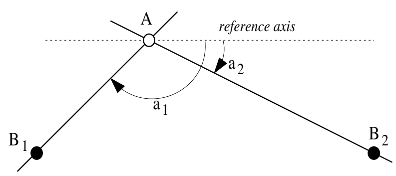

Fig. 2. The AOA technique.

The signal strength based technique exploits the fact that a signal loses its strength as a function of distance. Giving the power of a transmitter and a model of free-space loss, a receiver can determine the distance traveled by a signal. If three different such signals can be received, a receiver can determine its position in a way similar to the TOA technique. Application of this technique for cellular systems has been investigated by Figuel et al. [11] and Hata and Nagatsu [12]. The main criticism of Caffery and Stüber [7] is about the accuracy of the technique. This is due to transmission phenomena such as multi path fading and shadowing that cause important variation in signal strength.

All these techniques require line of sight propagation between the nodes involved in a signal measurement. Line of sight means that a non obstructed imaginary straight line can be drawn between the nodes. In other words, the accuracy is sensitive to radio propagation phenomena such as obstruction, reflection and scattering. With all the distance-based techniques (i.e. TOA, TDOA, signal strength) three position aware neighbors are required to determine the location of a position unware node, in a two dimensional model (e.g. latitude and longitude are determined), and four neighbors are required in a three dimensional model (e.g. altitude is determined as well). In the sequel, we augment the distance-based techniques with an algorithmic component that can resolve ambiguity in a two dimensional model when only two position aware neighbors are available. The ambiguity can also be resolved using knowledge about the trajectory when the wireless nodes are mobile or using the AOA technique. When the nodes are fixed and the technology required to apply the AOA technique is not available, the algorithm described in this paper can be used. We note that a similar algorithm is also possible in a three dimensional model.

3 Computing the Geographic Location

Consider a set S of n sensors in the plane. Further assume that the sensors have the same reachability radius r. We divide the set S of sensors into two subsets. A subset E of S consists of sensors equipped with GPS devices that enable them to determine their location in the plane. The set U of remaining sensors, i.e., $S \setminus E$, consists of sensors not equipped with GPS devices. (In pictures, the former are represented with solid circles and the latter with empty circles.) In this section, we consider the problem of determining the positions of sensors in a sensor network. In the beginning, we review the well-known *three neighbor algorithm* (3-NA) and conclude with an example illustrating why the algorithm does not necessarily compute the positions of the maximum possible number of nodes. Subsequently, we present an improvement, the *three/two neighbor algorithm* (3/2-NA). Essentially, this is an iteration of the 3-NA followed by an algorithm that uses only two neighbors as well as their distance one neighborhood.

3.1 Three neighbor algorithm and its deficiencies

If a sensor at A (see Figure 5) is not equipped with a GPS device, then it can determine its (x, y) coordinates using three neighboring nodes. After receiving

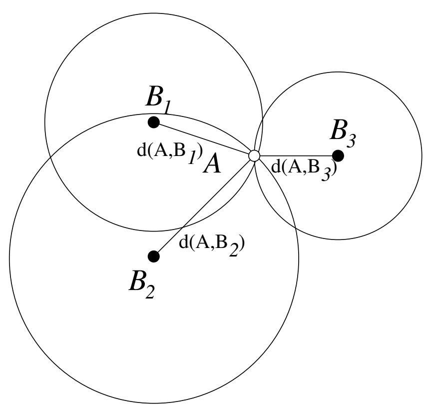

Fig. 3. A sensor at A not equipped with a GPS device can determine its position from the positions of its three neighbors B_1, B_2, B_3.

messages from B_1, B_2, and B_3, that include their position, node A can determine its distance from these nodes using a distance-based radiolocation method. Its position is determined as the point of intersection of three circles centered at B_1, B_2, and B_3 and distances $d(A, B_1), d(A, B_2)$, and $d(A, B_3)$, respectively.

The well-known 3-NA is as follows. Each node that is not equipped with a GPS device sends a position request message. A sensor that knows or can compute its position sends it to all its neighbors. A sensor that receives position messages from three different nodes, say B_1, B_2, and B_3 can calculate its position as in Figure 3.

Computing the position of the maximum number of nodes The 3-NA does not necessarily compute the positions of the maximum possible number of sensors without GPS devices. We illustrate this with a simple example depicted, in Figure 4. The left side of the picture depicts five nodes A, A', B_1, B_2, and C.

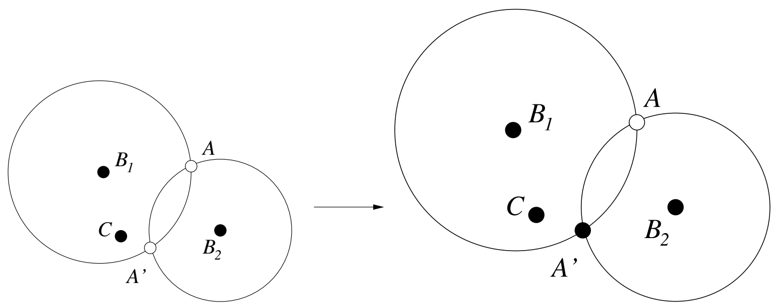

Fig. 4. Application of 3-NA will equip node A' with its (x, y)-coordinates, but it fails to do it with node A.

Node A' is within communication range of nodes B_1, B_2, and C. Node A is within communication range of nodes B_1 and B_2. Node A is out of the range of both nodes C and A'. Assume nodes B_1, B_2 and C know their (x, y)-coordinates. Application of the 3-NA will equip A' with its (x, y) coordinates (because it will receive messages from all three of its neighboring nodes B_1, B_2, C). This is depicted in the right side of the picture in Figure 4. However, node A will never receive three position messages and therefore will never be able to discover its (x, y)-coordinates. Nevertheless, we will see in the next algorithm that node A can indeed discover its position if additional information (i.e., the distance one neighborhood of its neighbors) is provided.

3.2 Three/Two neighbor algorithm

We now consider an extension of the 3-NA for the case where all the sensors have exactly the same reachability radius, say r.

On utilizing the distance one neighborhood Assume that we have concluded the execution of the 3-NA. For each node P, let $N(P)$ be the set of neighbors of P, i.e., the set of sensors within communication range of P. Suppose we have two nodes B_1 and B_2 (depicted in Figure 5) that know their position. The solid circles are determined by the reachability radius of the nodes. The

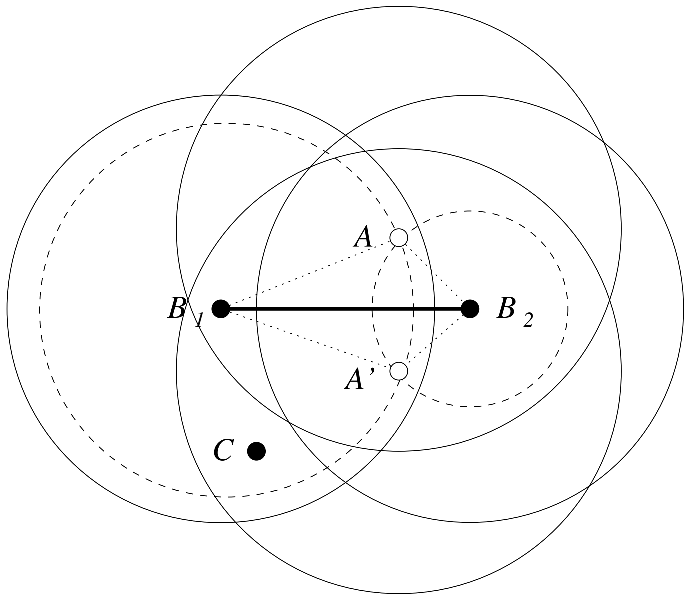

Fig. 5. A sensor not equipped with a GPS device can determine its coordinates using the positions of two neighbors B_1 and B_2 and their neighborhood information.

dashed circles are centered at B_1 and B_2 respectively. After using radiolocation they specify that the inquiring node must be located at one of their points of intersection (in this case either A or A'). Further assume that a given node X receives the positions of nodes B_1 and B_2 by radiolocation. On the basis of this information, X may be located in either position A or A'.

Lemma 1. *Suppose that both nodes A and A' receive from B_1 and B_2 the set $N(B_1) \cup N(B_2)$. If there is a sensor $C \in (N(B_1) \cup N(B_2)) \cap (N(A) \cup N(A'))$ that knows its position then both nodes A and A' can determine their position.*

Proof (of Lemma 1). We must consider two cases. In the first case, assume C is within range of both nodes A and A'. In this case the two nodes will receive position messages from all three nodes B_1, B_2, and C and can therefore determine their position. In the second case, assume C is within range of only one of the two nodes. Without loss of generality assume it is within range of A' but outside the range of A, i.e., $d(C, A') \leq r < d(C, A)$ (see Figure 5). Then A' can determine its position. Also, A can determine its position because it knows it must occupy one of the two positions A or A' and also can determine it cannot be node A' since its distance from C (a node whose position it has received) is outside its range. This completes the proof of Lemma 1. ■

It is now clear that Lemma 1 gives rise to the following 3/2-NA for computing sensor positions.

3/2-NA (Three/Two Neighbor Algorithm):

1. For each node as long as new nodes determine their position **do**
2. Each sensor executes the 3-NA algorithm and also collects the coordinates of all its neighbors.
3. **If** at the end of the execution of this algorithm a sensor has received the coordinates from only two neighbors, say B_1, B_2 **then**
   (a) it computes the distances from its current location to the sensors B_1, B_2 and also computes the coordinates of the two points of intersection A, A' of the circles centered at B_1, B_2, respectively;
   (b) it requests the coordinates of all the neighbors of B_1 and B_2 that are aware of their coordinates;
   (c) **if** $N(B_1) \cup N(B_1) \neq \emptyset$ then take any node $C \in N(B_1) \cup N(B_1)$ and compute $d(C, A), d(C, A')$; then the sensor occupies the position $X \in \{A, A'\}$ such that $d(X, C) > r$;

For any node P let $D(P; r)$ be the disc centered at P and with radius r, i.e., the set of points X such that $d(P, X) \leq r$. We can prove the following theorem.

Theorem 1. *The Three/Two Neighbor Algorithm terminates in at most* $(n-e)^2$ *steps, where* e *is the number of GPS equipped nodes.*

Proof of Theorem 1. First consider the three neighbor algorithm. At each iteration of the algorithm a sensor either waits until it receives three messages (of distances of its neighbors), or else it receives the coordinates of at least three neighboring sensors, in which case it computes its own coordinates and forwards it to all its neighbors. Let E_t be the number of sensors that know their coordinates by time t. Observe that initially $E_0 = E$ and $E_t \subseteq E_{t+1}$. If at any given time no new sensors are not GPS-equipped determines their coordinates (i.e., $E_t = E_{t+1}$) then no new non-equipped sensor will ever be added. It follows that the algorithm terminates in at most $n - e$ steps. After this step no new sensors will be equipped with their coordinates.

Now consider the three/two neighbor algorithm. Concerning correctness we argue as follows. Consider a sensor as above that has received position messages only from two neighbors, say B_1 and B_2. The sensor knows it is located at one of the points of intersection of the two circles (see Figure 5). Since $N(B_1) \cup N(B_2) \neq \emptyset$ and the sensor received no position message from any sensor in $N(B_1) \cup N(B_2)$ after the execution of the 3-NA algorithm it concludes that it must occupy position X, where $X \in \{A, A'\}$ such that $d(C, X) > r$. Since within each iteration at one new sensor computes its position the running time is as claimed. This completes the proof of Theorem 1. ■

Remark 1. There are several interesting issues arising in the algorithm 3/2-NA.

1. The sensors may not know the total number of sensors in the network participating in the positioning algorithm. In this case, they may have to guess an upper bound value n' and use this to execute the above algorithm or even execute the algorithm by increasing incrementally the value n'. The running time stated in Theorem 1 pre-supposes that the sensors know the value $n-e$.
2. The above algorithm will take a lot of messages. To increase efficiency, it may be a good idea that sensors localize their search within limited geographic boundaries.

3.3 Beyond distance one neighborhood

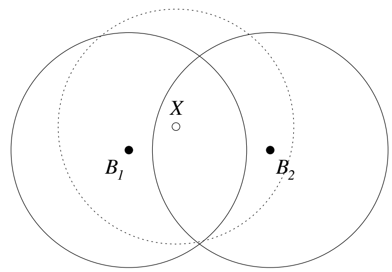

Fig. 6. The termination condition for the 3/2-NA.

When the 3/2-NA terminates no new node can compute its position. There is an improvement that can be made to the 3/2-NA. In Figure 5, this may happen when for some $k \geq 1$ the node C is at distance k hops from either B_1 or B_2. If the entire distance k neighborhood is being transmitted and C is within A''s range but outside A's range, then the node A can determine its position. This gives rise to the 3/2-NA(k) algorithm which is similar 3/2-NA algorithm except that now the nodes transmit their distance k neighborhood.

We define the distance k hop(s) neighborhood of a node P as follows. First, $N_1(P)$ is defined as $N(P)$. For $k = 2, 3, 4, \ldots$,

$$N_k(P) = \{A | A \in N_{k-1}(P) \vee \exists B \in N_{k-1}(P) \wedge A \in N(B)\}$$

The specific algorithm is as follows:

3/2-NA(k) (Three/Two Neighbor Algorithm):

1. For each node as long as new nodes determine their position **do**
2. Each sensor executes the 3-NA algorithm and also collects the coordinates of all its neighbors.
3. **If** at the end of the execution of this algorithm a sensor has received the coordinates from only two neighbors, say B_1, B_2 **then**

(a) it computes the distances from its current location to the sensors B_1, B_2 and also computes the coordinates of the two points of intersection A, A' of the circles centered at B_1, B_2, respectively;
(b) it requests the coordinates of all the neighbors of B_1 and B_2 that are aware of their coordinates;
(c) **if** $N_k(B_1) \cup N_k(B_1) \neq \emptyset$ then take any node $C \in N_k(B_1) \cup N_k(B_1)$ and compute $d(C, A), d(C, A')$; then the sensor occupies the position $X \in \{A, A'\}$ such that $d(X, C) > r$;

4 Simulations

Our approach is sufficiently general and may augment any distance-based geographic location method in use. For example, Capkun et al. [9] propose a GPS-free positioning algorithm for mobile ad hoc networks. In general, their algorithm succeeds in providing a common reference only for a subset of the total number of nodes. In this section, we provide simulations which confirm that our method increases the percentage of nodes that can compute their geographic location in an arbitrary sensor network.

4.1 Connectivity and coverage in random setting

Assume that n sensors (here assumed to be omnidirectional antennas) are dropped randomly and independently with the uniform distribution on the interior of a unit square. For any integer $k \geq 0$ and real number constant c let the sensors

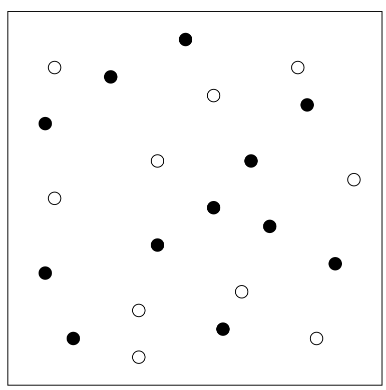

Fig. 7. Sensors dropped randomly and independently over a unit square region. Bolds (respectively, empty) circles denote sensors which are equipped (respectively, not equipped) with GPS devices.

have identical radius r, given by the formula

$$r = \sqrt{\frac{\ln n + k \ln\ln n + \ln(k!) + c}{n\pi}}. \tag{2}$$

A network is called k-connected if it cannot be disconnected after the removal of $k-1$ nodes. Then using the main result of Penrose [16, 17] we conclude that this is a threshold value for k-connectivity. Namely, we have the following theorem.

Theorem 2. *Consider sensors with reachability radius r given by Formula 2, and suppose that $k \geq 0$ is an integer and c is a real. Assume n sensors are dropped randomly and independently with the uniform distribution on the interior of a unit square. Then*

$$\begin{array}{l} \lim_{n\to\infty} \Pr[\textit{sensor network is } (k+1)\textit{-connected}] = e^{-e^{-c}}, \textit{ and} \\ \lim_{n\to\infty} \Pr[(k+1) \textit{ is the } \min \textit{ degree of the sensor network}] = e^{-e^{-c}}. \end{array} \tag{3}$$

■

Thus, for the radius chosen by Formula 2 not only is the network $(k+1)$-connected but within distance r, each node will have $k+1$ neighbors with high probability as indicated by Equations 3.

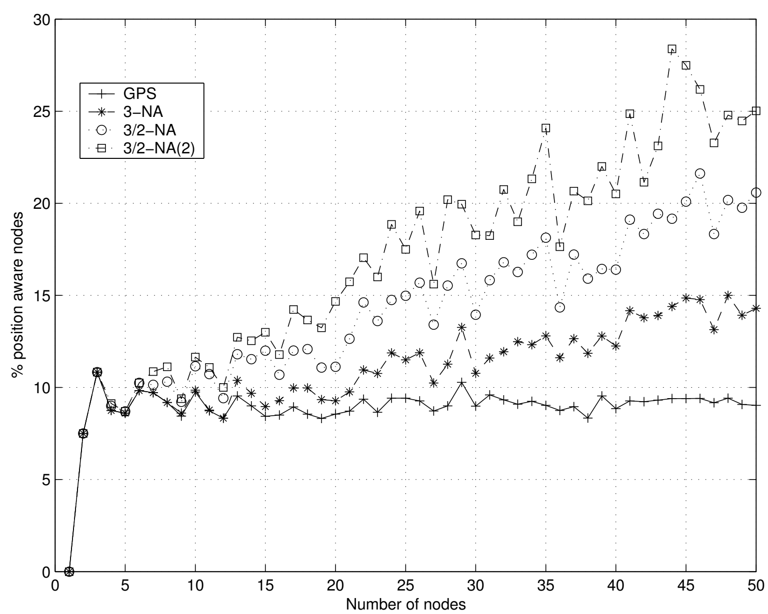

Fig. 8. 9% of the sensors are GPS equipped.

4.2 Experimental results

A simulation of the algorithms has been conducted. Sensors are spread randomly and independently with uniform distribution on a unit square. The communication range of each sensor is a circle centered at its position and of radius r as defined by Formula 2. Constants k and c are both set to value 1. If the number of nodes equipped with GPS devices is dense (as a proportion to the total), then we expect that with high probability every node will have three neighbors that are equipped with GPS devices. Therefore, the standard 3-NA algorithm is expected to enable all nodes to compute their position, with high probability. Therefore incremental differences will be more substantial in a sparse setting. Figure 8 pictures the results of the simulation of one to 50-sensor networks. An average of 9% of the sensors are GPS equipped and can determine their position independently of other sensors. Application the 3-NA, 3/2-NA (using one-hop neighbors only) and 3/2-NA(2) (using one or two-hop neighbors) all make an additional number of sensors aware of their position, up to 5% of the total number of sensors in each case. The simulation was run for 200 times for each network size.

Figure 9 pictures the results of the simulation of one to 50-sensor networks. An average of 28% of the sensors are GPS equipped and can determine their position independently of other sensors. The simulation was run for 200 times for each network size.

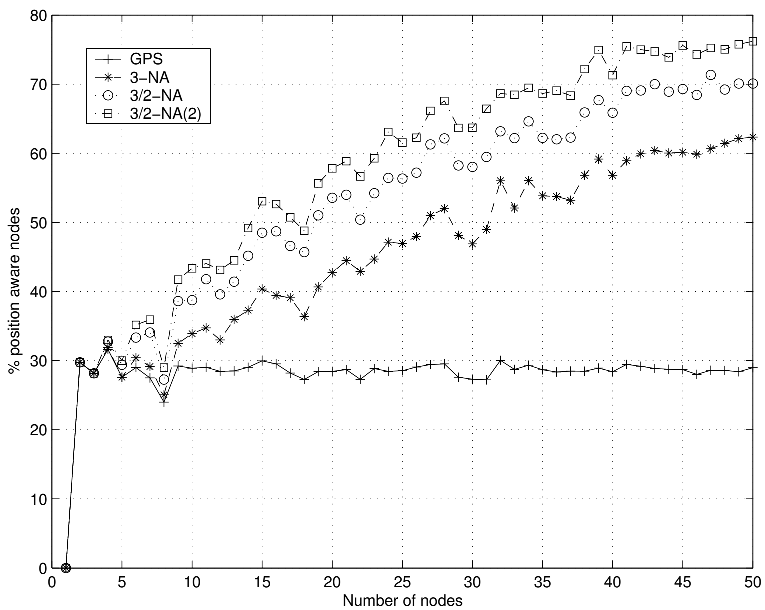

Fig. 9. 28% of the sensors are GPS equipped.

Application of the 3-NA makes the number of sensors aware of their position up to the double of the number of GPS equipped sensors. This is consistent with intuition since, with respect to Figure 8, more nodes are available to resolve loci of position unaware sensors. In addition, up to 10% and 7% more sensors can resolve their position using respectively the 3/2-NA (using one-hop neighbors only) and 3/2-NA(2) (using one or two-hop neighbors).

In both cases, the 3/2-NA increase significantly the number of sensors aware of their position.

5 Conclusion

In this paper, we have considered a new class of algorithms for improving any distance based geographic location method. Our technique may augment any existing algorithm by iterating a three-neighbor with a two-neighbor based calculation as well a taking into account the distance k neighbors of the given node. Our simulations show that 5% to 10% more sensors can resolve their position using the 3/2-NA and 3/2-NA(2).

Our analysis focused only on the two dimensional plane. However, it can be easily extended to the three dimensional space. In this case, the intersection of the spheres defined by the distance from four neighbors is required in order to determine the location of a node. The intersection of three spheres alone creates an ambiguity which can be resolved by looking at the distance k neighbors, just like the two dimensional case.

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