# Modeling of the Caltech Ducted Fan


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## 1 Introduction

This note details some of the modeling that went into the revitalization of the CDS 111 ducted fan experiment for the Spring quarter, 2000.

The Caltech Ducted Fan is a small flight control experiment whose dynamics are representative of either a Harrier in hover mode or a thrust vectored aircraft (such as the F18-HARV or X-31) in forward flight. A diagram of the experimental setup for the ducted fan is shown in Figure 1. It consists of a ducted fan engine with a high-efficiency electric motor and 6-inch diameter blade, capable of generating up to 9 Newtons of

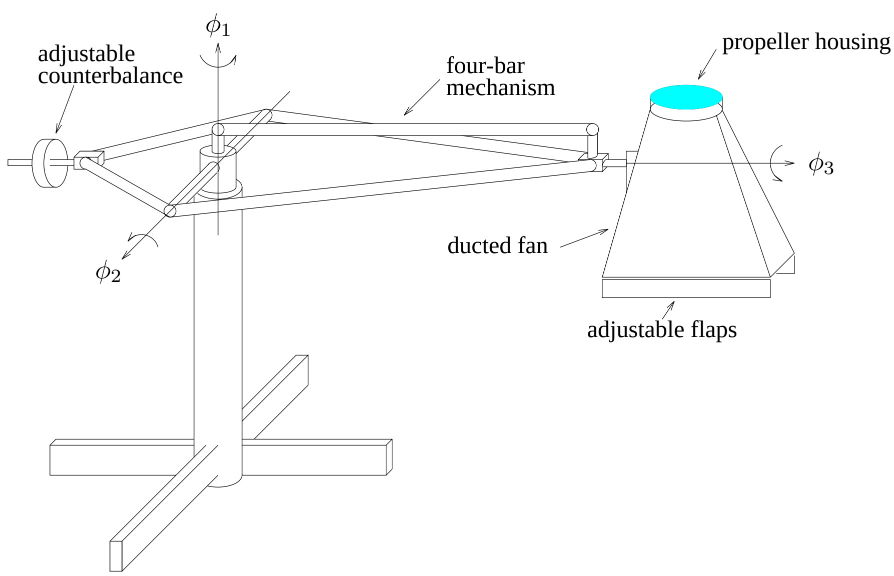


Figure 1: Caltech ducted fan with support stand.

thrust. Flaps on the fan allow the thrust to be vectored from side to side and even reversed. The engine is mounted on a three degree of freedom stand which allows horizontal and vertical translation as well as unrestricted pitch angle.

This system has been used for a number of studies and papers. A description of the overall design and control considerations is given in [2]. A comparison of several different linear and nonlinear controllers was performed by Kantner et al.[3] and a more focused comparison on LPV controllers is given in [1]. The application of differential flatness based controllers is reported in [4]. A document regarding the CDS 111 control design team efforts (Spring, 2000) is also available [1].

In this note we describe two different models for the ducted fan dynamics, the first being simpler but less accurate. For implementation, the models are constructed and simulated in the program Dymola [2]. Dymola models can be converted to the language Modelica [3], which provides an object-oriented approach to simulating physical systems (The conversion to Modelica is for future applications, as Dymola will ultimately be phased in favor of Modelica, which shows potential as a standard in modeling physical systems). We give parameter values corresponding to the current experimental setup and describe the methods used to obtain the values. Lastly, instructions are given to simulate the Dymola models and convert them into s-functions for implementation in Matlab Simulink.

# 2 The planar model

We begin with a simple planar model, shown in Figure 2, that ignores the stand dynamics. This model is useful for determining basic characteristics of the model and testing initial control designs. The equations of motion are first derived and the linearization given. The model in the Dymola software is documented in HTML format and can be found on the CDS 111 page (see [1]).

## 2.1 Equations of motion (analytic model)

Let $(x, y, \theta)$ denote the position and orientation of a point on the main axis of the fan that is a distance $l$ from the center of mass. (This point is where the stand would normally attach and hence it is useful to write the equations of motion in terms of the coordinates of this point rather than the center of mass.) We assume that the forces acting on the fan consist of a force $f_1$ perpendicular to the axis of the fan acting at a distance $r$ and a force $f_2$ parallel to the axis of the fan. Let $m$ be the (inertial) mass

[1] go to http://www.cds.caltech.edu/~murray/courses/cds111-s00

[2] go to http://www.dynasim.se/

[3] go to http://www.modelica.org/

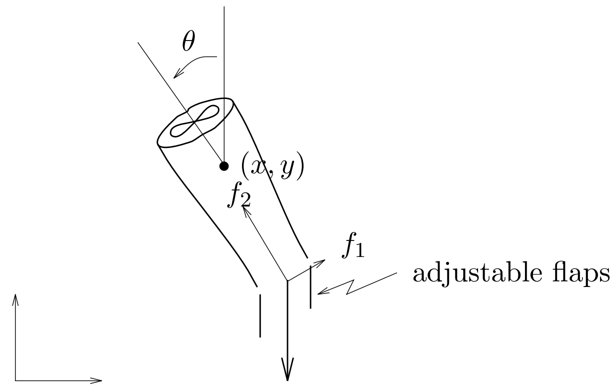


Figure 2: Planar ducted fan model.

of the fan, $J$ the mass moment of inertia, $g$ the gravitational constant, and $c_{d,x}(\theta, \dot{x})$ a function which models any drag and friction as a function of orientation. Then the equations of motion for the fan are given by:

$$
\begin{aligned}
m\ddot{x} &= f_1 \cos\theta - f_2 \sin\theta - c_{d,x}(\theta, \dot{x}) \\
m\ddot{y} &= f_1 \sin\theta + f_2 \cos\theta - mg - c_{d,y}(\theta, \dot{y}) \\
J\ddot{\theta} &= r f_1 - m_f g l \sin\theta - c_{d,\theta}(\theta, \dot{\theta})
\end{aligned}
\tag{1}
$$

where $m_f$ is the gravitational mass of the fan. It is convenient to redefine the inputs so that the origin is an equilibrium point of the system with zero input. If we let $u_1 = f_1$ and $u_2 = f_2 - m_s g$, where $m_s$ is the mass of the fan attached to the boom and counter weight, and model the drag terms as viscous friction, then the equations become

$$
\begin{aligned}
m\ddot{x} &= -m_s g \sin\theta - d_x \dot{x} + u_1 \cos\theta - u_2 \sin\theta \\
m\ddot{y} &= m_s g(\cos\theta - 1) - d_y \dot{y} + u_1 \sin\theta + u_2 \cos\theta \\
J\ddot{\theta} &= -m_f g l \sin\theta - d_\theta \dot{\theta} + r u_1.
\end{aligned}
\tag{2}
$$

These equations are referred to as the *planar ducted fan equations.*

For the Caltech ducted fan, representative values of the parameters are listed in Table 1. The maximum force that can be generated by the fan is approximately 10 Newtons in the axial direction and 2 Newtons in the sideways direction. A quick calculation shows that the fan cannot support its own weight with these limits on the maximum thrust, and hence a counterweight is used to offset part of the weight of the fan. This can be roughly incorporated into the dynamics given above by limiting the axial thrust to 5 Newtons instead of 10.

| Symbol | Description | Value | |
|---|---|---|---|
| $m$ | inertial mass of fan | 8.5 | kg |
| $\alpha$ | inertial mass of fan, x-axis | 8.62 | kg |
| $\beta$ | inertial mass of fan, y-axis | 8.33 | kg |
| $m_s$ | scale measured mass of fan (attached) | 0.38 | kg |
| $m_f$ | (gravitational) mass of fan | 2.25 | kg |
| $J$ | fan moment of inertia, $\phi_3$ axis | 0.0486 | kg m$^2$ |
| $r$ | nominal distance of flaps from fan pivot | 26.0 | cm |
| $l$ | center of mass offset for fan | 2.3 | cm |
| $d_x$ | viscous friction coefficient, x-direction | 0.3431 | kg/sec |
| $d_y$ | viscous friction coefficient, y-direction | 1.5623 | kg/sec |
| $d_\theta$ | viscous friction coefficient, $\theta$-direction | 0.00344 | N m sec |
| $g$ | gravitational constant | 9.81 | m/sec$^2$ |

Table 1: Parameter values for the planar ducted fan model.

The inertial mass $m$ is different about the x- and y-axes. Here, $m$ is computed as the average of the inertial masses of the fan about the x and y-axes, given by

$$m = \frac{1}{2}(\alpha + \beta), \quad \text{where} \quad \alpha = \frac{J_1 + (m_f r_f^2 + m_c r_c^2 + m_b r_b^2)}{r_f^2}$$
$$\text{and} \quad \beta = \frac{J_2 + m_f(r_f^2 + h^2) + m_b(r_b^2 + h^2) + m_c(r_c^2 + h^2)}{r_f^2},$$

where $J_1$ and $J_2$ are the moments of inertia of the fan/boom/counter weight system about $\phi_1$ and $\phi_2$ axes, respectively. It is likely more accurate to use $\alpha$ and $\beta$ as the $m$ term in the x and y dynamic equations, respectively. The viscous friction coefficients are computed as

$$d_x = \frac{b_1}{r_f^2}, \quad d_y = \frac{b_2}{r_f^2} \quad \text{and} \quad d_\theta = b_3.$$

Constants required for these calculations are defined in Table 2.

The linearization of the planar ducted fan model (2) provides a first approximation of the dynamics for testing initial control designs. The state space matrices that result from the linearization are provided in the data file `LinHov.mat` [4]. Note this file was revised (5/25/00) to use the individual inertial mass values in the x and y dynamic equations, rather than using an averaged inertial mass in both equations.

[4]go to http://www.cds.caltech.edu/~dunbar

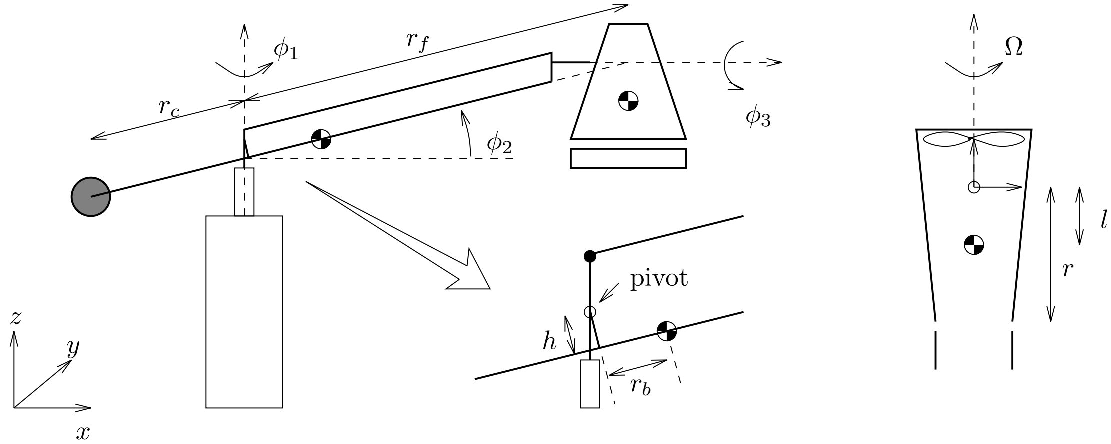


Figure 3: Ducted fan with simplified model of stand.

## 2.2 Planar ducted fan Dymola model

Although the planar ducted fan dynamics are derived analytically, writing the model also in Dymola serves as an example of how the software will be used later to include the more complicated stand dynamics. See [4] for documentation.

# 3 Ducted fan and stand model in Dymola

A much more accurate description of the dynamics is available by including some of the essential stand dynamics. We consider the simplified version of the stand shown in Figure 3. This model ignores some of the details of the stand geometry but captures most of the important effects. The main difference between this model and the previous model is the effect of the fan counterweight, which changes the effective mass and weight of the fan. In addition, due to the kinematics of the stand, the inertial of the fan about the vertical axis varies as a function of the altitude of the fan.

The three angular degrees of freedom are $\phi_1$, $\phi_2$ and $\phi_3$. Let $(r_f, m_f)$ be the distance from the stands vertical axis to the fan and the mass of the fan, respectively, and let $(r_c, m_c)$ be the distance and mass of the counterweight. We also incorporate the mass of the supporting strut, which we take to have mass $m_b$ and center of mass $r_b$ from the vertical axis. We assume that the boom (connecting the fan to the counterweight) has a center of along a line between the fan and the counterweight and that this line is a distance $h$ below the pivot point for the altitude axis. Figure 3 details the location of the center of mass of the fan with respect to the pin connection to the boom (with implied symmetry). It should be clear that only the fan itself is free to move in any of the three angular directions, while the boom and counter weight are constrained to the

$\phi_1$, $\phi_2$ directions.

The model in Dymola treats the counter weight, boom (four-bar linkage) and fan as three objects with inertial properties located at their centers of mass. As detailed in Section 4, these inertial properties were therefore identified. The HTML documentation on the Dymola model is available [5]. Note that the (nontrivial) fan blade/motor inertia is accounted for in the Dymola model as well.

# 4 New parameter identification of ducted fan

This section reports the methods used for identifying the parameters, and their values, required for modeling the ducted fan and stand in Dymola.

## 4.1 Collecting data

Initially, the fan was not observed to statically hang with symmetry in the vertical plane perpendicular to the boom axis. Therefore a small counter mass was added to the fan frame so as to reduce this asymmetry, as the modeling assumes symmetry. The fan was then removed from the boom and its mass was measured. The mass of the counter weight was also measured. The mass of the boom was taken as the value given in the old modeling document [6].

The fan was reconnected and the locations of the mass centers of the fan (assuming symmetry) and the counter weight were measured. The location of the boom mass center (assumed on the axis of fan and counter weight connection) was determined by statically measuring the mass of the fan end of the system at a measured distance and solving a moment balance equation. The quantities $r$ and $l$ were also identified by moment equations about the pinned axis and measuring mass at a measured distances. The value of $h$ was taken from the old modeling document.

For dynamic measurements, two translational springs were required. Their spring coefficients were estimated by statically measuring deflections of known masses. The two springs were henceforth assumed to behave linearly. For reference, their coefficients were estimated to be 285 N/m.

The geometry of the counter weight as a cylinder permitted the use of analytic expressions for it's mass moments of inertia. Namely,

$$J_1^c = (\frac{1}{4}m_c\bar{r}^2 + \frac{1}{12}m_c l_c^2) + m_c r_c^2 \;=\; \bar{J}_1^c + m_c r_c^2, \quad J_2^c = J_1^c \quad \text{and} \quad \bar{J}_2^c = \bar{J}_1^c,$$
$$J_3^c = \frac{1}{2}m_c\bar{r}^2, \quad \text{where} \quad \bar{r} = \text{radius}, \quad l_c = \text{length}.$$

[5]go to http://www.cds.caltech.edu/˜murray/courses/cds111-s00
[6]http://www.cds.caltech.edu/˜murray/courses/cds111-s98/

The $J_i$ values are about $\phi_i$ rotation axes.

To identify the remaing inertial properties of the fan and boom, as well as the viscous friction coefficients, dynamic measurements were taken. For the following set-ups, step responses of the full ducted fan system were collected:

1. Constrain to $\phi_3$ motion, i.e. underdamped oscillation of fan itself to step response.

2. Constrained to $\phi_1$ motion with springs in tension perpendicular to boom axis, located 123 cm from origin.

3. Same as 2. with the fan removed.

4. Constrained to $\phi_2$ motion with one spring in tension above and perpendicular to the boom axis, located 123 cm from origin. A mass was hung from the boom at 117 cm from origin to get better static deflection in the spring.

5. Same as 4. with fan removed, move spring below the boom at 117cm from origin and additional mass removed.

## 4.2 Freq and Damp Estimation (`freqdamp2.m`)

For the step responses collected for the set-ups described above, the mass moment of inertia and damping can be identified using the m-file `freqdamp2.m` in Matlab. The files `IDrun.m` and `IDrun2.m` detail the equations that model the step responses for each set-up and apply `freqdamp2.m`. These file are currently available [7].

### 4.2.1 Tabulated Results

The parameter values for the ducted fan and stand (boom and counter weight) are given in Table 2. All mass moment of inertia values are computed about the corresponding center of mass of each object. Although the $\bar{}$ notation was used to denote moment of inertia values at the center of mass in the last subsection, the bar is omitted in the table.

[7] http://www.cds.caltech.edu/~dunbar/

| Symbol | Description | Value | |
|---|---|---|---|
| $r_f$ | distance of fan center of mass from $\phi_2$ axis | 143.7 | cm |
| $r_c$ | distance of counter weight from $\phi_2$ axis | 49.8 | cm |
| $r_b$ | offset of boom center of mass | 20 | cm |
| $r$ | nominal distance of flaps from fan pivot | 26.0 | cm |
| $h$ | distance of boom axis from pivot point | 4.30 | cm |
| $m_f$ | mass of fan | 2.25 | kg |
| $m_c$ | counter weight mass | 6.43 | kg |
| $m_b$ | mass of boom | 2.87 | kg |
| $l$ | center of mass offset for fan | 2.30 | cm |
| $J_1^f$ * | fan mass moment of inertia | 1.784 | kg m$^2$ |
| $J_2^f$ | fan mass moment of inertia | 0.303 | kg m$^2$ |
| $J_3^f$ | fan mass moment of inertia | 0.0486 | kg m$^2$ |
| $J_1^c$ | counter weight mass moment of inertia | 0.01057 | kg m$^2$ |
| $J_2^c$ | counter weight mass moment of inertia | 0.01057 | kg m$^2$ |
| $J_3^c$ | counter weight mass moment of inertia | 0.0130 | kg m$^2$ |
| $J_1^b$ | boom mass moment of inertia | 3.291 | kg m$^2$ |
| $J_2^b$ | boom mass moment of inertia | 0.3027 | kg m$^2$ |
| $J_3^b$ | boom mass moment of inertia | 0.0486 | kg m$^2$ |
| $b_1$ | viscous friction coefficient, $\phi_1$ axis | 0.7085 | N m sec |
| $b_2$ | viscous friction coefficient, $\phi_2$ axis | 3.226 | N m sec |
| $b_3$ | viscous friction coefficient, $\phi_3$ axis | 0.00344 | N m sec |
| $J_m$ | inertia of motor and blade | $0.166 \times 10^{-3}$ | kg m$^2$ |
| $\Omega$ | angular speed of motor | 1047 | rad/sec |
| $g$ | gravitational constant | 9.81 | m/sec$^2$ |

Table 2: Parameter values for Caltech ducted fan (* - $J_i$ values at center of mass about $\phi_i$ rotation axis)

# 5 Simulating Dymola models

To open Dymola, execute Program → Dymola → Dymola Language → Dymola from the start menu on the PC. Note that Program → Dymola → Modelica Language → Dymola simulates Modelica models. Go to the Untitled model window and select File → OpenModel. The subfolder E:\Dymola\cds111 contains the models `planarDF.dym` and `ductedfan.dym`.

more to come

## 5.1 Export model to s-fuction implementation in Simulink

more to come

# References


[1] B. Bodenheimer, P. Bendotti, and M. Kantner. Linear parameter-varying control of a ducted fan engine. *International Journal of Robust and Nonlinear Control*, 1996. (to appear).

[2] H. Choi, P. Sturdza, and R. M. Murray. Design and construction of a small ducted fan engine for nonlinear control experiments. In *Proc. American Control Conference*, pages 2618–2622, 1994.

[3] M. Kantner, B. Bodenheimer, P. Bendotti, and R. M. Murray. An experimental comparison of controllers for a vectored thrust, ducted fan engine. In *Proc. American Control Conference*, 1995.

[4] M. van Nieuwstadt and R. M. Murray. Real time trajectory generation for differentially flat systems. In *Proc. IFAC World Congress*, 1996.