Joint Space-Time Interference Cancellation and Channel Shortening

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Abstract— **In this paper, we present a framework for the design of space-time equalizers that perform joint interference cancellation and channel shortening in multi-user multiple-input multiple-output frequency-selective channels. The space-time filter coefficients and the shortened channel vectors are jointly optimized to minimize the interference-plus-noise using training data. The receiver design is completed by using an appropriate Viterbi equalizer in the second stage for inter-symbol interference equalization. The design is adapted to two different detection schemes—independent detection and ordered successive interference cancellation. Simulation results show acceptable symbol error performance with both detection schemes.**

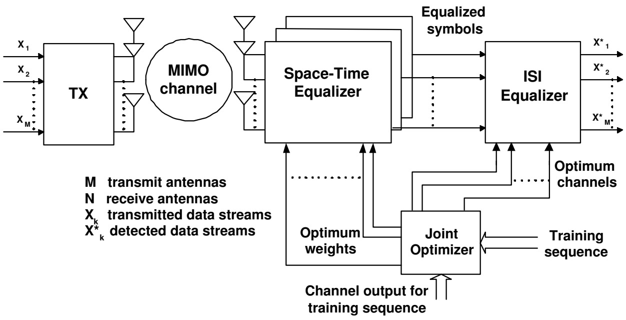

Fig. 1. Schematic for the two-stage receive structure

I. Introduction

In this paper, we consider a multi-user multiple-input multiple-output (MU-MIMO) communication system with frequency-selective channels. The receiver is assumed to be interested in the detection of one desired user, transmitting independent data streams on each of its transmit antennas. In this scenario, the receiver needs to suppress coantenna interference (CAI) from the different transmit streams, cochannel interference (CCI) induced by the multi-user environment, and inter-symbol interference (ISI). Various approaches to deal with CCI cancellation and ISI equalization have been suggested over the years. However, there are no practical receiver designs for a MU-MIMO system that can suppress CAI as well as CCI besides equalizing ISI.

A flexible approach for CAI cancellation and ISI equalization for multi-user single-input multiple-output (MU-SIMO) systems has been suggested in [1]. It presents a two-stage approach for separate CCI reduction and ISI equalization in slow frequency-selective channels. The channel output is received by an array of receive antennas, followed by a space-time filter used for CCI cancellation. A single-user Viterbi equalizer performs ISI equalization in the second stage. This approach can be extended to support MIMO users as shown in Figure 1, but the complexity of the Viterbi equalizer grows exponentially with the number of transmit antennas and the channel memory. One solution lies in reducing the effective channel memory using a pre-equalizer. This brings us into the realm of channel shortening, which has received extensive attention in the discrete multi-tone (DMT) transceivers and multicarrier systems literature [2] and [3]. Channel shortening results in noise coloring and the authors in [4] proposed a power complementarity constraint for broadband beamformers to preserve the whiteness of the channel noise. MIMO equalizers for channel shortening have been investigated in [5] and [6], but there was no attempt to reduce CAI/CCI.

In this paper, we present a framework for the design of space-time equalizers that perform joint CAI/CCI cancellation and channel shortening for MIMO channels using training data and use the same framework to adapt the design to two different detection schemes. The receiver design is completed by using an appropriate Viterbi equalizer in the ensuing stage for ISI equalization. A direct training data-based method is used instead of first estimating the channel with training sequences and then estimat-

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ing the equalizer coefficients to reduce the two-stage propagation of estimation error.

The paper is organized as follows. The signal model is introduced in Section II. The problem is formulated in terms of a cost function in Section III and the solutions are presented in Section IV. The performance of the receiver is demonstrated in Section V.

II. MIMO Signal Model

We consider one desired user (source) with M_t transmit antennas, F MIMO interferers and one receiver with M_r receive antennas in a slow frequency-selective channel. The data model is first developed by including only the desired user and then it is extended to accommodate the F interferers. The length of the channel between each transmit antenna of the desired user and each receive antenna, which includes the pulse-shaping filter, propagation channel and receive filters, is assumed to be equal to $Q+1$. The sample of the received signal at the m_rth receive antenna at time instant k, $r_k^{m_r}$ is

$$r_k^{m_r} = \sum_{m_t=1}^{M_t} \sum_{q=0}^{Q} h_q^{m_r,m_t} x_{k-q}^{m_t} + n_k^{m_r} \tag{1}$$

where $x_k^{m_t}$ is the data symbol transmitted from the m_tth antenna at the kth instant, $h_q^{m_r,m_t}$ is the channel of memory Q between the m_tth transmit antenna and the m_rth receive antenna and $n_k^{m_r}$ is the additive white Gaussian noise (AWGN) added by the channel to the m_rth output. The symbol duration and the symbol power (or energy) are assumed to be 1 for simplicity, i.e, $\mathcal{E}[x_k x_k^*] = 1$. The n_k's are independent and identically distributed (i.i.d.) with distribution $\mathcal{CN}(0, \sigma_n^2)$ and are uncorrelated with the input symbols. If the samples received at the m_rth antenna over a block of K_e+1 symbol periods are stacked together to form a tall vector, (1) can be rewritten as

$$\underline{\mathbf{r}}_k^{m_r} = \underline{\mathbf{H}}^{m_r} \underline{\mathbf{x}}_k + \underline{\mathbf{n}}_k^{m_r} \tag{2}$$

where $\underline{\mathbf{r}}_k^{m_r} = [r_k^{m_r} \; r_{k-1}^{m_r} \; \cdots \; r_{k-K_e}^{m_r}]^T$. This is the receive vector at the m_rth antenna consisting of $r_k^{m_r}$ and its K_e delayed versions. The corresponding noise vector, $\underline{\mathbf{n}}_k^{m_r}$, is $[n_k^{m_r} \; n_{k-1}^{m_r} \; \cdots \; n_{k-K_e}^{m_r}]^T$. To create $\underline{\mathbf{x}}_k$, the $Q+K_e$ previously transmitted data symbols from each transmit antenna below $x_k^{m_t}$ are stacked to form $\mathbf{x}_k^{m_t} = [x_k^{m_t} \; x_{k-1}^{m_t} \; \cdots \; x_{k-Q-K_e}^{m_t}]^T$, $m_t = 1, \ldots, M_t$, and then these vectors for all transmit antennas are concatenated one below the other to form a single column vector, $\underline{\mathbf{x}}_k = [(\mathbf{x}_k^1)^T \; (\mathbf{x}_k^2)^T \; \cdots \; (\mathbf{x}_k^{M_t})^T]^T$. The channel matrix, $\underline{\mathbf{H}}^{m_r}$, is

$$\begin{bmatrix} \mathbf{h}_0^{m_r} \; 0 \; \cdots \; 0 \; \mathbf{h}_1^{m_r} \; 0 \; \cdots \; 0 \; \cdots \; \mathbf{h}_Q^{m_r} \; 0 \; \cdots \; 0 \\ 0 \; \mathbf{h}_0^{m_r} \; 0 \; \cdots \; 0 \; \mathbf{h}_1^{m_r} \; 0 \; \cdots \; 0 \; \cdots \; \mathbf{h}_Q^{m_r} \; 0 \; \cdots \\ \ddots \\ 0 \; \cdots \; 0 \; \mathbf{h}_0^{m_r} \; 0 \; \cdots \; 0 \; \mathbf{h}_1^{m_r} \; 0 \; \cdots \; 0 \; \cdots \; \mathbf{h}_Q^{m_r} \end{bmatrix}$$

where $\mathbf{h}_q^{m_r} = [h_q^{m_r,1} \; h_q^{m_r,2} \; \cdots \; h_q^{m_r,M_t}]$ for $q = 0, \ldots, Q$.

The space-time data model with one user can be completed by stacking the receive vectors corresponding to the M_r receive antennas on top of each other to yield a tall column vector, $\mathbf{r}_k = [(\underline{\mathbf{r}}_k^1)^T \; (\underline{\mathbf{r}}_k^2)^T \; \cdots \; (\underline{\mathbf{r}}_k^{M_r})^T]^T$, and including s snapshots of the input vector $\underline{\mathbf{x}}_k$ to form a fat input matrix, $\mathbf{X} = [\underline{\mathbf{x}}_k \; \underline{\mathbf{x}}_{k+1} \; \cdots \; \underline{\mathbf{x}}_{k+s}]$, and the corresponding output matrix, $\mathbf{R} = [\mathbf{r}_k \; \mathbf{r}_{k+1} \; \cdots \; \mathbf{r}_{k+s}]$. Thus, we have

$$\mathbf{R} = \mathbf{HX} + \mathbf{N} \tag{3}$$

where $\mathbf{N}$ is the noise matrix, constructed in the same way as $\mathbf{R}$, and $\mathbf{H}$ is a block Toeplitz matrix, similar in structure to the Toeplitz matrix $\underline{\mathbf{H}}^{m_r}$. If the $\mathbf{h}_q^{m_r}$ vectors are stacked up to create $\overline{\mathbf{H}}_q = [\mathbf{h}_q^1 \; \mathbf{h}_q^2 \; \cdots \; \mathbf{h}_q^{M_r}]^T$, then we can write

$$\mathbf{H} = \begin{bmatrix} \overline{\mathbf{H}}_0 \; \mathbf{0} \; \cdots \; \mathbf{0} \; \overline{\mathbf{H}}_1 \; \mathbf{0} \; \cdots \; \mathbf{0} \; \cdots \; \overline{\mathbf{H}}_\mathbf{Q} \; \mathbf{0} \; \cdots \; \mathbf{0} \\ \mathbf{0} \; \overline{\mathbf{H}}_0 \; \mathbf{0} \; \cdots \; \mathbf{0} \; \overline{\mathbf{H}}_1 \; \mathbf{0} \; \cdots \; \mathbf{0} \; \cdots \; \overline{\mathbf{H}}_\mathbf{Q} \; \mathbf{0} \; \cdots \\ \vdots \qquad \ddots \qquad \vdots \\ \mathbf{0} \; \cdots \; \mathbf{0} \; \overline{\mathbf{H}}_0 \; \mathbf{0} \; \cdots \; \mathbf{0} \; \overline{\mathbf{H}}_1 \; \mathbf{0} \; \cdots \; \mathbf{0} \; \cdots \; \overline{\mathbf{H}}_\mathbf{Q} \end{bmatrix}$$

where $\mathbf{0}$ is a column vector of zeros of length M_r.

The F cochannel interferers can now be added to $\mathbf{R}$ to yield the final MIMO equation,

$$\mathbf{Y} = \mathbf{HX} + \sum_{f=1}^{F} \mathbf{H}_f \mathbf{X}_f + \mathbf{N} \tag{4}$$

where $\mathbf{H}_f$ and $\mathbf{X}_f$ are the channel and transmit matrices of the fth interferer and are of the same form as $\mathbf{H}$ and $\mathbf{X}$, respectively.

III. Problem Formulation

Let $\mathbf{X}$ in (4) be the input matrix corresponding to the training sequences transmitted on the M_t transmit antennas of the desired source and let $\mathbf{Y}$ in (4) be the associated output of this training data received at the receive antenna array. The training input matrix $\mathbf{X}$ is chosen to have full rank. We equalize the channel output with a space-time equalizer of K_e+1 taps and denote it by $W = [\mathbf{w}^1 \; \mathbf{w}^2 \; \cdots \; \mathbf{w}^{M_t}]$. Here, each $\mathbf{w}^u$ is a vector of size $M_r(K_e+1) \times 1$ and represents the column of the MIMO equalizer which is used to equalize the sequence transmitted from the uth transmit antenna. We denote the conjugate transpose of a vector/matrix as $(\cdot)^H$ and write

$$\mathbf{W}^H \mathbf{Y} = \widetilde{\mathbf{Z}}^H \mathbf{X} + \sum_{f=1}^{F} \mathbf{W}^H \mathbf{H}_f \mathbf{X}_f + \mathbf{W}^H \mathbf{N} \tag{5}$$

where $\widetilde{\mathbf{Z}}$ is the effective channel matrix for the desired source after equalization, and can be expressed in the following form

$$\begin{bmatrix} \widetilde{\mathbf{z}}^{1,1} & \widetilde{\mathbf{z}}^{1,2} & \dots & \widetilde{\mathbf{z}}^{1,M_t} \\ \widetilde{\mathbf{z}}^{2,1} & \widetilde{\mathbf{z}}^{2,2} & \dots & \widetilde{\mathbf{z}}^{2,M_t} \\ \vdots & & \ddots & \vdots \\ \widetilde{\mathbf{z}}^{M_t,1} & \widetilde{\mathbf{z}}^{M_t,2} & \dots & \widetilde{\mathbf{z}}^{M_t,M_t} \end{bmatrix}.$$

The vector $\widetilde{\mathbf{z}}^{v,u}$ represents the effective channel vector for the sequence transmitted from the vth antenna when $\mathbf{w}^u$ is used to equalize the uth transmit sequence. To accommodate channel shortening, we set $(\widetilde{\mathbf{z}}^{v,v})^H = [\mathbf{0}_{1\times\delta}\ (\mathbf{z}^{v,v})^H\ \mathbf{0}_{1\times(K_e+L-\nu-\delta)}]$, where $\mathbf{z}^{v,v}$ is the shortened channel of length $\nu+1$, for the vth transmit stream and δ is a parameter in the range $0 \le \delta \le K_e + L - \nu$, representing the optimum delay for the shortened channel. When the transmit sequence from the vth antenna is detected by first equalizing it by $\mathbf{w}^v$, $\widetilde{\mathbf{z}}^{v,u}$ for $u \neq v$ represent the effective channels for the co-antenna interferers. These channels will be called *cross-channels* henceforth.

The objective is to design the MIMO equalizer $\mathbf{W}$ for performing CAI/CCI cancellation to conform with the detection strategy and obtain the shortened effective channels for the different transmit streams, using direct training data. A joint optimization is performed to obtain $\mathbf{W}$ and $\mathbf{Z}$ by minimizing the objective function shown in (6) subject to different constraints depending on the detection algorithm. The cost function is presented in its most general form below

$$\sum_{u=1}^{M_t} \|(\mathbf{w}^u)^H\mathbf{Y} - \sum_{v=1}^{p}(\widetilde{\mathbf{z}}^{v,u})^H\mathbf{X}\|_2^2. \tag{6}$$

The value of p ranges between 1 and M_t, depending on the type of detection to be used. The technique of separation of variables is used to obtain the solution for this optimization problem by solving for $\mathbf{W}$ first, conditioned on $\mathbf{Z}$. This is followed by plugging the solution back into the objective function and solving for $\mathbf{Z}$.

The error between the equalized output $\mathbf{W}^H\mathbf{Y}$ and the effective channel output $\widetilde{\mathbf{Z}}_p^H\mathbf{X}$, where the form of $\widetilde{\mathbf{Z}}_p$ varies as we change the value of p in (6), is given by the equalization error matrix

$$\mathbf{E}_p = \mathbf{W}^H\mathbf{Y} - \widetilde{\mathbf{Z}}_p^H\mathbf{X}. \tag{7}$$

We optimize $\mathbf{W}$ to minimize the least square error in equalization, LSE($\mathbf{E}_p$), defined as the trace($\mathbf{E}_p\mathbf{E}_p^H$). This is exactly equivalent to the expression in (6). We can solve this by first differentiating LSE($\mathbf{E}_p$) $= (\mathbf{W}^H\mathbf{Y} - \widetilde{\mathbf{Z}}_p^H\mathbf{X})(\mathbf{W}^H\mathbf{Y} - \widetilde{\mathbf{Z}}_p^H\mathbf{X})^H$ with respect to $\mathbf{W}$ and setting the result to zero to obtain

$$\mathbf{W}_{opt}^H = \widetilde{\mathbf{Z}}_{p,opt}^H\mathbf{X}\mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}. \tag{8}$$

At this point, it is helpful to recall that $\mathbf{X}$ represents the training data matrix and $\mathbf{Y}$ is the corresponding training output matrix. It only remains to find $\widetilde{\mathbf{Z}}_{p,opt}^H$ and this is done in the following section with different constraints on $\widetilde{\mathbf{Z}}_p$ that helps adapt the design to different detection schemes.

IV. Solutions/Algorithms

A. Independent Detection

In this detection scheme, each of the transmit streams is detected independently by treating all the other transmit streams as interference. The cost function in (6) can now be modified to yield the following objective function, in which all the cross-channels are minimized,

$$\min_{\mathbf{W},\widetilde{\mathbf{Z}}} \sum_{u=1}^{M_t} \|(\mathbf{w}^u)^H\mathbf{Y} - (\widetilde{\mathbf{z}}^{u,u})^H\mathbf{X}\|_2^2. \tag{9}$$

This objective function can be decoupled by minimizing $\|(\mathbf{w}^u)^H\mathbf{Y} - (\widetilde{\mathbf{z}}^{u,u})^H\mathbf{X}\|_2^2$ for $u = 1, \dots, M_t$. Each $\mathbf{z}^{u,u}$ is constrained to have unit energy to avoid a trivial solution (zero vector). Substituting (8), we have

$$\min_{\widetilde{\mathbf{z}}^{u,u}} \|(\widetilde{\mathbf{z}}^{u,u})^H\mathbf{X}\mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}\mathbf{Y} - (\widetilde{\mathbf{z}}^{u,u})^H\mathbf{X}\|_2^2 \tag{10}$$

subject to $\|\widetilde{\mathbf{z}}^{u,u}\|_2^2 = 1$, for $u = 1, \dots, M_t$, i.e., for each transmit sequence.

As each $\widetilde{\mathbf{z}}^{u,u}$ has the shortened channel $\mathbf{z}^{u,u}$ embedded in it, with δ zeros preceding it , the constraint is equivalent to $\|\mathbf{z}^{u,u}\|_2^2 = 1$ and (10) can be written as

$$\min_{\mathbf{z}^{u,u}} \|(\mathbf{z}^{u,u})^H\mathbf{X}_\delta^u\mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}\mathbf{Y} - (\mathbf{z}^{u,u})^H\mathbf{X}_\delta^u\|_2^2 \tag{11}$$

where $\mathbf{X}_\delta^u = [\mathbf{x}_{k,\delta}^u\ \mathbf{x}_{k+1,\delta}^u\ \cdots\ \mathbf{x}_{k+s,\delta}^u]$ is a sub-matrix corresponding to the $\delta+1, \dots, \delta+\nu+1$ rows of the training matrix of the uth transmit sequence. Simplifying, we get

$$\min_{\mathbf{z}^{u,u}} (\mathbf{z}^{u,u})^H(\mathbf{X}_\delta^u)^*\mathbf{P}_{\mathbf{Y}^H}^*(\mathbf{X}_\delta^u)^T\mathbf{z}^{u,u} \tag{12}$$

subject to $(\mathbf{z}^{u,u})^H\mathbf{z}^{u,u} = 1$, where $\mathbf{P}_{\mathbf{Y}^H} = \mathbf{I} - \mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}\mathbf{Y}$ is the projection matrix. By the Rayleigh-Reitz theorem [7], $(\mathbf{z}_{opt}^{u,u})^H$ is given by any scalar multiple of the eigenvector corresponding to the smallest eigenvalue of $(\mathbf{X}_\delta^u)^*\mathbf{P}_{\mathbf{Y}^H}^*(\mathbf{X}_\delta^u)^T$. The optimum value of δ is found by calculating the value of the objective function in (12) for all possible values of δ and choosing the one which minimizes the function. $(\mathbf{w}_{opt}^{u})^H$ can now be found by substituting $(\mathbf{z}_{opt}^{u,u})^H$ and $\mathbf{X}_{\delta_{opt}}^{u}$ in (8).

The same algorithm is repeated for all transmit sequences. The form of the objective function used is interesting as it absorbs the CAI, CCI, residual interference due to shortening and AWGN in the error vector between the equalized output and the shortened channel output, thereby enabling us to perform joint CAI/CCI cancellation and channel shortening. To distinguish this from the normal interference-plus-noise, we shall call this the all-interference-plus-noise term. The signal-to-all-interference-plus-noise ratio will be denoted by SAINR. Each transmit sequence is detected by a single-channel Viterbi equalizer in the second stage of the receiver. The Viterbi equalizer uses the shortened effective channel and equalized output for each transmit stream to suppress the ISI. This two-stage receiver structure is similar in concept to the one proposed in [1] in which the objective function maximized by the authors is the post-equalization SINR. It can be shown that if we simply minimize the all-interference-plus-noise (with the constraint proposed in this paper), instead of maximizing the SAINR, the computational complexity is reduced and the performance of the system is not significantly affected.

B. Ordered Successive Interference Cancellation

The space-time equalizer of Section IV-A nulls out all the co-antenna interferers and is basically a linear receiver. It is possible to improve symbol error rate performance of a linear receiver by using ordered successive interference cancellation (OSIC) as proposed in [8]. In this approach, transmit streams are detected in succession and the interference contributed by the already detected streams is subtracted out from the remaining streams. This progressively reduces the number of co-antenna interferers at each stage, thereby improving the efficacy of the detection process. The order in which the streams are detected is optimized to further elevate the performance.

The space-time equalizer, $\mathbf{W}_{opt}$, can be designed to perform OSIC with some modifications in the objective function. An arbitrary ordering $\{u_1, u_2, \ldots, u_{M_t}\}$ is assumed for convenience and the optimal ordering criterion is discussed at a later stage. It is possible to decouple the objective function in (6) in this case too as the equalizer vector and effective channel for each transmit sequence are found independently of the others. For transmit sequence u_1, $(\mathbf{z}_{opt}^{u_1,u_1})^H$ and $(\mathbf{w}_{opt}^{u_1})^H$ are found as in Section IV-A. For the next transmit sequence, $(\mathbf{z}_{opt}^{u_2,u_2})^H$ as well as $(\mathbf{z}_{opt}^{u_1,u_2})^H$, which is the cross-channel due to the interference from transmit sequence u_1, are found using the following version of the objective function

$$\min_{\mathbf{w}^{u_2},\widetilde{\mathbf{z}}^{u_1,u_2},\widetilde{\mathbf{z}}^{u_2,u_2}} \|(\mathbf{w}^{u_2})^H\mathbf{Y} - (\widetilde{\mathbf{z}}^{u_1,u_2})^H\mathbf{X} - (\widetilde{\mathbf{z}}^{u_2,u_2})^H\mathbf{X}\|_2^2 \tag{13}$$

subject to $\|[(\widetilde{\mathbf{z}}^{u_1,u_2})^H \ (\widetilde{\mathbf{z}}^{u_2,u_2})^H]\|_2^2 = 1$
or equivalently,

$$\min_{\mathbf{w}^{u_2},\widetilde{\mathbf{z}}^{u_2}} \|(\mathbf{w}^{u_2})^H\mathbf{Y} - (\widetilde{\mathbf{z}}^{u_2})^H\mathbf{X}\|_2^2 \tag{14}$$

subject to $\|\widetilde{\mathbf{z}}^{u_2}\|_2^2 = 1$.

The vector $(\widetilde{\mathbf{z}}^{u_2})^H$ is formed by concatenating the effective channels for transmit sequences u_1 and u_2. It can now replaced by $(\mathbf{z}^{u_2})^H = [(\widetilde{\mathbf{z}}^{u_1,u_2})^H \ (\mathbf{z}^{u_2,u_2})^H]$, where $\mathbf{z}^{u_2,u_2}$ is the shortened channel embedded in $\widetilde{\mathbf{z}}^{u_2,u_2}$, and (8) is substituted to yield

$$\min_{\mathbf{z}^{u_2}} \|(\mathbf{z}^{u_2})^H\mathbf{X}_{\delta}^{(u_{1,2})}\mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}\mathbf{Y} - (\mathbf{z}^{u_2})^H\mathbf{X}_{\delta}^{u_{1,2}}\|_2^2 \tag{15}$$

with the constraint $\|\mathbf{z}^{u_2}\|_2^2 = 1$. $\mathbf{X}_{\delta}^{u_{1,2}}$ is formed by stacking the vector, $\mathbf{X}^{u_1} = [\mathbf{x}_k^{u_1} \ \mathbf{x}_{k+1}^{u_1} \ \cdots \ \mathbf{x}_{k+s}^{u_1}]$ on top of $\mathbf{X}_{\delta}^{u_2}$, which is of the same form as $\mathbf{X}_{\delta}^{u}$ in (11). Only the desired transmit sequence's channel is shortened and not the cross-channel. It should be noted that $\mathbf{X}_{\delta}^{u_{1,2}}$ is the part of $\mathbf{X}$ which is actually multiplied by the cross-channel vector $(\widetilde{\mathbf{z}}^{u_1,u_2})^H$ and the shortened desired channel vector $(\mathbf{z}^{u_2,u_2})^H$. This can now be simplified and solved as before and thus, $(\mathbf{z}^{u_2})^H$ is given by the eigenvector corresponding to the smallest eigenvalue of $(\mathbf{X}_{\delta}^{u_{1,2}})^*\mathbf{P}_{\mathbf{Y}^H}^*(\mathbf{X}_{\delta}^{u_{1,2}})^T$ where δ is optimized as before. $(\mathbf{w}_{opt}^{u_2})^H$ is found by substituting $(\mathbf{z}^{u_2})^H$ and $\mathbf{X}_{\delta}^{u_{1,2}}$ in (8). Having found $(\mathbf{z}^{u_2})^H$, $(\widetilde{\mathbf{z}}^{u_1,u_2})^H$ and $(\mathbf{z}^{u_2,u_2})^H$ can now be extracted from it.

For the third transmit sequence, the cross-channels from the u_1 and u_2 transmit sequences are found along with the shortened channel for the u_3th sequence. Likewise, the effective cross-channels due to all the previously considered transmit sequences and the corresponding equalizer vectors for each subsequent transmit stream are found. In this design, $(\widetilde{\mathbf{Z}}_{p,opt})^H$ is designed to be an upper triangular matrix, with an increasing number of terms included in the inner summation of the objective function.

Choosing the transmit sequence with the best post-processing *metric* at each stage results in the globally optimum ordering [8]. The metric, in this case, is simply the value of the objective function $\min_{\mathbf{w}^{u_i},\widetilde{\mathbf{z}}^{u_i}} \|(\mathbf{w}^{u_i})^H\mathbf{Y} - (\widetilde{\mathbf{z}}^{u_i})^H\mathbf{X}\|_2^2$ in the u_ith stage. So, in the u_ith stage, this value, representing the all-interference-plus-noise, is evaluated for all remaining transmit sequences, using their optimum equalizer vectors and effective channels, and the transmit sequence with the overall minimum value is selected to represent the u_ith state in the ordering.

The detection process is completed by employing a single channel Viterbi equalizer for each transmit sequence as done for the case of independent detection. There is, however, a key difference. Having detected the transmit sequence which is first in the optimal ordering, the interference corresponding to it is subtracted out from the equalized output for the second sequence in the ordering using the effective cross-channel vector. This is a simple extension of the approach presented in Section IV-A and shows the flexibility of the design framework in adapting to different detection schemes.

V. Simulation Results

For the purpose of simulations, a 2×2 MIMO system is considered with a channel memory of 2, i.e. $L = 2$. The channel is assumed to be Rayleigh fading. The entries of $\mathbf{H}$ are i.i.d. with distribution $\mathcal{CN}(0, 1)$. The number of taps of the equalizer is chosen to be 4, i.e., $K = 4$. The effective channel will thus be 7 taps long without any channel shortening. A block fading model is used, where the channel is assumed to be constant over a frame of symbols of length 100. The simulation results are obtained by averaging the performance over 100 channel realizations. In Figure 2, the probability of symbol error with OSIC and independent detection is plotted versus input SNR. We can get a symbol error rate of 0.01 at approximately 12 dB input SNR with both the detection schemes. We validate our claim for CCI cancellation in Figure 3. The set-up is a two user scenario. The SAINR of the desired user is plotted with the carrier-to-interference ratio (CIR) for independent detection with and without channel shortening. It can be seen that the SAINR for the shortened channel starts overshooting 10 dB between 10 to 15 dB of CIR. This shows reasonable cochannel rejection at moderate to high CIR values.

As a final remark, we note that the OSIC symbol error performance can be improved by using constraints that are optimal for symbol error performance using the same framework presented here. This is an area that needs further investigation.

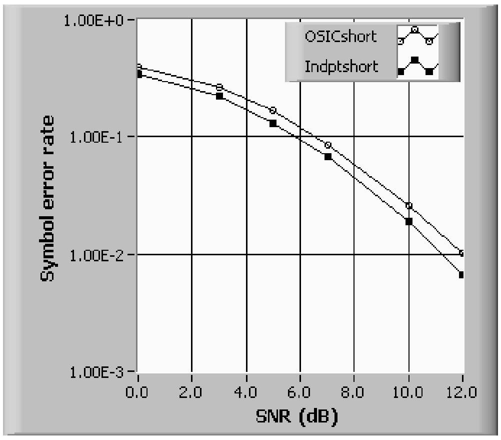

Fig. 2. Comparison of probability of symbol error for independent detection and ordered successive interference cancellation for a shortened channel of length 4

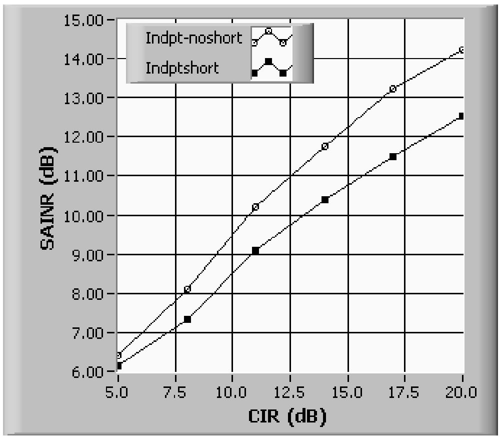

Fig. 3. SAINR of desired user versus CIR in a two-user scenario for independent detection with no shortening and shortening to a length 4 channel

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