Combining raised cosine windowing and per tone equalization for RFI mitigation in DMT receivers

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***Abstract*— Discrete multitone (DMT) offers an elegant way to achieve high capacity, dividing the spectrum into small bands and processing these individually. The per tone equalizer (PTEQ) optimizes the capacity for each band individually, thus optimizing the whole. However, it provides little protection against narrow band radio frequency interference (RFI), being spread over all tones because of the high side lobes of the DFT filter bank used in the receiver. The use of windowing functions limits this noise spreading, but is difficult to combine with the PTEQ. This paper describes a method to combine the PTEQ with a raised cosine window, while keeping the complexity reasonable. Extensions to other windowing functions are also given.**

I. Introduction

DISCRETE multitone (DMT) achieves near-Shannon capacity by dividing the available spectrum into small bands. Carriers (tones) in these bands are (de)modulated in the digital domain, through a discrete fourier transform (DFT), in practice carried out using the fast fourier transform (FFT) algorithm [1]. Equalization is facilitated by a *cyclic prefix* (CP) preceding each symbol [2]. In case the channel impulse response's length does not exceed the CP length, the linear convolution with the channel impulse response can be described as a circular one. The equalization of each tone can then be done easily throug a one-tap frequency domain equalizer (FEQ), consisting of a multiplication and phase shift for each tone individually.

Since the prefix does not carry any useful information, it is kept a short as possible, implying that often the channel impulse response's length exceeds the cyclic prefix, such that the aforementioned condition for easy equalization does not hold, and intercarrier interference (ICI) results ([3], [4], [5]). Classic DMT receiver schemes make use of a time domain equalizer (TEQ) to shorten the channel such that the cascade of the channel and TEQ is shorter than the CP. This TEQ is mostly implemented as a finite impulse response (FIR) filter of length T.

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In [6] a new receiver structure, based on so-called *per tone equalization* (PTEQ), has been developed as an alternative to the classical TEQ. For each tone separately, a T-taps FEQ is constructed that maximizes the SNR on that tone. It can be shown that the PTEQ of length T offers an upper bound for any TEQ design of the same length.

The spectrum occupied by high speed digital subscriber line (DSL) modems overlaps with the bands used for radio communications. AM broadcast stations and amateur radio transmitters introduce radio frequency interference (RFI) impairing the DSL-receiver. Window functions can be used to reduce the effect of spectral leakage of such narrow band RFI. Limiting the window size to the DFT size N would inevitably lead to the loss of orthogonality between the tones ([7], [8]). This can be overcome by extending the window size with a length μ.

In [9], a procedure is given to calculate an optimum window, given an existing TEQ. It has been shown [10] that applying a time domain window is equivalent to applying a fixed per-tone equalizer with different equalizers for different tones. However, the straightforward implementation of the windowing operation as a per tone equalization is computationally demanding, as the number of taps needed for such implementation increases from T to $T+\mu$. In [11], an efficient method for the combination of a fixed trapezoidal window and the PTEQ was proposed. This article describes a nontrivial extension to the more common *raised cosine* window function. In section II, the new receiver structure is introduced, section III compares the new method to previous techniques. Finally in section IV the proposed technique as well as extension to other window functions are discussed.

II. PTEQ and the Raised Cosine Window

Let N be the DFT size, and ν the cyclic prefix length. At time k, the tones $i \in [0, \frac{N}{2}-1]$ are carrying the frequency-domain information $X_i^{(k)}$. The data model is adopted from [6]. A T-taps equalizer $\bar{\mathbf{w}}_i$ is used for each tone i individually. The received time domain samples are denoted as y and are grouped in a column vector $\mathbf{y}^{(k)}$ of length $\mu+N+T-1$:

$$\mathbf{y}^{(k)} = \left[\begin{array}{ccc} y_k & \cdots & y_{k+N+T+\mu-2} \end{array}\right]^T.$$

The *raised cosine* window (shown in fig. 1) is defined as:

$$\left[\begin{array}{ccccccccc} b_0 & \cdots & b_{\mu-1} & 1 & \cdots & 1 & b_\mu & \cdots & b_{2\mu-1} \end{array}\right]^T,$$

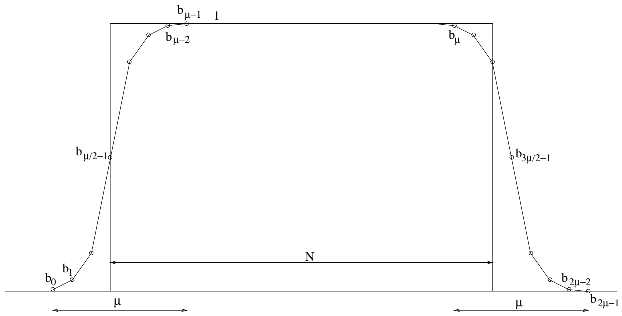

Fig. 1. *raised cosine* window

$$\text{with } b_x = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi(x+1)}{2\mu}\right)\right).$$

Please note the symmetry property:

$$b_x + b_{x+\mu} = 1, \text{ for } 0 \le x \le (\mu - 1), \tag{1}$$

more specifically: $b_{\mu-1} = 1$, and $b_{2\mu-1} = 0$. For each tone i, the T-taps per tone equalizer $\bar{\mathbf{w}}_i$ is determined solving the following minimization problem:

$$\min_{\bar{\mathbf{w}}_i} J(\bar{\mathbf{w}}_i) = \min_{\bar{\mathbf{w}}_i} \mathcal{E}\left\{\left|\bar{\mathbf{w}}_i^{\mathrm{T}}\mathbf{F}_{rc,i}\mathbf{y}^{(k)} - X_i^{(k)}\right|^2\right\}, \tag{2}$$

with

$$\mathbf{F}_{rc,i} = \begin{bmatrix} \boxed{\quad\mathcal{F}_{rc}(i,:)\quad} & \cdots & 0 \\ & \ddots \quad \ddots & \\ 0 \quad \cdots & & \boxed{\quad\mathcal{F}_{rc}(i,:)\quad} \end{bmatrix},$$

$$\mathcal{F}_{rc}(i,:) = [\, b_0 \;\; b_1\alpha_i \;\; \cdots \;\; b_{\mu-1}\alpha_i^{\mu-1} \;\; \alpha_i^{\mu} \;\; \cdots \;\; \alpha_i^{N-1} \;\; b_{\mu}\alpha_i^{N} \;\; \cdots \;\; b_{2\mu-1}\alpha_i^{\mu+N-1} \,],$$

$$\alpha_i = e^{\frac{-j2\pi(i-1)}{N}}$$

The vector $\mathcal{F}_{rc}(i,:)$ corresponds to a combination of a windowing operation, folding (to length N) and pointwise multiplication with the i^{th} row of the DFT matrix. From (2), it looks as if a T taps equalizer requires the computation of T complete (windowed) DFTs at the symbol rate. However, because of the structure in $\mathbf{F}_{rc,i}$, every row (except the last one) can be written as a linear combination of the lower rows and some correction terms. This corresponds to the transformation of (2) into another equivalent but simpler problem. For the elaboration, define the differences of the window as:

$$D_{rc} = [\; d_0 \quad d_1 \quad \ldots \quad d_{\mu-1} \;],$$

$$d_x = \begin{cases} b_0 & x = 0 \\ b_x - b_{x-1} & 1 \le x \le \mu - 1 \end{cases}$$

and, for tone i, the modulated differences as

$$D_{rc,i} = [\; d_0 \quad d_1\alpha_i \quad \cdots \quad d_{\mu-1}\alpha_i^{\mu-1} \;].$$

If we define

$$\mathbf{P}_i = \begin{bmatrix} 1 & \alpha_i & \ldots & \alpha_i^{(T-1)} \\ 0 & 1 & \ddots & \ddots \\ \vdots & \ddots & \ddots & \alpha_i \\ 0 & \ldots & 0 & 1 \end{bmatrix}, \text{ and}$$

$$\mathbf{M}_{rc,i} = \begin{bmatrix} \boxed{D_{rc,i}} & 0 \;\cdots\; 0 & \boxed{\text{-}D_{rc,i}} & & \cdots & 0 \\ & \ddots & & \ddots & & \\ 0 \;\cdots & \boxed{D_{rc,i}} & 0 \;\cdots\; 0 & & \boxed{\text{-}D_{rc,i}} & 0 \\ 0 \;\cdots & & \boxed{\quad\mathcal{F}_{rc}(i,:)\quad} & & & \end{bmatrix},$$

then the matrix $\mathbf{F}_{rc,i}$ can be rewritten as

$$\mathbf{F}_{rc,i} = \mathbf{P}_i\mathbf{M}_{rc,i}. \tag{3}$$

Now we can substitute $\bar{\mathbf{w}}_i^{\mathrm{T}}$ by $\bar{\mathbf{v}}_i^{\mathrm{T}}$, defined as $\bar{\mathbf{v}}_i^{\mathrm{T}} = \bar{\mathbf{w}}_i^{\mathrm{T}} \cdot \mathbf{P}_i$, such that (2) can be rewritten as a function of $\bar{\mathbf{v}}_i$

$$\min_{\bar{\mathbf{v}}_i} J(\bar{\mathbf{v}}_i) = \min_{\bar{\mathbf{v}}_i} \mathcal{E}\left\{\left|\bar{\mathbf{v}}_i^{\mathrm{T}}\mathbf{M}_{rc,i}\mathbf{y}^{(k)} - X_i^{(k)}\right|^2\right\} \tag{4}$$

The part related to the first $T-1$ rows of $\mathbf{M}_{rc,i}$ can be further simplified. If we define $\bar{\mathbf{u}}_i = \bar{\mathbf{v}}_i(1:T-1)$, $R = \bar{\mathbf{v}}_i(T)\mathcal{F}_{rc}(i,:)\mathbf{y}(T-1:N+T-1+\mu-1)$, $\mathbf{I}$ the unity matrix of size $(\mu + T - 2)$, $\mathbf{O}$ the zero matrix of size $(\mu + T - 2 \times N - \mu - T + 2)$, and $\mathbf{0}$ the zero matrix of size $(\mu + T - 2 \times 1)$, then (4) can be rewritten as (5) (see below):

If we define the vector of difference terms

$$\boldsymbol{\Delta} = [\; \Delta_k \quad \cdots \quad \Delta_{k+T+\mu-3} \;]^T, \; \Delta_x = (y_x - y_{x+N}),$$

we can rewrite the $\bar{\mathbf{u}}_i$-related part of (5) as

$$\bar{\mathbf{u}}_i^{\mathrm{T}} \begin{bmatrix} \boxed{D_{rc,i}} & \cdots & 0 \\ \ddots & \ddots & \\ 0 \;\cdots & & \boxed{D_{rc,i}} \end{bmatrix} \begin{bmatrix} \Delta_k \\ \Delta_{k+1} \\ \vdots \\ \Delta_{k+T+\mu-3} \end{bmatrix} \tag{6}$$

Being the differences of (half of a period of) a cosine function, the differences D_{rc} resemble (half a period of) a sinusoid. Accordingly, $D_{rc,i}$ can be seen as the first μ elements of the i^{th} row of the DFT-matrix, with a sinusoidal window applied. Under the condition that the DFT size N is an even integer multiple of the window extension μ, i.e.

$$N = 2l.\mu, \tag{7}$$

$$\min_{\mathbf{v}_i} J(\mathbf{v}_i) = \min_{\mathbf{v}_i} \mathcal{E}\left\{\left|\bar{\mathbf{u}}_i^{\mathrm{T}} \begin{bmatrix} \boxed{D_{rc,i}} & \cdots & 0 \\ \ddots & \ddots & \\ 0 \;\cdots & & \boxed{D_{rc,i}} \end{bmatrix} \cdot \underbrace{[\; \mathbf{I} \mid \mathbf{O} \mid -\mathbf{I} \mid \mathbf{0} \;]\,\mathbf{y}^{(k)}}_{\boldsymbol{\Delta}} + R - X_i^{(k)}\right|^2\right\}. \tag{5}$$

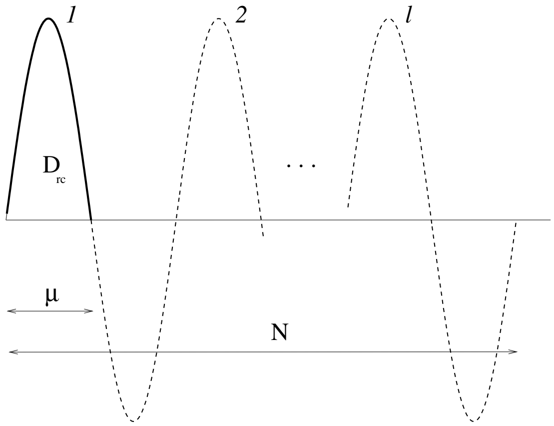

Fig. 2. Extending D_{rc} to a sinusoidal window

with l integer, the sinusoidal window can be extended to full length yielding $D_{rc,i,ext}$ (fig. 2):

$$D_{rc,i,ext} = \underbrace{[\, D_{rc} \; -D_{rc} \;\; D_{rc} \;\; \cdots \;\; -D_{rc} \,]}_{l \text{ repetitions}} \odot \mathcal{F}_N(i,:),$$

with $\mathcal{F}_N$ the DFT matrix of size N, $\mathcal{F}_N(i,:)$ the i^{th} row of this matrix and $\odot$ the pointwise multiplication operator.

We can now rewrite (6) as:

$$D_{rc,i,ext} \left[\begin{array}{cccc} \Delta_k & \Delta_{k+1} & \cdots & \Delta_{k+T-2} \\ \Delta_{k+1} & \Delta_{k+2} & \cdots & \Delta_{k+T-1} \\ \vdots & \vdots & & \vdots \\ \Delta_{k+\mu-1} & \Delta_{k+\mu} & \cdots & \Delta_{k+T+\mu-3} \\ \hline & O & & \\ & \vdots & & \\ & O & & \end{array} \right] \bar{\mathbf{u}}_i. \quad (8)$$

The previous expression (8) represents nothing else than a linear combination of a series of sinusoidally weighted DFTs of zero-padded Δ-sequences. It is generally known that the output of a windowed DFT for which the window is a sinusoid with an integer number of repetitions l, can be written as a linear combination of outputs of the corresponding non-windowed DFT [12]. Indeed, the windowed DFT essentially boils down to a multiplication in the time domain. This can be written as a convolution in the frequency domain. Since the frequency domain representation of the window (sinusoid) is a set of impulses, the frequency domain representation of the windowed DFT is equal to the linear combination of two outputs of the non-windowed DFT.

More specifically,

$$D_{rc,i,ext} = -j\frac{\mathcal{F}_N(i-l,:) - \mathcal{F}_N(i+l,:)}{2} \quad (9)$$

(Please note that possible negative or overflow indices of matrix $\mathcal{F}_N$ should be folded back). In other words, instead of calculating another (sinusoidally) windowed DFT and use its output at tone i as an input for the equalizer, we can also calculate the corresponding non-windowed DFT, and use the outputs at tones $(i-l)$ and $(i+l)$. In a last step, this new series of DFTs can be calculated recursively, in a manner comparable to the transition from (2) to (4): the DFT of every column of the Δ-matrix in (8), except the last one, can be written as a linear combination of the DFT of more righthand columns, and the correction terms $\Delta_k \cdots \Delta_{k+T-3}$, $\Delta_{k+\mu} \cdots \Delta_{k+T+\mu-3}$. This leads to the following minimization problem:

$$\min_{\bar{\mathbf{w}}_{i,u}} J(\bar{\mathbf{w}}_{i,u}) = \min_{\bar{\mathbf{w}}_{i,u}} \mathcal{E}\left\{ \left| \bar{\mathbf{w}}_{i,u}^{\mathrm{T}} \mathbf{F}_{rc,i,u} \mathbf{y}^{(k)} - X_i^{(k)} \right|^2 \right\}, \quad (10)$$

with $\mathbf{F}_{rc,i,u}$ defined in equation (11) at the bottom of the page.

The index $_u$ has been added to $\mathbf{F}_{rc,i}$ and $\bar{\mathbf{w}}_i$ to indicate that (10) corresponds to an *unconstrained* version of (2). Indeed, $\mathbf{F}_{rc,i}$ is fully contained in the row space of $\mathbf{F}_{rc,i,u}$ but not vice-versa. This is due to the fact that the outputs of the non-windowed DFT can also be combined linearly in another manner than the one expressed in (9). Moreover, because the window D_{rc} is not of full length N, two difference terms are needed for each additional sliding (windowed) DFT.

Obviously, this implies that the dimensionality of the problem is increased. If one denotes the number of rows in matrix $\mathbf{F}_{rc,i,u}$ as $T_{rc,u}$, it follows from (11) the following relationship holds:

$$T_{rc,u} = 2(T-2) + 3 = 2T - 1.$$

In comparison to a previous implementation [10], the method is most attractive in the most common case that $T \ll \mu$. The rows of the matrix $\mathbf{F}_{rc,i,u}$ correspond (from bottom to top) to

- the original windowed DFT output at tone i
- the two additional DFT outputs at frequencies cyclically symmetric around i. These are obtained from the DFT of the last column of the Δ-matrix in (8). Together, they form a sinusoidally windowed DFT as described by (9).
- the $2(T-2)$ Δ's needed to compute the other (sliding) DFT's from (8), namely Δ_k,..., Δ_{k+T-3}, and $\Delta_{k+\mu}$,..., $\Delta_{k+T+\mu-3}$.

A signal flow graph of this scheme is shown in fig. 3.

$$\mathbf{F}_{rc,i,u} = \left[\begin{array}{ccccccc} \mathrm{I}_{T-2} & & \mathrm{O}_{T-2,N-T+2} & & -\mathrm{I}_{T-2} & \mathrm{O}_{T-2,\mu} & \mathrm{O}_{T-2,1} \\ \mathrm{O}_{T-2,\mu} & & \mathrm{I}_{T-2} & & \mathrm{O}_{T-2,N-T+2} & -\mathrm{I}_{T-2} & \mathrm{O}_{T-2,1} \\ 0 & \cdots & \boxed{\mathcal{F}_N(i-\frac{N}{2\mu},1:\mu)} & 0 \;\cdots & 0 & \boxed{-\mathcal{F}_N(i-\frac{N}{2\mu},1:\mu)} & 0 \\ 0 & \cdots & \boxed{\mathcal{F}_N(i+\frac{N}{2\mu},1:\mu)} & 0 \;\cdots & 0 & \boxed{-\mathcal{F}_N(i+\frac{N}{2\mu},1:\mu)} & 0 \\ 0 & \cdots & \multicolumn{4}{c}{\boxed{\mathcal{F}_{rc}(i,:)}} & \end{array} \right] \quad (11)$$

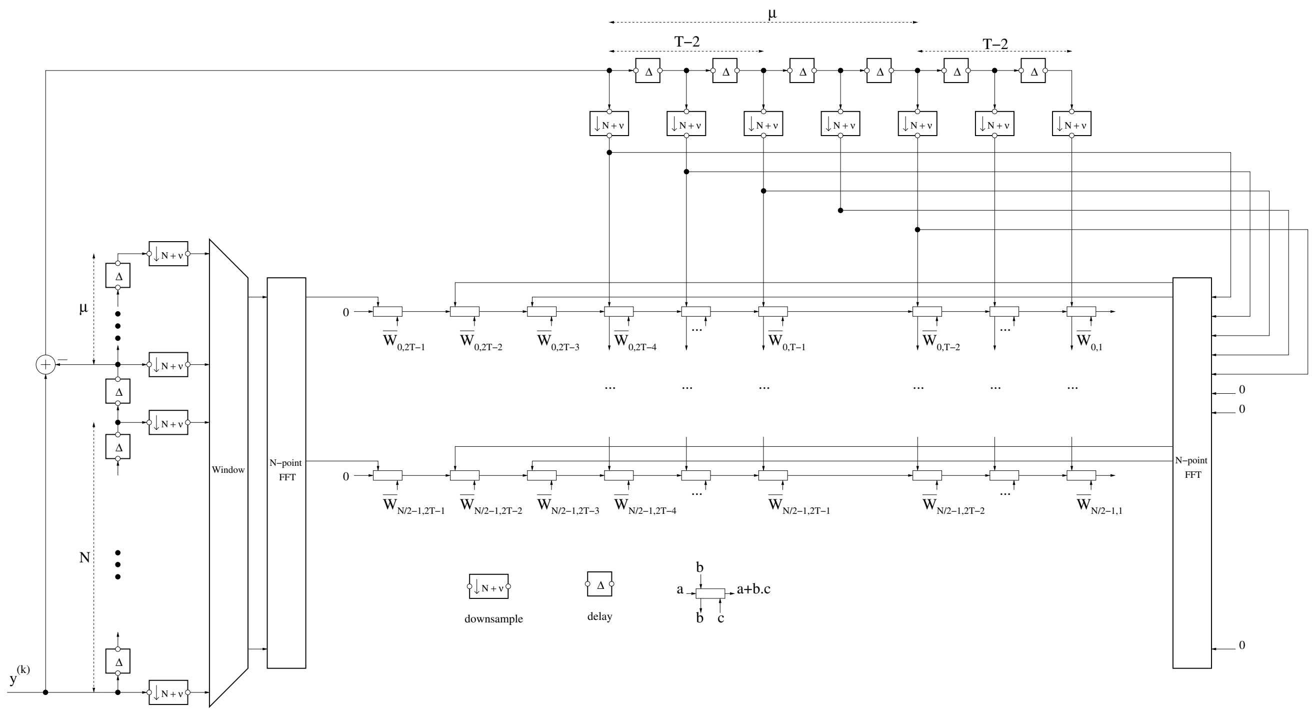

Fig. 3. Signal flow graph. Note that $\bar{\mathbf{w}}_{i,m}$ denotes the m^{th} element of the equalizator $\bar{\mathbf{w}}_{i,u}$ for tone i

III. Simulation results

The proposed technique has been compared to the PTEQ without windowing, and a more recent technique, combining the PTEQ and a trapezoidal window [11]. A typical scenario for an Asymmetrical DSL (ADSL) system was simulated, using a standard loop T1.601#13 with additive white gaussian noise (AWGN) at -140dBm/Hz. The used tones are from tone 38 up to tone 256, the symbol size N is 512 and the prefix length is 32. All simulations have been done using a recursive least squares algorithm (RLS). The PTEQ uses $T = 12$ taps, the method using the trapezoidal window uses $T_{t,u} = 11$ taps, and the new technique uses $T_{rc,u} = 11$ taps, such that the complexity is comparable. The window extension $\mu = 16$. After (training based) filter tap convergence, the adaptation is stopped, and an RFI noise source emerges at time instant 300, at 740kHz, with power -90dBm. The results are shown in figure 4. In the absence of RFI, the three techniques are comparable. However, in case RFI emerges, the new technique is clearly superior to the others. The PTEQ capacity nearly halves, whereas the combination with the raised cosine window proves to be more robust. The combination of PTEQ and a trapezoidal window is somewhere in between.

IV. Conclusions

A new technique for the combination of windowing functions and the per-tone equalizer has been proposed. The concept is based on a previous method, combining the PTEQ and a trapezoidal windowing function, but is more general. In summary, for each tone i, the optimal equalization is determined by

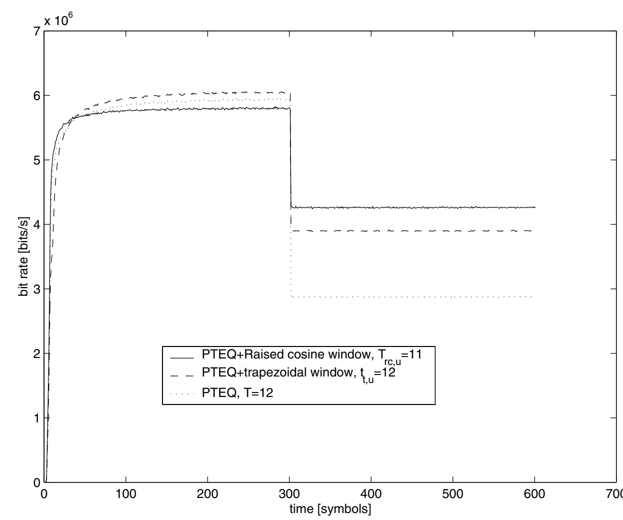

Fig. 4. RLS simulations: PTEQ vs combination of PTEQ and trapezoidal window and raised cosine window. RFI emerges at time 300

- the i^{th} output of the windowed DFT of the received symbol
- two outputs of the DFT of a vector of difference terms $\left[\begin{array}{cccccc} \Delta_k & \ldots & \Delta_{k+\mu-1} & 0 & \ldots & 0 \end{array}\right]$ (padded with zeroes), symmetrical around i. The distance to i is determined by the length of the head (tail) of the window and the size of the DFT.
- the difference terms $\Delta_k \ldots \Delta_{k+T-3}$, and $\Delta_{k+\mu} \ldots \Delta_{k+T+\mu-3}$.

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The main idea is to split the windowing function into two parts: the flat part, and the leading and ending taper. The first (flat) part does not pose any difficulties. The inputs corresponding to the tapers are combined in the form of difference terms. This leads to the calculation of a series of taper difference windowed DFTs. (In the case of the raised cosine window, the taper differences are a mere sinusoid). This windowed DFT can be expressed as a linear combination of the corresponding non-windowed DFTs. In case of the raised cosine window, this is described by (9). As such, the method can be extended to any other window satisfying the symmetry property (1). The number of taps needed depends only on the number of non-windowed DFT outputs necessary to do the frequency domain windowing. In case of the raised cosine window the total number of taps equals $2T - 1$. Compared to previous implementations, this method is attractive in the most common case that $T \ll \mu$.

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