MULTI-SPLIT ADAPTIVE FILTERING

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ABSTRACT

In this paper a novel transversal split filter configuration is proposed and the split optimum Wiener filter is introduced, as well as the symmetric and antisymmetric linear phase Wiener filter. The approach consists of combining the idea of split filtering with a linearly-constrained optimization scheme. Then, the continuously split procedure is introduced and the multi-split adaptive filter is derived. It is also shown that such structure can be viewed as a Hadamard-domain adaptive filter when the number of coefficients is set to a power of two. Simulation results obtained with the normalized LMS algorithm are presented and compared with DCT-domain adaptive filter.

1. INTRODUCTION

In order to improve the convergence rate and reduce the computational burden, the split adaptive filter has emerged as an interesting solution. The fundamental principles were introduced when Delsarte and Genin proposed a split Levinson algorithm for real Toeplitz matrices in [1]. By identifying the redundancy of computing the set of the symmetric and antisymmetric parts of the predictors, they reduced by half the number of multiplications in the standard Levinson algorithm. Subsequently, they extended the technique to classical algorithms in linear prediction theory (Schur, lattice and normalized lattice algorithms) [2].

An LMS adaptive split filter for AR modeling (linear prediction) was proposed in [3] and generalized to a so-called unified approach [4], by the introduction of the continuously split and the corresponding application in a general transversal filtering problem.

An alternative representation of the split FIR filtering theory has been provided in [5], based on the linearly-constrained optimization approach. It consists of imposing the symmetry and the antisymmetry conditions to the impulse responses of two filters connected in parallel, by means of an appropriate set of linear constraints implemented with the so-called GSC structure.

The present paper applies the continuously split process in the novel approach proposed in [5], giving rise to a multi-split adaptive filtering structure. Such scheme differs from the work in [4] in two important aspects: *i*) as far as the theoretical derivation is concerned, the proposed GSC representation allows us to introduce the split optimum Wiener filter, as well as its symmetric and antisymmetric parts; and *ii*) the adaptive procedure is more general, since it can be carried out for any number of filter coefficients N. In the particular case of $N=2^M$, where M is assumed to be an integer, our solution can also be implemented by a Hadamard (or Walsh) transform scheme.

The paper is organized as follows. In the next section the principles of the split transversal filter is recalled and the linearly-constrained approach is presented together with the GSC structure. The split Wiener filter and the linear-phase Wiener filter are then introduced. Section 3 proceeds with the continuous split in the proposed approach and states its connections with a transformed structure using a Hadamard mapping of the input signal when $N=2^M$. The multi-split adaptive filter is considered in Section 4, where the previous structure is updated with a normalized LMS algorithm. Simulation results and some comparisons with a DCT transformed adaptive filter are provided in Section 5. Finally, Section 6 presents our conclusions.

2. SPLIT TRANSVERSAL FILTERING

Let us consider the classical scheme of an adaptive transversal filter as shown in Figure 1, in which the N-by-1 tap-weight vector of the filter $\mathbf{w}(n)=[w_0(n), ..., w_{N-1}(n)]^t$ has been split into its symmetric and antisymmetric parts:

$$\mathbf{w}(n) = \mathbf{w}_s(n) + \mathbf{w}_a(n), \tag{1}$$

where $\mathbf{w}_s(n) = ½ [\mathbf{w}(n) + \mathbf{J}\mathbf{w}(n)]$, $\mathbf{w}_a(n) = ½ [\mathbf{w}(n) - \mathbf{J}\mathbf{w}(n)]$ and $\mathbf{J}$ is the reflection matrix. Without loss of generality, all the parameters have been assumed to be real valued.

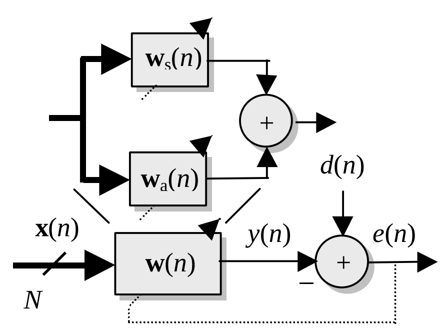

Figure 1: Split adaptive transversal filtering.

The symmetry and antisymmetry conditions of $\mathbf{w}_s(n)$ and $\mathbf{w}_a(n)$ can be easily introduced through a linearly-constrained approach as follows [5]. It consists of making:

$$\mathbf{C}_s = \begin{bmatrix} \mathbf{I}_K \\ \mathbf{0}_K^t \\ -\mathbf{J}_K \end{bmatrix}_{N \mathrm{x} K}, \quad \mathbf{C}_a = \begin{bmatrix} \mathbf{I}_K & \mathbf{0}_K \\ \mathbf{0}_K^t & \sqrt{2} \\ \mathbf{J}_K & \mathbf{0}_K \end{bmatrix}_{N \mathrm{x} K+1}, \qquad (2)$$

$\mathbf{f}_s=\mathbf{0}_K$, $\mathbf{f}_a=\mathbf{0}_{K+1}$, for N odd ($K=(N-1)/2$), or

$$\mathbf{C}_s = \begin{bmatrix} \mathbf{I}_K \\ -\mathbf{J}_K \end{bmatrix}, \quad \mathbf{C}_a = \begin{bmatrix} \mathbf{I}_K \\ \mathbf{J}_K \end{bmatrix}, \qquad (3)$$

$\mathbf{f}_s=\mathbf{f}_a=\mathbf{0}_K$, for N even ($K=N/2$), and of imposing:

$$\mathbf{C}_s^t \mathbf{w}_s(n) = \mathbf{f}_s \text{ and } \mathbf{C}_a^t \mathbf{w}_a(n) = \mathbf{f}_a, \qquad (4)$$

in a constrained optimization process of the mean-square value of the estimation error $e(n)$, which is defined as the difference between the desired response $d(n)$ and the filter output $y(n)$.

Now, using the GSC structure with the symmetry and antisymmetry constraints, the split filtering scheme in Figure 1 turns into the form represented by Figure 2 (N even). This simple structure arises from the fact that: *i*) since $\mathbf{f}_s=\mathbf{f}_a=\mathbf{0}_{N/2}$, the filters of the GSC structure which satisfy the symmetric and antisymmetric constraints are equal to zero; and *ii*) one of the possible signal blocking matrices of the symmetric part is the antisymmetry constraint matrix $\mathbf{C}_a$ itself ($\mathbf{C}_a^t\mathbf{C}_s=\mathbf{0}_{N/2\mathrm{x}N/2}$), and vice versa for the antisymmetric part.

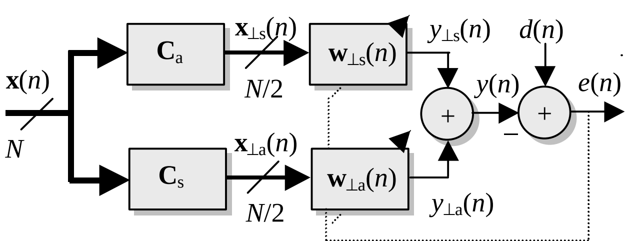

Figure 2: GSC implementation of the split filter.

It is interesting to observe that the vectors $\mathbf{w}_{\perp s}(n)$ and $\mathbf{w}_{\perp a}(n)$ are merely composed of the first $N/2$ coefficients of $\mathbf{w}_s(n)$ and $\mathbf{w}_a(n)$. The pre-multiplication of $\mathbf{w}_{\perp s}(n)$ by $\mathbf{C}_a$ yields $\mathbf{w}_s(n)$ and of $\mathbf{w}_{\perp a}(n)$ by $\mathbf{C}_s$ yields $\mathbf{w}_a(n)$. The estimation error is then given by

$$e(n) = d(n) - \mathbf{w}_{\perp s}^t(n)\mathbf{C}_a^t\mathbf{x}(n) - \mathbf{w}_{\perp a}^t(n)\mathbf{C}_s^t\mathbf{x}(n), \qquad (5)$$

where $\mathbf{x}(n) = [x(n), ..., x(n-N+1)]^t$ denotes the N-by-1 tap-input vector. In the mean-squared-error sense, the vectors $\mathbf{w}_{\perp s}(n)$ and $\mathbf{w}_{\perp a}(n)$ are chosen to minimize the following cost function:

$$\begin{aligned} F &= \mathrm{E}\{e^2(n)\} \\ &= s_d^2 - \mathbf{w}_{\perp s}^t\mathbf{C}_a^t\mathbf{p} - \mathbf{p}^t\mathbf{C}_a\mathbf{w}_{\perp s} + \mathbf{w}_{\perp s}^t\mathbf{C}_a^t\mathbf{R}\mathbf{C}_a\mathbf{w}_{\perp s} + \\ &\quad - \mathbf{w}_{\perp a}^t\mathbf{C}_s^t\mathbf{p} - \mathbf{p}^t\mathbf{C}_s\mathbf{w}_{\perp a} + \mathbf{w}_{\perp a}^t\mathbf{C}_s^t\mathbf{R}\mathbf{C}_s\mathbf{w}_{\perp a} + \\ &\quad + \mathbf{w}_{\perp s}^t\mathbf{C}_a^t\mathbf{R}\mathbf{C}_s\mathbf{w}_{\perp a} + \mathbf{w}_{\perp a}^t\mathbf{C}_s^t\mathbf{R}\mathbf{C}_a\mathbf{w}_{\perp s}, \end{aligned} \qquad (6)$$

where s_d^2 is the variance of $d(n)$, $\mathbf{R}$ is the N-by-N correlation matrix of $\mathbf{x}(n)$, and $\mathbf{p}$ is the N-by-1 cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$. Taking into account that $\mathbf{R}$ is centrosymmetric, it is easy to show by direct substitution of (3) (or (2) if N is odd) that the last two terms in (6) fall to zero. In other words, $y_{\perp s}(n)$ and $y_{\perp a}(n)$ are totally uncorrelated and, consequently, the symmetric and antisymmetric parts can be optimized separately. Thus, the optimum solution is given by

$$\mathbf{w}_{\perp s}^{\mathrm{opt}} = (\mathbf{C}_a^t\mathbf{R}\mathbf{C}_a)^{-1}\mathbf{C}_a^t\mathbf{p} \text{ and } \mathbf{w}_{\perp a}^{\mathrm{opt}} = (\mathbf{C}_s^t\mathbf{R}\mathbf{C}_s)^{-1}\mathbf{C}_s^t\mathbf{p}, \qquad (7)$$

and the scheme of Figure 2 corresponds to the split Wiener filter:

$$\mathbf{w}_{\mathrm{Wiener}}^{\mathrm{opt}} = \mathbf{w}_s^{\mathrm{opt}} + \mathbf{w}_a^{\mathrm{opt}}, \qquad (8)$$

where

$$\mathbf{w}_s^{\mathrm{opt}} = \mathbf{C}_a\mathbf{w}_{\perp s}^{\mathrm{opt}} \text{ and } \mathbf{w}_a^{\mathrm{opt}} = \mathbf{C}_s\mathbf{w}_{\perp a}^{\mathrm{opt}}. \qquad (9)$$

The filter $\mathbf{w}_s^{\mathrm{opt}}$ is the true optimum linear phase Wiener filter, having both constant group delay and constant phase delay (symmetric impulse response). On the other hand, the filter $\mathbf{w}_a^{\mathrm{opt}}$ is a second type of optimum "linear phase" Wiener filter (affine phase filter), having only the constant group delay (antisymmetric impulse response).

3. MULTI-SPLIT AND HADAMARD TRANSFORM

For ease of presentation, let $N=2^M$, where M is an integer number greater than one. Now, if each branch in Figure 2 is considered separately, the transversal filters $\mathbf{w}_{\perp s}(n)$ and $\mathbf{w}_{\perp a}(n)$ can also be split into their symmetric and antisymmetric parts. By proceeding continuously with this process and also splitting the resulting filters, we arrive, after M steps with 2^{m-1} splitting operations ($m=1, 2, \ldots, M$), at the multi-split scheme shown in Figure 3. $\mathbf{C}_{sm}$ and $\mathbf{C}_{am}$ are 2^{M-m+1}-by-2^{M-m} matrices such as in (3) and $w_{\perp i}(n)$, for $i=0, 1, \ldots, N-1$, are the single parameters of the resulting zero-order filters.

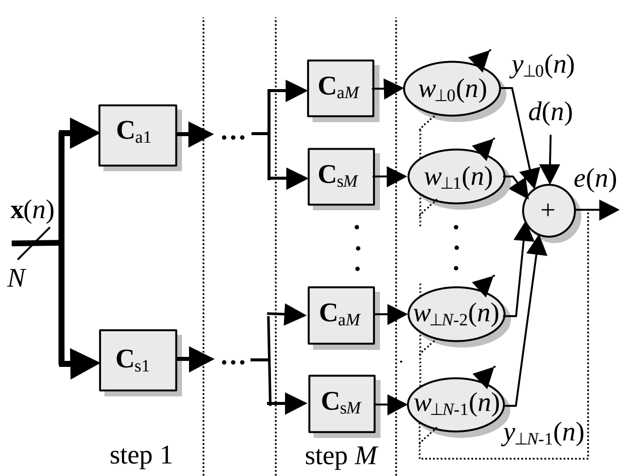

Figure 3: Multi-split adaptive filtering.

The above multi-split scheme can be viewed as a linear transformation of $\mathbf{x}(n)$ denoted by

$$\mathbf{x}_{\perp}(n) = \mathbf{T}^{t}\mathbf{x}(n)\,, \qquad (10)$$

where

$$\mathbf{T} = \begin{bmatrix} \mathbf{C}_{aM}^{t}\,\mathbf{C}_{aM-1}^{t}\cdots\mathbf{C}_{a1}^{t} \\ \mathbf{C}_{sM}^{t}\,\mathbf{C}_{aM-1}^{t}\cdots\mathbf{C}_{a1}^{t} \\ \vdots \\ \mathbf{C}_{sM}^{t}\,\mathbf{C}_{sM-1}^{t}\cdots\mathbf{C}_{s1}^{t} \end{bmatrix}^{t} . \qquad (11)$$

It can be verified by direct substitution that $\mathbf{T}$ is a matrix of +1's and –1's, in which the inner product of any two distinct columns is zero. However, at this stage, it is worth pointing out that $\mathbf{T}$ does not transform the vector $\mathbf{x}(n)$ into a corresponding input vector of uncorrelated variables. This is true only between the $N/2$ first and last variables of $\mathbf{x}_{\perp}(n)$, as a consequence of the split process in step 1. Therefore, the single parameters in Figure 3 cannot be optimized separately by the mean-squared error criterion.

An interesting point to mention is that the columns of $\mathbf{T}$ can be permuted which amounts to a re-arrangement of the single-parameters in Figure 3 in different sequences. Then, there are $N!$ possible permutations. The remarkable result is that one of them turns $\mathbf{T}$ into the N-order Hadamard matrix $\mathbf{H}_N$, so that the multi-split scheme can be represented in the compact form shown in Figure 4.

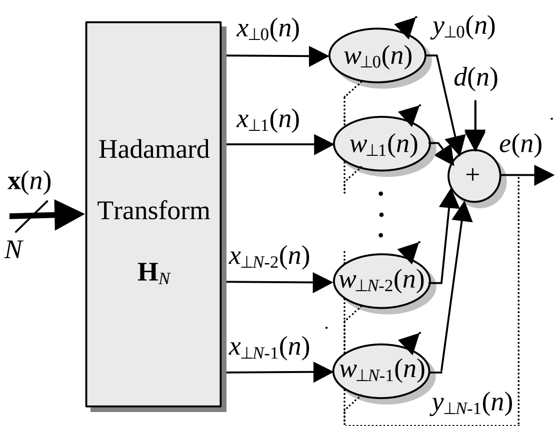

Figure 4: Hadamard transform of the input $\mathbf{x}(n)$.

4. MULTI-SPLIT ADAPTIVE FILTERING

In the adaptive context, due to its stochastic nature, the LMS algorithm can be directly applied for updating the single parameters, with no increase in the computational complexity. Furthermore, it can be done in the normalized form of Table I, as in the case of DCT-LMS algorithm [6].

The Hadamard matrix of order $2M$ can be constructed from $\mathbf{H}_M$ as follows:

$$\mathbf{H}_{2M} = \begin{bmatrix} \mathbf{H}_M & \mathbf{H}_M \\ \mathbf{H}_M & -\mathbf{H}_M \end{bmatrix}. \qquad (12)$$

Starting with $\mathbf{H}_1$=[1], such expression provides $\mathbf{H}_2$ $\mathbf{H}_4$ $\mathbf{H}_8$, and all Hadamard matrices the orders of which are powers of two.

Table 1: MS-LMS algorithm.

➢ *Selection of parameters*:

$$m = \frac{1}{2N} \text{ (step-size) and } 0<g<1 \text{ (forgetting factor)}$$

➢ *Initialization*:

For i=0,1, …, N-1, set: $w_{\perp i}(0)$=0 and $r_i(0)$=0

➢ *Updating*:

For i=0,1, …, N-1 and n=1,2, …, compute:

1) **T** *transform*: $x_{\perp i}(n) = \mathbf{t}_i^{t}\mathbf{x}(n)$

where $\mathbf{t}_i$ is the $(i+1)^{\text{th}}$ column vector of $\mathbf{T}$

2) *LMS algorithm*:

$$y(n) = \sum_{i=0}^{N-1} x_{\perp i}(n) w_{\perp i}(n)$$

$$e(n)=d(n)-y(n)$$

$$r_i(n) = g\; r_i(n-1) + \frac{1}{n}\left(x_{\perp i}^{2}(n) - g\; r_i(n-1)\right)$$

$$w_{\perp i}(n+1) = w_{\perp i}(n) + \frac{m}{r_i} x_{\perp i}(n) e(n)$$

As regards the linear phase adaptive filtering, the input samples for updating the $N/2$ single-parameters are given by $\mathbf{x}_{\perp s}(n) = \mathbf{H}_{2^{M-1}}\mathbf{C}_{a1}^{t}\mathbf{x}(n)$, for symmetric impulse response constraint, or by $\mathbf{x}_{\perp a}(n) = \mathbf{H}_{2^{M-1}}\mathbf{C}_{s1}^{t}\mathbf{x}(n)$ for antisymmetric impulse response constraint.

It is important to stress that the use of the Hadamard transform is conditioned to $N=2^M$. Otherwise, $\mathbf{T}$ is not composed only of +1's and –1's. Nevertheless, the representation of the multi-split filtering in Figure 4 holds with $\mathbf{T}$ instead of $\mathbf{H}$, which requires a number of multiplication operations proportional to N.

It is also worth pointing out that all split and multi-split transversal filtering theory developed above can be applied in linear prediction making $d(n)=x(n)$ or $d(n)=x(n-N)$.

Finally, the procedure can be extended to complex parameters by considering, from (1), that the reflection matrix $\mathbf{J}$ also operates the complex conjugation. Furthermore, the operations of transposition in the corresponding equations may be substituted by Hermitian transpositions.

5. SIMULATION RESULTS

To evaluate the performance of the multi-split (MS)-LMS algorithm, the same adaptive equalization system in [6, chap.9] (Figure 5) is used. The input channel $x(n)$ is binary, with $b(n)=\pm 1$, and the impulse response of the channel is described by the raised cosine:

$$c_j = \begin{cases} \frac{1}{2}(1+\cos(\frac{2p}{S}(j-2))), & j=1,2,3 \\ 0, & \text{otherwise} \end{cases}, \qquad (13)$$

where S controls the eigenvalue spread $\chi(\mathbf{R})$ of the correlation matrix of the tap inputs in the equalizer, with $\chi(\mathbf{R})=6.0782$ for $S= 2.9$ and $\chi(\mathbf{R})=46.8216$ for $S= 3.5$. The sequence $v(n)$ is an additive white noise that corrupts the channel output with variance $s_v^2 = 0.001$, and the equalizer has N=11 coefficients.

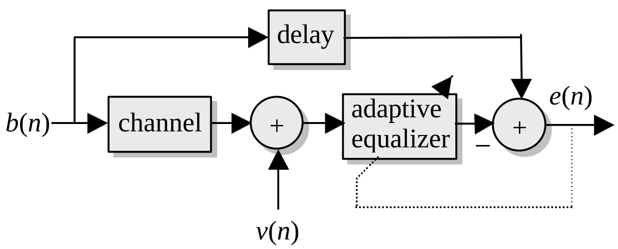

Figure 5: Adaptive equalizer for simulation.

Figure 6 show a comparison of the ensemble-averaged error performances of the DCT-LMS, MS-LMS, standard LMS and RLS algorithms for $\chi(\mathbf{R})=6.0782$ and $\chi(\mathbf{R})=46.8216$. The good performance of the MS-LMS algorithm can be observed in terms of convergence rate, compared with the standard LMS algorithm. Furthermore, it can be observed that the MS-LMS algorithm is somewhat sensitive to variations in the eigenvalue spread (more than the DCT tranform). This shows clearly that the multi-split approach does not orthoganalize the input data vector. Nevertheless, its simplicity justifies its application, rather than that of the DCT transform.

6. CONCLUSIONS

It has been shown that the split transversal filtering is an exact solution of a linearly-constrained optimization problem and can be implemented by means of a parallel GSC structure. Thus, the split Wiener filter can be introduced together with its symmetric and antisymmetric linear phase parts. Based on this result, the multi-split adaptive filter is proposed as an alternative technique for improving the convergence rate and reducing the computational burden. The novel approach is generic since it can be used for any value of filter order N, for complex and real parameters and can be extended to the linear prediction case. For a power of two value of N this proposition corresponds to a Hadamard transform domain adaptive filter. Simulation results illustrate its good performance when compared with a similar scheme based on DCT when the normalized LMS algorithm is employed in both cases.

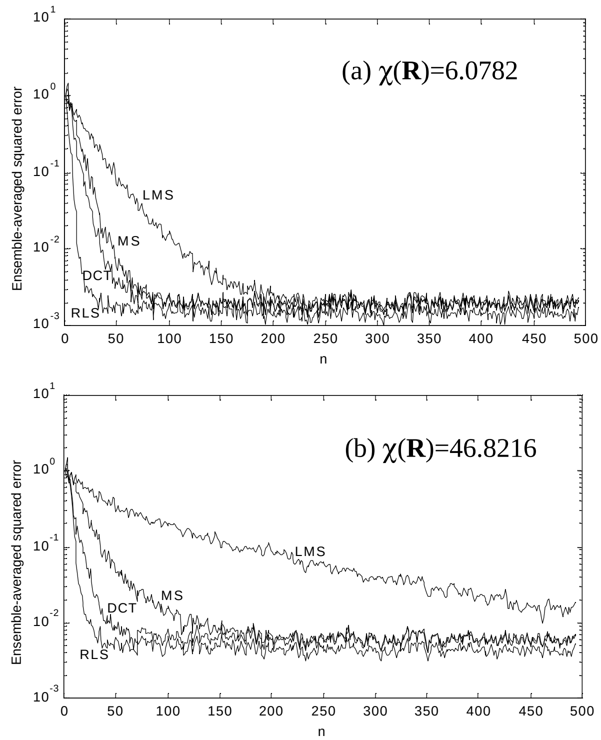

Figure 6: Learning curves.

7. ACKNOWLEDGMENT

The authors are grateful to Prof. Max Gerken, from São Paulo University in Brazil, for stimulating discussions on the subject of this paper.

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