On Power-Laws in SDH Transport Networks

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Abstract— **It is has been previously demonstrated [4] that Internet topologies which were once considered unstructured networks with no global design processes actually follow power-laws, both at the router level and the AS (Autonomous System) domain level. This discovery has very wide implications on network research as well as network and protocol design. The Internet is not the only network instance to exhibit power laws however; in this paper we present evidence for similar power laws also existing in transport layer topologies; in this case a real world deployed SDH (Synchronous Digital Hierarchy) network. The existence of such traits is unexpected as transport technologies are planned and engineered, in contrast to the rather looser planning and dynamic routing of the Internet. SDH networks are globally designed with multiple hierarchical levels and a specific structure, whereas the Internet is a growing collection of networks under independent control. Data is presented to demonstrate the conformance to power laws of the SDH network, the possible effects of the physical layer and the extent to which the topology remains scale-free throughout the network's hierarchy. The possible sources of the traits are discussed and contrasted to those hypothesized for the Internet.**

Keywords - SDH; transport networks; topology; power-law; self-organising; emergence;

I. Introduction

SDH (Synchronous Digital Hierarchy) networks are used as the transport layer for many different network technologies and provide guaranteed capacity between end-points. They consist of end-to-end circuits that connect users and offer guarantees on available bandwidth, delay and various other service elements such as availability. The need for guaranteed levels of service requires that the end-to-end route for allocated demand capacity is pre-calculated when the network is dimensioned and planned. The paths are not dynamically allocated and do not change. The adjacency to the physical layer restricts the planning of SDH networks greatly with the connectivity being limited by physical layer connectivity.

The Internet in contrast is a collection of IP (Internet Protocol) networks. Each network will route traffic across itself to peering points where the traffic enters another network. Internally these networks comprise of nodes whose routing strategies are often dynamic, using protocols such as OSPF (Open Shortest Path First). The inter-network routing decisions are governed by pre-determined routing policies and implemented in routing protocols such as BGP (the Border Gateway Protocol). The policies are devised by negotiation between network operators. The connectivity is much less restricted because transport networks, such as SDH, provide the end-to-end capacity on which the IP traffic is carried.

In a heterogeneous, dynamic, adaptive system like the Internet it may not be surprising to find power-laws and a self-organizing topology, but in a designed and highly structured hierarchical network like an SDH network it may not be expected. Here we will see how a real world deployed SDH network does in fact follow a number of power-laws quite closely and try to examine possible sources.

II. TRANSPORT NETWORKS

Transport networks exist to transport bits between nodes. An example of such a transport layer technology is SDH. It serves as a capacity packaging structure to allow for the easier management and control of capacity. SDH is a TDM (Time Division Multiplexing) technology designed to multiplex bit flows into larger flows. SDH networks comprise of frames called STMs (Synchronous Transfer Module) into which VCs (Virtual Container) are packaged. STM flows are then transmitted over the physical layer. The VCs provide the node-to-node data transfer with hard guarantees on delay, jitter and available capacity. The concatenation of VCs through a network forms an end-to-end circuit.

SDH can be seen in Fig. 1 as part of a hypothetical simplified multi-layer network. Here SDH is used to transport Leased Line, ATM (Asynchronous Transfer Mode) and IP (e.g. POS, Packet over SDH/SONET) traffic. The physical layer below SDH is a WDM (Wavelength Division Multiplexing) optical network. The figure includes fibre and ducts as they are also layers with their own topologies. SDH's dependence on physical connectivity means that it is influenced directly by available light paths in WDM fibres and fibre placement and therefore the duct connectivity.

The wide variety of network technologies and end-user applications that use SDH for transport mean that the capacity demands on SDH can vary widely in magnitude and topology. To plan an SDH network [1][2] there are many considerations such as available resources (physical layer capacity), network requirements (required capacity, resilience [2][3]) and network structure (SDH hierarchies). Resilience in SDH is achieved by having main and standby circuits for a demand. If any part of a circuit fails then one of the standby circuits are used instead. For easier planning SDH capacity is often arranged in rings to provide alternate paths.

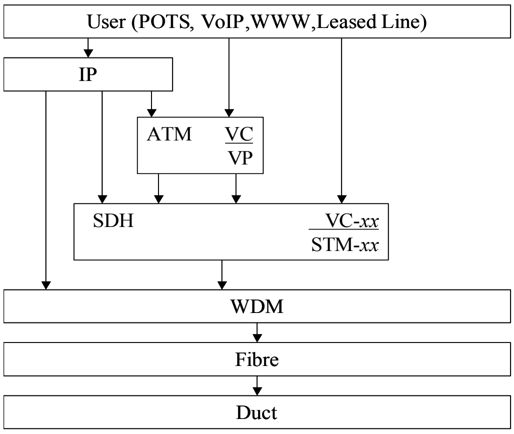

Figure 1. A simplified multi-layer network

The SDH topology is therefore defined by user demand, network architecture (rings, hierarchies) and available physical capacity. The need for rings and the use of hierarchies leads to explicit structure in the topology; a deliberately planned structure, not an unstructured, evolved ad-hoc topology.

III. INTERNET TOPOLOGY POWER-LAWS

Providing demands to the transport network are network layer technologies such as IP. The Internet is the largest example of such networks and is formed by the peering of many smaller such networks. They are not planned to the extent of SDH networks and do not have inherent structure like SDH but do also exhibit hierarchies with LAN (local area networks), MANs (metropolitan area networks) and WANs (wide area networks) [15]. Such an ad-hoc concatenation of loosely planned networks would not be expected to have distinctive topological traits but it does. Faloutsos et al. discovered [4] that the Internet followed four power laws when they examined three instances of inter-domain topologies and one instance of a node-level topology. The node-level topology was the connectivity of the actual IP network routers, while inter-domain topologies were formed from the connectivity of AS (Autonomous System) domains which are the basic network elements in BGP routing. AS domains describe sub-networks of routers which are under the control of specific organisations. The following four laws were found to hold at both the node-level and the BGP domain-level:

Power-Law 1 (rank exponent): The outdegree (connections from a node) was found to be proportional to the rank of a node, to the power of a constant. The rank being the position of the node in a table sorted (numerically decreasing) by the outdegree of the node.

Power-Law 2 (outdegree exponent): The frequency of an outdegree is proportional to the outdegree to the power of a constant.

Power-Law (approximation) 3 (hop-plot exponent): The total number of pairs of nodes within h hops of each other, is proportional to the number of hops to the power of a constant. This is more of an approximation since it only holds for values of h that are much less than the network diameter.

Power-Law 4 (eigenvalue exponent): The sorted eigenvalues (decreasing order) of the adjacency matrix (an N by N matrix (where N is the number of nodes) which is 1 when the two nodes are connected and 0 otherwise) are proportional to the index into the list, to the power of a constant. The power law was shown to hold for only the 20 or so largest eigenvalues.

IV. THE SDH NETWORK

As an example of a transport network a real-world, country wide, deployed SDH network was considered. The network data examined consisted of a list of end-to-end circuits and the sites that they traverse. Each node in the data was a geographic site, rather than actual SDH rack equipment. Therefore nodes in this topology were not SDH equipment (ADMs or Cross-Connects), but rather the physical locations of the equipment. This resulted in loss of measurement resolution but did provide an accurate topology of the allocated capacity between sites, effectively the connectivity of POPs (points of presence). The superposition of these VCs constituted the STM topology. The data described the route of circuits across this topology but did not include the customer end-points and therefore only described the core of the network (including the ingress and egress nodes). All circuits had the same return path as the forward path.

The network consisted of a few thousand sites and had an average number of links per node of 1.77. The data available described an entire topology rather than partial measurements or projections onto the network. In discovering power-laws in the Internet Faloutsos et al. used measurements of the Internet [5] which did not necessarily form a complete map [6]. This data on the other hand is a complete topology, including dedicated capacity allocated to standby protection circuits.

A. Network Topology

In Fig. 2 to Fig. 5 we can see equivalent plots for the SDH network that Faloutsos et al. made [4] of the Internet. It is evident that the SDH topologies also follow the same power laws. In Fig. 2 we can see the degree rank which follows a power-law with a R^2 value of 0.929. There is a trend to a lower than expected degree at the highest ranks. Degree, rather than outdegree, has been used throughout as all links are bi-directional. Various values have been normalised to protect certain network information: Rank is normalised to network size and degree is normalised to number of links. The exponents and R^2 are not affected. The network shows much better conformance to power-laws 2 (Fig. 3) and 3 (Fig. 4), while the eigenvalue power-law (Fig. 5) shows very good conformance to the 30 largest eigenvalues.

In an attempt to extract information about possible structure in the SDH topology measurements of clustering co-efficient were also made. The cluster co-efficient [7] is a measure of the connectivity of the neighbourhood of nodes surrounding a given node. This clustering metric of a node is defined as the number of links in the neighbourhood (a central node and all its immediately adjacent nodes) of that node divided by the total number of possible links in the neighbourhood ((S+1).S/2, where S is the number of neighbours). In Fig. 6 it was found

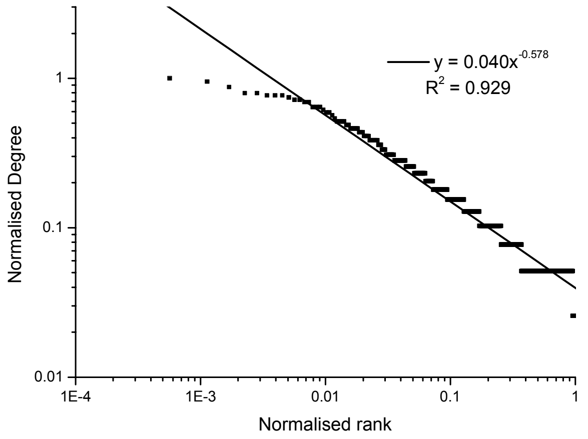

Figure 2. Power-Law 1: The degree rank

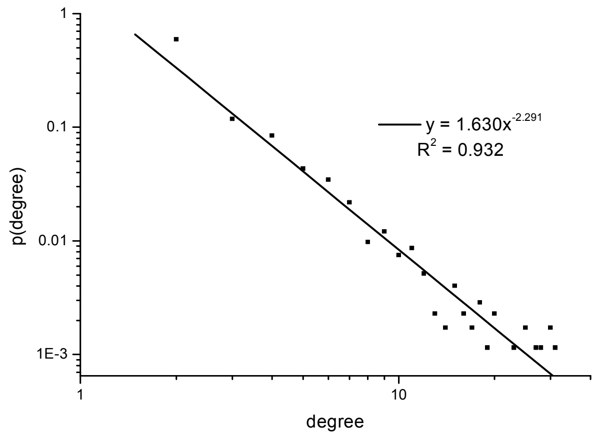

Figure 3. Power-law 2: The degree distribution

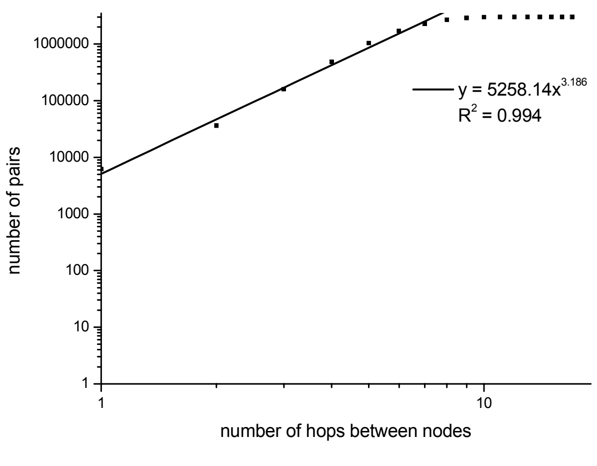

Figure 4. Power-Law 3: The approximation of number of pairs of nodes within a given number of hops

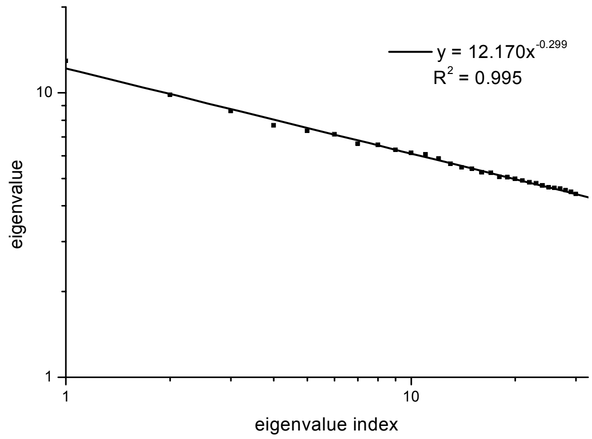

Figure 5. Power-Law 4: The 30 largest sorted eigenvalues of the adjacency matrix

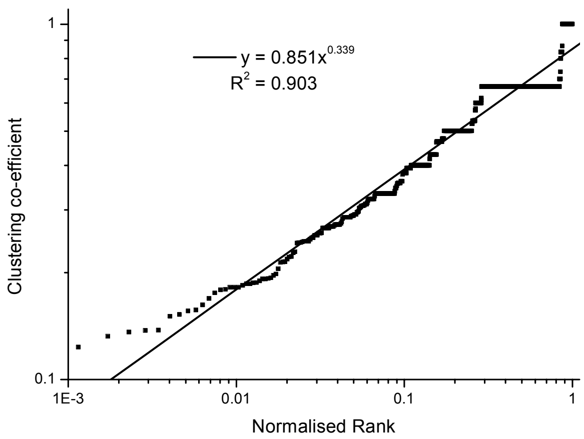

Figure 6. Power-Law 5: The rank of clustering metric

that when ranked in increasing order the co-efficients also followed a power law (Power-Law 5), albeit with some deviation at the low extremity. The exponent values of the five power-laws were therefore: -0.578 (R^2=0.929), -2.291 (R^2=0.932), 3.186 (R^2=0.994), -0.299 (R^2=0.995) and 0.339 (R^2=0.903) respectively.

To investigate the expected dependence that SDH connectivity may have on the physical layer we can see the geographic distance distribution in Fig. 7. The probability of a link is the fraction of all possible node pairs with a given distance between them that were actually linked (rather than a plot of link length distribution). The distance was calculated from the original SDH geography data that was in an arbitrary Cartesian co-ordinate system. One mile (1.61 kilometres) is approximately 177 distance units (one kilometre is 110 distance units). The histogram has a bin size of 500 distance units.

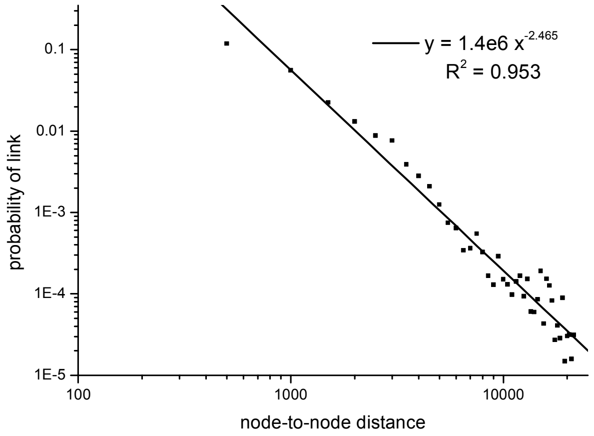

Figure 7. The probability distribution of a link existing over a given distance. Distances have been grouped in 500 distance unit bins.

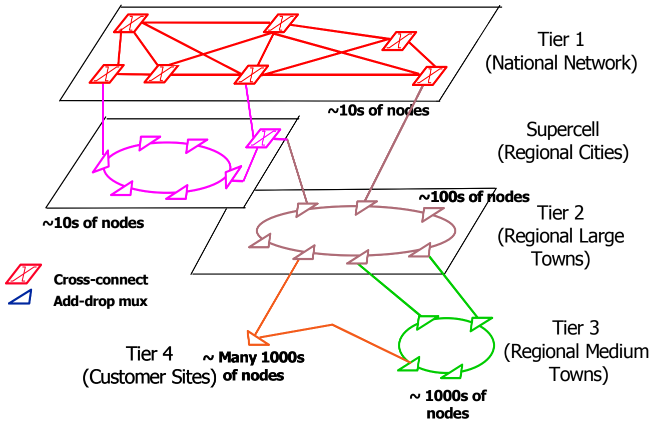

Figure 8. The planned structure of the SDH network. Diagram reproduced with permission of British Telecommunications PLC.

B. *Network Geography and Hierarchy*

To further understand the features of the above plots we should examine the actual planned structure of the SDH hierarchy, which can be seen in Fig. 8. The network consists of four tiers, each of which have various sizes, topologies and cover varying geographic distances. The lower tiers are predominantly ring based structures while the Tier 1 national network is a highly connected mesh of nodes.

This knowledge of the hierarchy can start to explain elements of the clustering co-efficient rank distribution. The high number of nodes with a higher than expected clustering co-efficient which are responsible for the upward turning tail could be attributed to the lack of customer site nodes connected to the edge nodes which would otherwise form the hubs of star topologies. Similarly the large number of nodes with high clustering co-efficients in the highly meshed core could be responsible for the plateau to the right of the graph.

To attempt to examine the effect on topology of the tiers the network's geographic space was divided into a uniform grid of squares. These squares formed a topology whose connectivity followed that of the enclosed SDH nodes. By varying the size of the squares we hope to encapsulate villages, towns, cities and eventually the whole network.

In Fig. 9 we can see the effect of increasing the width M (in the same distance units as used previously) of each square in the grid against the exponent of the degree distribution power-law. As M increases to encapsulate entire population areas the connectivity of these areas still maintains a power-law relationship and the exponent decreases. It is not until the grid size reaches 4000 (about 36 kilometres) that the conformance (R^2 value) to the power-law drops below 0.9. At this scale we are examining the connectivity of towns and small cities, but large cities could still span multiple squares. This can be further seen in Fig. 10 where the number of individual nodes is plotted against the grid size. At a grid size of 4000, just before R^2 drops below 0.9 the topology consists of only 10% of the original number of nodes.

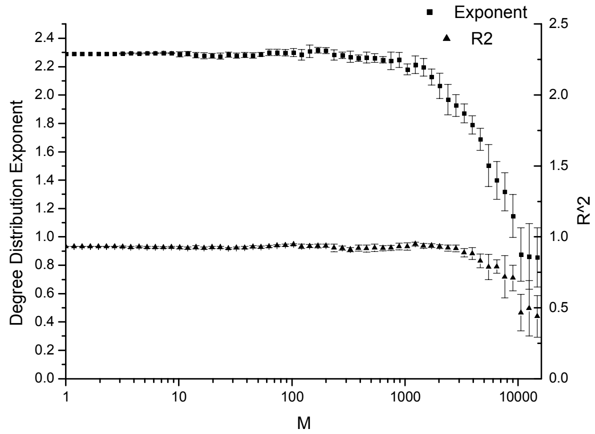

Figure 9. The degree distribution exponent (power-law 2) for various grid sizes, M (distance units), and the conformance to the power law, R^2 on the right axis.

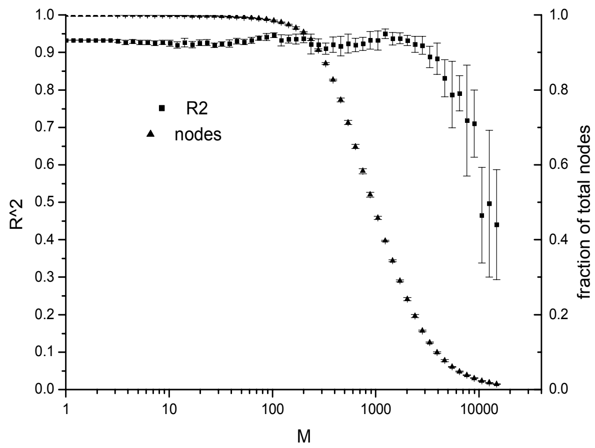

Figure 10. Conformance to power-law 2 (left axis) and the topology size (right axis) against grid size, M.

The grouping of nodes into a uniform grid in such a way is equivalent to the box-counting method of finding fractal dimension [8]. The fractal dimension of the geographic node distribution was found to be 1.46. This demonstrated that the nodes have a scale-invariant geographic distribution [9].

V. DISCUSSION AND CONCLUSION

The existence of power-laws in the transport topology is unexpected considering its highly structured nature. It contains many ideal structures that by themselves do not follow power-laws.

The strong dependence on the physical layer may be partly responsible but connectivity based purely on geography may not necessarily create these traits either. Waxman [10] used a geography based topology generator but used a uniformly random distribution of nodes rather than the fractal distribution here present. The strong reliance on the physical layer rules out network evolution based solely on preferential link attachment [11].

To try to explain possible sources we should consider the design processes involved. While the network is globally planned it constantly undergoes growth and evolution. As new demands are presented to the network engineer these demands must be added with minimal disruption to the existing network, and definitely not with a global re-configuration of the entire network. There are therefore micro-scale forces that influence the topology, not just the macro-scale global design algorithms. When Faloutsos et al. [4] theorized as to the sources of the power-laws in the Internet they suggested co-operative and antagonistic forces working against each other. In the case of the Internet this could be network size growth and the dynamic shortest path routing [13] by OSPF. In the SDH case, network growth exists as one force and the network engineer's actions as another. The engineer's actions are however on much longer time-scales than OSPF and much restricted by the physical layer, explicit structures and so on. The theory does however have merit as it has been known for such co-evolving systems to self-organise and exhibit power-law traits [12]. It was shown that a further requirement of such self-organisation is heterogeneity [14]. Both the Internet and this SDH network are certainly heterogeneous in terms of link capacities, demand topologies and underlying network topologies.

Because of the long time-scales and restrictions we hypothesise that the cause of power laws in transport networks could be feedback from the adjacent layers; the physical layer and the network layer. The demands made by the network layer must be satisfied by reconfiguration in the transport layer and eventual transmission across the physical layer. The physical layer in return must adapt (within restrictions) to carry transport layer demands (fibres are added or upgraded). The eventual goal of the network engineer is the optimisation of resources; to carry the most traffic using the least possible resources.

The applicability of this discovery is widespread, from network performance analysis and scalability investigations to SDH network modeling and the generation of realistic demands for WDM planners. Rather than use ideal traffic models, more realistic models that generate power-law networks can be used [16]. A number of power-law compliant topology generators already exist [11][17], although they do not necessarily create networks with similar power-law exponents.

This paper has demonstrated the existence of power-law traits in one snapshot of a SDH transport network and examined the possible effect geography, imposed structure and hierarchy could have on the topology. We also hypothesised as to possible sources for the Power-Law traits and compared them to those theorised for the Internet.

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