A Nonlinear Iterative Learning Controller for a Finite-Time Convergence against Initial State Error

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Abstract

In this paper, a nonlinear iterative learning control(ILC) algorithm is proposed for a LTI system and it is shown that the effect of the initial state error can be reached to zero in a given finite time. It is also shown that the bound of error reduction can be effectively contrlled by tuning gains of the proposed controller. In order to confirm validity of the proposed ILC algorithm, an example is presented.

1 Introduction

The iterative learning control (ILC) method has been found to be a good alternative when detailed knowledge about the plant is not available and the required task is repetitive[1, 2, 3]. The ILC algorithm is generally expressed in the following form:

$$u_{k+1}(t) = u_k(t) + f\left(e_k(\cdot)\right)(t), 0 \le t \le T,$$

where $u_k(t)$ is the control input and f is a functional of error function $e_k(t)$, $0 \le t \le T$, between the actual output and the desired output at the k-th iteration. Most of the results up to now on the ILC are in the form of linear control algorithm. In general, linear control algorithms are more simple to implement and easier to analyze than nonlinear algorithms. However, a nonlinear controller may possess capability to overcome limitations of a linear controller and improve the system performance. In this paper, we consider the effect of the initial state error and show that the proposed nonlinear ILC algorithm can improve the performance against the initial state error.

Consider the linear system described by Eqn. (1).

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

Here, $x \in R^n, u \in R^r$ and $y \in R^m$ denote state, input and output, respectively. A, B and C are matrices with appropriate dimensions and it is assumed that CB is a full rank matrix. Let $x_d(\cdot)$ be the desired state trajectory and $y_d(\cdot)$ be the corresponding output trajectory. Assume that $y_d(\cdot)$ and $x_d(\cdot)$ are continuously differentiable on $[0, T]$. Lee and Bien[4] showed that when the ILC algorithm Eqn. (2) is applied to the system Eqn. (1), the output trajectory converges to the form in Eqn. (3).

$$u_{k+1}(t) = u_k(t) + \Gamma\left(\dot{e}_k(t) - Re_k(t)\right) \tag{2}$$

$$\lim_{k\to\infty} y_k(t) = y_d(t) + e^{Rt}C\left(x_0 - x_d(0)\right) \tag{3}$$

Here, $y_k(\cdot), u_k(\cdot)$ and $e_k(\cdot)$ are output trajectory, control input trajectory, and output error trajectory at k-th iteration, respectively. Eqn. (3) shows that the effect of initial state error can be controlled by gain R of the ILC algorithm, and the error asymptotically converges to zero if R is chosen such that all eigenvalues have negative real parts. In the paper, it is shown that when the proposed nonlinear ILC algorithm is applied to the system Eqn. (1), the error can be reached to zero in a finite time and can be bounded by an exponential function.

In the sequel, for n-dimensional Euclidean space R^n, $\|x\|_\infty$ denotes sup-norm of a vector $x = (x_1, \cdots, x_n)^T$ defined by $\|x\|_\infty = \sup_{1\le i\le n} |x_i|$ and $\{x\}_i$ denotes the i-th element of x. For an $n \times r$ matrix A with elements a_{ij}, $\|A\|_\infty$ is defined by $\|A\|_\infty = \max_{1\le i\le n}\sum_{j=1}^{r}|a_{ij}|$. As in the notation $u_k(t)$, the subscript k denotes the iteration number.

2 Nonlinear ILC algorithm for a Finite-Time Convergence

In this section, the effect of initial state error for LTI systems is shown. Let there be given a desired trajectory $y_d(t)$, $t \in [0, T]$, and a finite time $t_s \le T$ for a LTI system described by Eqn. (1). The problem is to find a control input $u(t), 0 \le t \le T$, such that the error $e_k(t)$ is reached to zero in a given finite time t_s and is exponentially bounded.

The following ILC algorithm is proposed as a solution.

$$u_{k+1}(t) = u_k(t) + \Gamma\left(\dot{e}_k(t) + Re(t)^{q/p}\right) \tag{4}$$

where $e_k(t) = y_d(t) - y_k(t)$, $R = diag\{r_i\}$, and $p, q(p > q)$ are positive odd integers.

Now, the convergence of the ILC algorithm (4) will be shown.

Theorem 1 *Suppose that the update law Eqn. (4) is applied to the system Eqn. (1) and the initial state at each iteration can be different from the desired initial state, i.e.,* $x_k(0) = x_0 \neq x_d(0)$*, for* $k = 0, 1, 2, \cdots$*. If*

$$\|I - \Gamma CB\|_\infty \leq \rho < 1$$

then,

$$\lim_{k\to\infty} \{y_k(t)\}_i = \begin{cases} \{y_d(t)\}_i + \{C(x_0 - x_d(0))\}_i \\ \times \left[1 - r_i \left\{C^{\frac{p-q}{p}}(x_0 - x_d(0))\right\}_i^{\frac{q-p}{p}} t\right]^{\frac{p}{p-q}} \\ , 0 \leq t \leq t_s^i \\ \\ \{y_d(t)\}_i, \quad t_s^i < t \leq T \end{cases}$$

$$t_s^i = \frac{p}{r_i(p-q)} \{C(x_0 - x_d(0))\}_i^{\frac{p-q}{p}}.$$

Theorem 1 implies that if the initial state error is the same at each iteration, the output trajectory can be exactly estimated from the information about the desired output trajectory, the initial output error value, and the learning controller parameters p, q and R. Note that if the parameters p and q are chosen as $p = q$, the algorithm becomes PD-type as in Eqn. (2) and the output trajectory converges to Eqn. (3). From this observation, the proposed nonlinear ILC algorithm is considered as an extension of the PD-type ILC algorithm. Assume that the initial state error at each iteration is bounded such that

$$\|x_0 - x_d(0)\|_\infty \leq \Delta.$$

Then, for a given finite time t_s, if we know the bound Δ, we can choose the parameters p, q and r_i such that $t_s^i \leq t_s, 1 \leq i \leq m$. It is easily seen that the converged output trajectory is satisfying the following:

$$\begin{aligned} |\{C(x_0 - x_d(0)\}_i| - r_i t &\leq |\lim_{k\to\infty} \{y_k(t) - y_d(t)\}_i| \\ &\leq |\{C(x_0 - x_d(0)\}_i| e^{-r_i t}. \end{aligned}$$

Note that r_i determine the exponential bound.

3 Numerical Example

The following example is given to illustrate efficiency of the proposed algorithms.

Consider the following dynamic system.

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{aligned}$$

The desired output trajectory and the initial state are given as follows.

$$\begin{aligned} y_d(t) &= 3t(1-t), 0 \leq t \leq 1 \\ x_k(0) &= x_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \end{aligned}$$

According to Theorem 1, one can know that the best choice of Γ is 1. Under the assumption of 30% uncertainty of the system parameters, Γ can be chosen as 1/1.3. Fig. 1 shows the output trajectory at 20th iteration. The bottom line shows the desired trajectory and the top line shows the exponential bound.

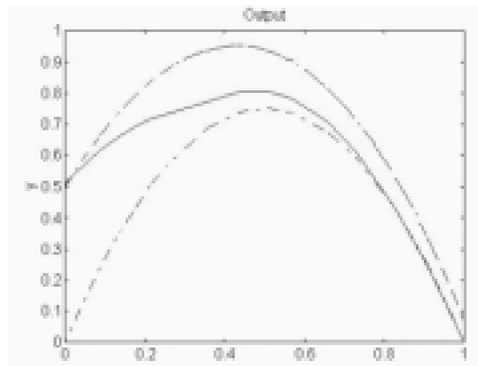

Figure 1: output trajectory

4 Concluding Remark

In this paper, robustness of a nonlinear ILC algorithm was investigated against initial state error. It was shown that the proposed nonlinear ILC algorithm could more effectively control the effect of the initial state error. Robustness of the nonlinear ILC algorithm is open to further investigateion for nonlinear systems and against random initial state error.

References

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