Matched–Filter– and MMSE–Based Iterative Equalization with Soft Feedback for QPSK Transmission

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Abstract— **In this paper, two novel equalization algorithms applying soft–decision feedback, designed for quaternary phase–shift keying (QPSK) transmission are introduced. The first one uses a matched filter (MF) as a front end and is derived from [1] where it was originally developed for binary transmission. The second one is an improved version of the first, employing a minimum mean–squared error (MMSE) filter in each iteration in order to refine the data estimates. The rule for generating soft decisions is adapted continuously to the current state of the algorithm. In most cases, standard DFE methods are clearly outperformed. In contrast to the algorithm of [1], a performance close to that of maximum–likelihood sequence estimation can be obtained for the MMSE–based scheme for a broad class of channels.**

Keywords— **Equalization, Soft–Decision Feedback, Iterative Sequence Estimation, MMSE filtering**

I. Introduction

It is well known that maximum–likelihood sequence estimation (MLSE) using the Viterbi algorithm (VA) [2] is the optimum scheme for equalization of dispersive channels producing intersymbol interference (ISI). However, the complexity of the VA becomes prohibitively high for long channel impulse responses (CIR's), and suboptimum schemes have to be applied in this case, like decision–feedback equalization (DFE) or delayed decision–feedback sequence estimation (DDFSE) [3]. Because a minimum–phase impulse response is essential for suboptimum trellis–based equalizers, in general, a prefilter is necessary which transforms the CIR into its minimum–phase equivalent [4]. Even with optimized prefilter, a very high number of states might be necessary for reduced–state equalizers such as DDFSE in order to approach the performance of maximum–likelihood sequence estimation (MLSE).

In [1], an iterative algorithm for binary phase–shift keying (BPSK) transmission has been introduced, which requires no minimum–phase response but employs a matched filter (MF) as a front end and performs even better than optimized DDFSE with high number of states. However, this holds only for long random CIR's. For CIR's of moderate length and/or with well–defined shape, an error floor occurs and the performance of MLSE cannot be achieved.

In this paper, we first generalize the algorithm of [1] to quaternary phase–shift keying (QPSK) transmission and then introduce a scheme using minimum mean–squared error (MMSE) filtering instead of a MF. It is demonstrated that with this modification, high performance is also obtained for CIR's of only moderate length. A similar MMSE–based scheme has been proposed in [5] for synchronous CDMA transmission. But contrary to the MMSE scheme proposed in this paper, ISI was not considered in [5] and the width of the MMSE filter was equal to the block length (i.e., CDMA symbol length). In this paper, we use a sliding window MMSE filter, which results in a noticeably lower complexity compared to filtering of a whole transmission block, while high performance is maintained.

In [6], [7], [8] iterative equalization algorithms ("turbo equalization") have been derived from the turbo decoding approach [9]. In contrast to the approach in this paper in each iteration channel decoding is exploited in order to refine the data estimates. Using these approaches obviously leads to a higher complexity due to the decoding process involved in each iteration. Iterative equalization without including a decoder like in this paper was also proposed in [10], [11]. But contrary to our approach, only hard estimates or soft decisions generated by simplified rules are fed back in order to cancel ISI leading to a suboptimum performance.

The paper is organized as follows. First, we introduce the system model in Section II. In Section III, we develop an iterative receiver algorithm for QPSK based on matched filtering. In Section IV we derive an improved version of the first scheme using MMSE filters in each iteration. Simulation results for both algorithms and various channels are given in Section V and compared with the corresponding results for DFE and a performance bound for MLSE [2].

II. System Model

A packet transmission according to Fig. 1 with QPSK and Gray mapping is considered. In the following, all signals are represented by their complex–valued baseband equivalents. The discrete–time received signal is given by

$$r[k] = \sum_{\kappa=0}^{q_h} h[\kappa]\, a[k-\kappa] + n_0[k], \qquad (1)$$

where $a[k] \in \{\pm 1 \pm \mathrm{j}\}$ is the transmitted QPSK coefficient at discrete time k, $h[\cdot]$ denotes the causal CIR of order q_h comprising transmit filter, channel, and continuous–time receiver input filter, and $n_0[k]$ is additive complex white Gaussian noise with variance $\sigma_{n_0}^2$. In general, the CIR is not of minimum phase. We assume that ideal channel state information is available at the receiver. In order to simplify notation, the discrete time index $k = 1$ is assigned to the first of L data symbols in each burst.

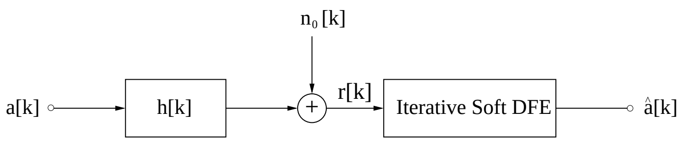

Fig. 1. Model of transmission.

For the MMSE–based algorithm we use a matrix vector notation for simplicity, which is introduced in the following. First, we define a convolution matrix $\boldsymbol{H}$ of size $(L + q_h)\times L$ whose entries in the ith row and jth column are $\boldsymbol{H}_{(i,j)} = h[i - j]$. Then, the received signal vector

$$\boldsymbol{r} = [\, r[1], r[2], \ldots, r[L + q_h]\,]^T \tag{2}$$

can be expressed as

$$\boldsymbol{r} = \boldsymbol{H} \cdot \boldsymbol{a} + \boldsymbol{n}_0 \,, \tag{3}$$

where the definitions

$$\boldsymbol{a} = [\, a[1], a[2], \ldots, a[L]\,]^T \tag{4}$$

and

$$\boldsymbol{n}_0 = [\, n_0[1], n_0[2], \ldots, n_0[L + q_h]\,]^T \tag{5}$$

have been used. Furthermore, a truncated version of $\boldsymbol{r}$ is used to derive the sliding window MMSE filter of the MMSE–based algorithm. We consider q_w received samples before and $q_w + q_h$ received samples after k_0, respectively, for estimation of $a[k_0]$. Collecting these samples in a vector $\boldsymbol{r}_{k_0,q_w}$, we obtain

$$\boldsymbol{r}_{k_0,q_w} = \boldsymbol{H}_{k_0,q_w} \cdot \boldsymbol{a}_{k_0,q_w} + \boldsymbol{n}_{k_0,q_w} \,, \tag{6}$$

with

$$\begin{aligned}\boldsymbol{r}_{k_0,q_w} \;&=\; [\, r[k_0 - q_w], \ldots, r[k_0], \ldots, r[k_0 + q_w], \\ &\qquad \ldots, r[k_0 + q_w + q_h]\,]^T,\end{aligned} \tag{7}$$

$\boldsymbol{H}_{k_0,q_w\,(i,j)} = h[q_h + i - j]$, where $\boldsymbol{H}_{k_0,q_w}$ has size $(2q_w + 1 + q_h)\times(2q_w + 1 + 2q_h)$,

$$\begin{aligned}\boldsymbol{a}_{k_0,q_w} \;&=\; [\, a[k_0 - q_w - q_h], \ldots, a[k_0], \\ &\qquad \ldots, a[k_0 + q_w + q_h]\,]^T,\end{aligned} \tag{8}$$

and

$$\begin{aligned}\boldsymbol{n}_{k_0,q_w} \;&=\; [\, n[k_0 - q_w], \ldots, n[k_0], \ldots, n[k_0 + q_w], \\ &\qquad \ldots, n[k_0 + q_w + q_h]\,]^T.\end{aligned} \tag{9}$$

We assume that a sufficient number of tail symbols are sent prior and after each block, i.e., $a[k] = 0$ is valid for $k \notin \{1, 2 \ldots, L\}$. This means that e.g. $\boldsymbol{a}_{1,q_w}$ contains $q_w + q_h$ leading zeros and $\boldsymbol{a}_{L,q_w}$ contains $q_w + q_h$ trailing zeros.

III. Matched Filter Based Iterative Soft Decision Interference Cancellation (MF ISDIC)

For the MF–based algorithm, in front of equalization a matched filter $h^*[-k]$, whose coefficients in practice result immediately from channel estimation, is applied to the received signal. Its output signal $u[k]$ can be written as

$$u[k] = \sum_{\kappa=-q_h}^{q_h} \rho_{hh}[\kappa]\, a[k - \kappa] + n[k], \tag{10}$$

with the filter autocorrelation sequence of the CIR $\rho_{hh}[\kappa] = h[\kappa] * h^*[-\kappa]$ and $n[k] = h^*[-k] * n_0[k]$. Application of a matched filter as prefilter concentrates the energy of the overall impulse response in time instant $\kappa = 0$. However, due to noncausal ISI, i.e., precursors of significant energy occur, conventional (noniterative) DFE cannot be employed.

In each iteration of the algorithm, soft–decision feedback is performed for cancellation of pre- and postcursor ISI in a sequential manner starting from $k = 1$ up to $k = L$,

$$y_\mu[k] = u[k] - \sum_{\kappa=1}^{q_h} \left(\rho_{hh}[\kappa]\, \hat{a}^{\mathrm{s}}_\mu[k - \kappa] + \rho_{hh}[-\kappa]\, \hat{a}^{\mathrm{s}}_{\mu-1}[k + \kappa]\right) . \tag{11}$$

Here, $\mu > 0$ is the number of the current iteration, and $\hat{a}^{\mathrm{s}}_\mu[k]$ denotes the soft decision on $a[k]$ after iteration μ. It should be noted that the soft decisions $\hat{a}^{\mathrm{s}}_\mu[k]$ minimize the mean–squared error (MMSE) after feedback in the current iteration under the assumption that the sum of noise and ISI can be modeled as Gaussian random variable with zero mean and uncorrelated inphase and quadrature components (which is used in the following), cf. [12], [13]. The soft decisions $\hat{a}^{\mathrm{s}}_\mu[k]$ are initialized according to

$$\hat{a}^{\mathrm{s}}_0[k] = 0 \,, \quad k \in \{1, 2, \ldots, L\} \,. \tag{12}$$

After soft cancellation of ISI, a soft decision for QPSK symbol $a[k]$ is calculated by

$$\begin{aligned}\hat{a}^{\mathrm{s}}_\mu[k] \;&=\; \mathcal{E}\left\{a[k] | y_\mu[k]\right\} \\ &=\; \mathcal{E}\left\{\mathrm{Re}\left\{a[k]\right\} | \,\mathrm{Re}\left\{y_\mu[k]\right\}\right\} \\ &\qquad + \mathrm{j}\, \mathcal{E}\left\{\mathrm{Im}\left\{a[k]\right\} | \,\mathrm{Im}\left\{y_\mu[k]\right\}\right\} \\ &=\; \tanh\left(\frac{\rho_{hh}[0]}{\hat{\sigma}^2_{I,\mu}[k]}\, \mathrm{Re}\left\{y_\mu[k]\right\}\right) \\ &\qquad + \mathrm{j}\tanh\left(\frac{\rho_{hh}[0]}{\hat{\sigma}^2_{Q,\mu}[k]}\, \mathrm{Im}\left\{y_\mu[k]\right\}\right)\end{aligned} \tag{13}$$

and used for ISI cancellation in the next time steps of the current iteration. In (13), $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real part and the imaginary part of complex numbers, respectively. $\hat{\sigma}^2_{I,\mu}[k]$ is the estimated variance of the error signal after soft cancellation influencing the inphase component of $y_\mu[k]$ and $\hat{\sigma}^2_{Q,\mu}[k]$ denotes the estimated variance of the error signal influencing the quadrature component of $y_\mu[k]$. Both may be derived analogously to [14] resulting in

$$\hat{\sigma}^2_{I,\mu}[k] \;=\; = \frac{\sigma^2_n}{2} + \sum_{\kappa=1}^{q_h} \left(\left(\mathrm{Re}\left\{\rho_{hh}[\kappa]\right\}\right)^2 \left(1 - \left(\mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_\mu[k - \kappa]\right\}\right)^2\right.\right.$$

$$+1-\left(\mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\mu-1}[k+\kappa]\right\}\right)^2\Big)$$
$$+\left(\mathrm{Im}\left\{\rho_{hh}[\kappa]\right\}\right)^2\Big(1-\left(\mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\mu}[k-\kappa]\right\}\right)^2$$
$$+1-\left(\mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\mu-1}[k+\kappa]\right\}\right)^2\Big)\Big)\,, \quad (14)$$

$$\hat{\sigma}^2_{Q,\mu}[k] = \\ = \frac{\sigma_n^2}{2}+\sum_{\kappa=1}^{q_h}\Big(\left(\mathrm{Im}\left\{\rho_{hh}[\kappa]\right\}\right)^2\Big(1-\left(\mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\mu}[k-\kappa]\right\}\right)^2$$
$$+1-\left(\mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\mu-1}[k+\kappa]\right\}\right)^2\Big)$$
$$+\left(\mathrm{Re}\left\{\rho_{hh}[\kappa]\right\}\right)^2\Big(1-\left(\mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\mu}[k-\kappa]\right\}\right)^2$$
$$+1-\left(\mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\mu-1}[k+\kappa]\right\}\right)^2\Big)\Big)\,, \quad (15)$$

where the variance of $n[k]$ is:

$$\sigma_n^2 = \rho_{hh}[0]\sigma^2_{n_0}\,. \quad (16)$$

After the current iteration has arrived at the position of the last data symbol, a new iteration starts. The algorithm stops if soft decisions remain essentially unchanged from one iteration to the next, i.e.

$$\max_{k\in\{1,2,\ldots,L\}}\left|\mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\mu}[k]\right\}-\mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\mu-1}[k]\right\}\right| < \epsilon \quad \text{and}$$
$$\max_{k\in\{1,2,\ldots,L\}}\left|\mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\mu}[k]\right\}-\mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\mu-1}[k]\right\}\right| < \epsilon\,, \quad (17)$$

with a small constant ϵ, or the iteration number exceeds a prescribed limit $\mu_{\max}$. Hard estimates for the data symbols are obtained from the signs of the inphase and quadrature soft decisions of last iteration μ_{stop}:

$$\hat{a}[k] = \mathrm{sign}\left(\mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\mu_{\mathrm{stop}}}[k]\right\}\right)+\mathrm{j}\,\mathrm{sign}\left(\mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\mu_{\mathrm{stop}}}[k]\right\}\right) \\ k\in\{1,2,\ldots,L\}. \quad (18)$$

IV. MMSE Based Iterative Soft Decision Interference Cancellation (MMSE ISDIC)

For derivation of the MMSE–based algorithm, we have to introduce two vectors,

$$\bar{\boldsymbol{a}}^s_{k,q_w,\mu} = [\,\hat{a}^s_{\mu}[k-q_w-q_h],\ldots,\hat{a}^s_{\mu}[k-1],0, \\ \hat{a}^s_{\mu-1}[k+1],\ldots,\hat{a}^s_{\mu-1}[k+q_w+q_h]\,]^T \quad (19)$$

and

$$\hat{\boldsymbol{a}}^s_{k,q_w,\mu} = [\,\hat{a}^s_{\mu}[k-q_w-q_h],\ldots,\hat{a}^s_{\mu}[k-1],\hat{a}^s_{\mu-1}[k], \\ \hat{a}^s_{\mu-1}[k+1],\ldots,\hat{a}^s_{\mu-1}[k+q_w+q_h]\,]^T, \quad (20)$$

which are both used for calculation of the estimate $\hat{a}^s_{\mu}[k]$.

In each iteration and for each $k\in\{1,2,\ldots,L\}$ the following steps have to be done. In order to obtain an estimate for the desired coefficient $a[k]$ intersymbol interference from other coefficients is removed from the vector $\boldsymbol{r}_{k,q_w}$ in the best possible way using the latest soft estimates in vector $\bar{\boldsymbol{a}}^s_{k,q_w,\mu}$:

$$\boldsymbol{r}_{k,q_w,\mu} = \boldsymbol{r}_{k,q_w}-\boldsymbol{H}_{k,q_w}\cdot\bar{\boldsymbol{a}}^s_{k,q_w,\mu} \\ = \boldsymbol{H}_{k,q_w}\cdot\left(\boldsymbol{a}_{k,q_w}-\bar{\boldsymbol{a}}^s_{k,q_w,\mu}\right)+\boldsymbol{n}_{k,q_w}\,. \quad (21)$$

This leads to a vector $\boldsymbol{r}_{k,q_w,\mu}$ which contains significantly less interference compared to $\boldsymbol{r}_{k,q_w}$ when the soft estimates contained in $\bar{\boldsymbol{a}}^s_{k,q_w,\mu}$ get better from iteration to iteration. To calculate an estimate $\hat{a}^s_{\mu}[k]$ from the vector $\boldsymbol{r}_{k,q_w,\mu}$ an MMSE filter is used. Obviously, when calculating this MMSE filter, it is advantageous to utilize prior knowledge contained in the vector $\boldsymbol{r}_{k,q_w,\mu}$. Therefore, the MMSE filter $\boldsymbol{w}^H_{k,q_w,\mu}$ is determined based on conditioned expectations:

$$\boldsymbol{w}^H_{k,q_w,\mu} = \mathcal{E}\left\{a[k]\cdot\boldsymbol{r}^H_{k,q_w,\mu}|\boldsymbol{u}_{k,q_w,\mu}\right\} \\ \cdot\left(\mathcal{E}\left\{\boldsymbol{r}_{k,q_w,\mu}\cdot\boldsymbol{r}^H_{k,q_w,\mu}|\boldsymbol{u}_{k,q_w,\mu}\right\}\right)^{-1}$$

The vector $\boldsymbol{u}_{k,q_w,\mu}$ is defined as

$$\boldsymbol{u}_{k,q_w,\mu} = [\,u_{\mu}[k-q_w-q_h],\ldots,u_{\mu}[k-1],0, \\ u_{\mu-1}[k+1],\ldots,u_{\mu-1}[k+q_w+q_h]\,]^T\,, \quad (22)$$

where $u_{\mu}[k]$ is the output of the MMSE filter:

$$u_{\mu}[k] = \boldsymbol{w}^H_{k,q_w,\mu}\boldsymbol{r}_{k,q_w,\mu}\,. \quad (23)$$

The initialization of $u_{\mu}[k]$ is done according to:

$$u_0[k] = 0\,,\quad \forall k. \quad (24)$$

Neglecting the statistical dependencies arising between the filter outputs $u_{\mu}[k]$ and $\hat{a}^s_{\mu}[l]$, $a[l]$ for $k\neq l$ and between $u_{\mu}[k]$ and $n[l]$ $\forall l,k$ in course of the iterations to allow analytical tractability in general, calculation of $\boldsymbol{w}^H_{k,q_w,\mu}$ leads to:

$$\boldsymbol{w}^H_{k,q_w,\mu} = \sigma_a^2\left(\boldsymbol{H}_{k,q_{w\,(:,q_w+1)}}\right)^H\cdot\Big(\boldsymbol{H}_{k,q_w} \\ \cdot\mathcal{E}\left\{\left(\boldsymbol{a}_{k,q_w}-\bar{\boldsymbol{a}}^s_{k,q_w,\mu}\right)\left(\boldsymbol{a}_{k,q_w}-\bar{\boldsymbol{a}}^s_{k,q_w,\mu}\right)^H|\boldsymbol{u}_{k,q_w,\mu}\right\} \\ \cdot\boldsymbol{H}_{k,q_w}+\sigma_n^2\boldsymbol{I}\Big)^{-1}. \quad (25)$$

Here, $\boldsymbol{H}_{k,q_{w\,(:,q_w+1)}}$ denotes the (q_w+1)th column of matrix $\boldsymbol{H}_{k,q_w}$. For the expectation in (25) we get:

$$\mathcal{E}\left\{\left(\boldsymbol{a}_{k,q_w}-\bar{\boldsymbol{a}}^s_{k,q_w,\mu}\right)\left(\boldsymbol{a}_{k,q_w}-\bar{\boldsymbol{a}}^s_{k,q_w,\mu}\right)^H|\boldsymbol{u}_{k,q_w,\mu}\right\} = \\ \mathrm{diag}\left(\sigma_a^2-|\hat{a}^s_{\mu}[k-q_w]|^2,\ldots,\sigma_a^2-|\hat{a}^s_{\mu}[k-1]|^2,\sigma_a^2,\right. \\ \left.\sigma_a^2-|\hat{a}^s_{\mu}[k+1]|^2,\ldots,\sigma_a^2-|\hat{a}^s_{\mu}[k+q_w]|^2\right)\,, \quad (26)$$

where we have used $\hat{\boldsymbol{a}}^s_{k,q_w,\mu} = \mathcal{E}\{\boldsymbol{a}_{k,q_w}|\boldsymbol{u}_{k,q_w,\mu}\}$, cf. (13). Applying the derived MMSE filter $\boldsymbol{w}^H_{k,q_w,\mu}$ to the vector $\boldsymbol{r}_{k,q_w,\mu}$ we get

$$u_{\mu}[k] = \boldsymbol{w}^H_{k,q_w,\mu}\boldsymbol{r}_{k,q_w,\mu} = b_{k,q_w,\mu}a[k]+i_{k,q_w,\mu}\,, \quad (27)$$

with a bias $b_{k,q_w,\mu}$ produced by the MMSE filter and a distortion $i_{k,q_w,\mu}$ composed of the sum of residual interference and channel noise. The bias term can be expressed as

$$b_{k,q_w,\mu} = \boldsymbol{w}^H_{k,q_w,\mu}\boldsymbol{H}_{k,q_{w\,(:,q_w+1)}}\,. \quad (28)$$

Multiplying the filter output with a factor $1/b_{k,q_w,\mu}$ we get the unbiased signal:

$$\frac{u_{\mu}[k]}{b_{k,q_w,\mu}} = \frac{\boldsymbol{w}^H_{k,q_w,\mu}\boldsymbol{r}_{k,q_w,\mu}}{b_{k,q_w,\mu}} = a[k]+\frac{i_{k,q_w,\mu}}{b_{k,q_w,\mu}}\,. \quad (29)$$

The power of the residual interference and noise after bias compensation is calculated in the following. According to [15]:

$$b_{k,q_w,\mu} = \frac{\mathrm{SNR}_{\mathrm{unbiased}}}{\mathrm{SNR}_{\mathrm{unbiased}} + 1}\,, \tag{30}$$

where $\mathrm{SNR}_{\mathrm{unbiased}}$ is defined as:

$$\mathrm{SNR}_{\mathrm{unbiased}} = \frac{\sigma_a^2}{\mathcal{E}\left\{\left|\frac{i_{k,q_w,\mu}}{b_{k,q_w,\mu}}\right|^2\right\}}\,. \tag{31}$$

Applying straightforward calculations, we get the power of the residual interference and noise after bias compensation from Eqs. (30) and (31):

$$\mathcal{E}\left\{\left|\frac{i_{k,q_w,\mu}}{b_{k,q_w,\mu}}\right|^2\right\} = \sigma_a^2\,\frac{1-b_{k,q_w,\mu}}{b_{k,q_w,\mu}}\,. \tag{32}$$

We finally are able to calculate $\hat{a}^s_\mu[k]$,

$$\begin{aligned}\hat{a}^s_\mu[k] &= \tanh\left(\frac{b_{k,q_w,\mu}}{1-b_{k,q_w,\mu}}\,\mathrm{Re}\left\{\frac{u_\mu[k]}{b_{k,q_w,\mu}}\right\}\right)\\ &\quad +\mathrm{j}\tanh\left(\frac{b_{k,q_w,\mu}}{1-b_{k,q_w,\mu}}\,\mathrm{Im}\left\{\frac{u_\mu[k]}{b_{k,q_w,\mu}}\right\}\right), \end{aligned} \tag{33}$$

where we have assumed that inphase and quadrature component of $\frac{i_{k,q_w,\mu}}{b_{k,q_w,\mu}}$ are uncorrelated and have identical variance $\frac{\sigma_a^2}{2}\,\frac{1-b_{k,q_w,\mu}}{b_{k,q_w,\mu}}$ (with $\sigma_a^2 = 2$). The stopping criterion is the same as in (17). Hard estimates are calculated again according to (18), and initialization is done according to (12).

V. Numerical Results and Discussion

In the following, the performance of the proposed algorithms is compared for three channels, A, B, and C. The discrete–time impulse response of channel A, which is time–invariant, is shown in Fig. 2. It consists of 25 randomly generated real–valued taps. Such an impulse response may be encountered e.g. in a high–rate transmission over a multipath channel with path weights of equal average power, cf. [16]. Channels B and C are time–variant and have complex–valued coefficients.

Channel B consists of 15 Rayleigh fading taps with exponentially decreasing power. The corresponding power delay profile is given in Fig. 3. Channel C is a Rayleigh fading channel with 15 taps of equal average power. In Figs. 4 to 7 simulation results for both algorithms are shown. In all figures, the respective matched filter bound and the performance of zero–forcing (ZF) DFE [17] is given. For simulations, data block length has been chosen to $L = 256$, number of iterations maximally tolerated to $\mu_{\max} = 100$, and $\epsilon = 10^{-3}$. We use $q_w = q_h$ in the MMSE–based algorithm in order to have a sufficient filter length for interference suppression.

Fig. 4 shows the performance of both algorithms for channel A. In Fig. 5 the performance of the algorithms is shown for the Rayleigh fading channel B with ideal power control, which can be understood as normalizing $\sum_{k=0}^{q_h} |h[k]|^2$ to unity. Figs. 6 and 7 show the performance of both algorithms for the Rayleigh fading channel C with ideal and without power control. The matched filter bound for the non power controlled channel C in Fig. 7 is calculated according to [18] whereas the matched filter bound in all other figures is simply given by the AWGN channel performance. For the fixed channel A, we have also included a performance bound for MLSE [2],

$$\mathrm{BER} \approx \mathrm{Q}\left(\sqrt{d^2_{\min}\,\frac{E_b}{\mathcal{N}_0}}\right), \tag{34}$$

where the normalized squared Euclidean distance $d^2_{\min}$ has been calculated to $d^2_{\min} = 2$ for this channel using the Dijkstra algorithm [19]. But is has to be mentioned, that MLSE is barely feasible due to the huge complexity.

As can be seen from the simulation results the MMSE–based scheme is superior to ZF DFE for all channels and only for high signal–to–noise ratios ZF DFE performs better than the MF–based scheme. Obviously the MMSE–based algorithm clearly outperforms the MF–based one for all considered channels. In [1] it has been observed that the MF–based scheme needs a quite long CIR without well-defined shape in order to exhibit a very good performance close to that of MLSE. Now, using the MMSE–based scheme, it is possible to achieve a good performance even for CIR's of moderate length. Furthermore, even for channel B which is characterized by quite structured CIR's, the performance gap between MMSE ISDIC and MLSE is limited to ≈ 0.5 dB in the region of BER $\geq 10^{-5}$.

As shown in this paper, iterative soft decision interference cancellation provides a very good performance when applied to equalization. A further extension of the MMSE–based scheme introduced here would be to replace the linear MMSE estimation filter with a widely linear MMSE estimation filter according to [20]. This idea was utilized in [5] for CDMA transmission and as shown there, a noticeable gain compared to a MMSE ISDIC might be achievable.

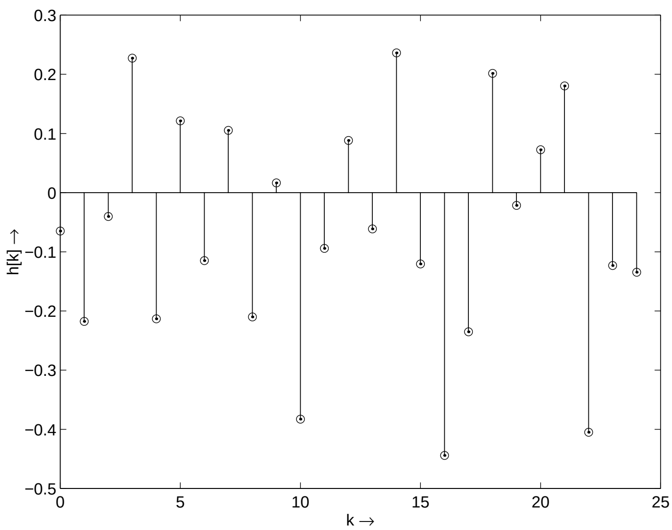

Fig. 2. Impulse response of channel A ($q_h = 24$).

VI. Concluding Remarks

Two iterative soft–decision feedback equalization algorithms have been introduced and analyzed. The performance of MLSE

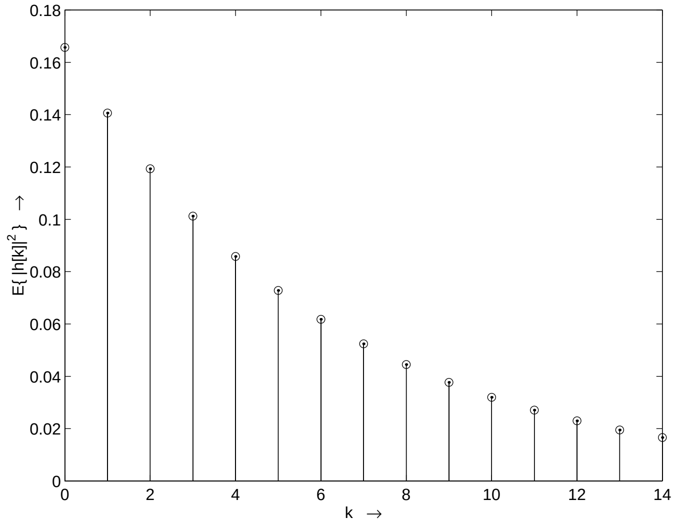

Fig. 3. Power delay profile of channel B ($q_h = 14$).

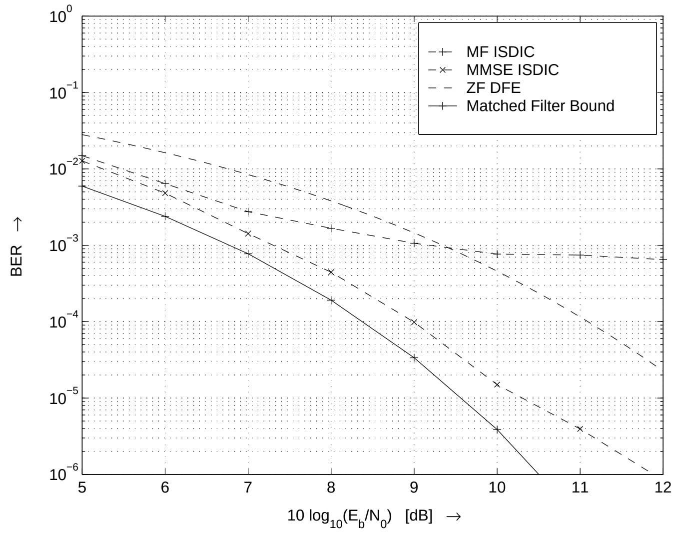

Fig. 5. BER vs. $E_b/\mathcal{N}_0$ for channel B (Rayleigh fading) with ideal power control for ISDIC with MF and MMSE approach.

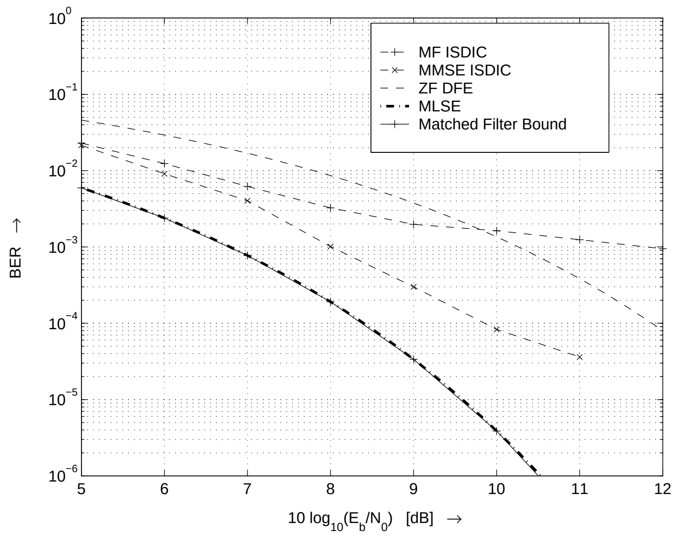

Fig. 4. BER vs. $E_b/\mathcal{N}_0$ for static channel A for ISDIC with MF and MMSE approach.

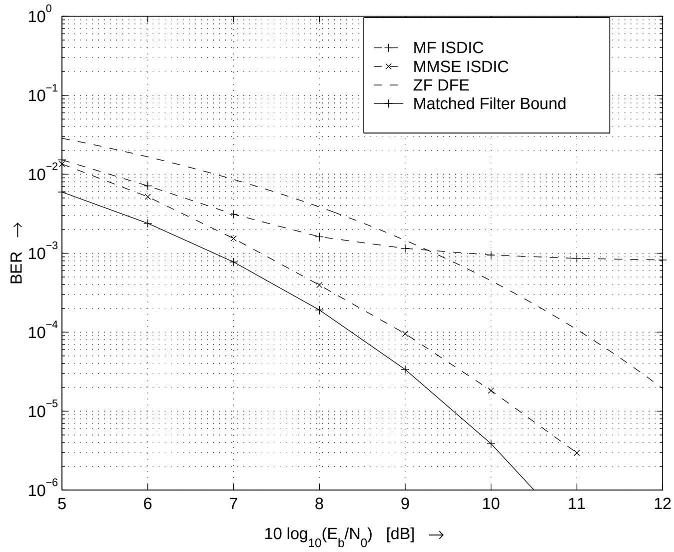

Fig. 6. BER vs. $E_b/\mathcal{N}_0$ for channel C (Rayleigh fading) with ideal power control for ISDIC with MF and MMSE approach.

can be approached by the MMSE–based algorithm very closely even for relatively short CIR's. The MF–based scheme has a low complexity, but needs long CIR's to approach the performance of MLSE as stated in [1]. The main computational load of the MMSE scheme consists in L inversions of a $(2q_w + 1 + q_h) \times (2q_w + 1 + q_h)$ matrix per block and iteration. In this paper, a block–oriented transmission has been assumed, but the algorithms can also be accommodated to continuous transmission, cf. [1].

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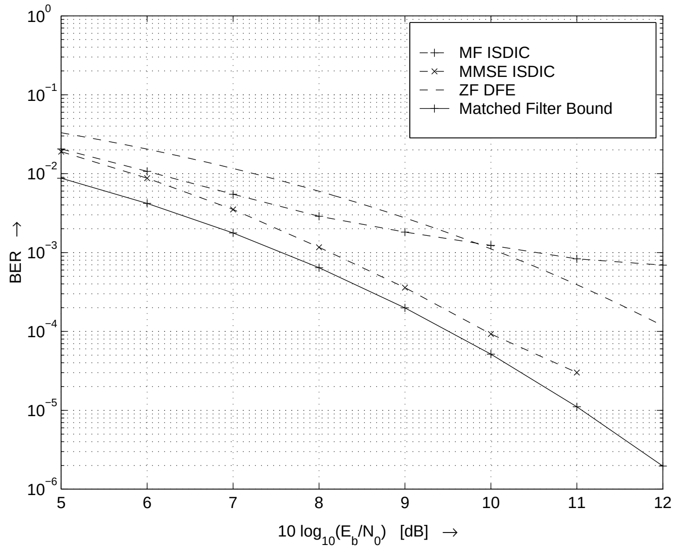

Fig. 7. BER vs. $E_b/\mathcal{N}_0$ for channel C (Rayleigh fading) without power control for ISDIC with MF and MMSE approach.

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