Towards on-line underwater vehicle trajectory estimation using diffusion-based observers

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***Abstract*— This paper extends previous work [6],[7] on the estimation of underwater vehicle trajectories using Gyro-Doppler (body-fixed velocities) and acoustic signals (earth-fixed positions). The approach consists of diffusion-based observers processing a whole trajectory segment at a time, allowing the consideration of important practical problems such as different information update rates, outages, and outliers in a very simple framework. We detail issues related to real-time applications, such as implementation and convergence. A theorem guaranteeing stability of the observer and simulation examples are given.**

I. Introduction

Knowing the horizontal position of an underwater vehicle precisely enough, at the bottom of the sea, is of great importance, not only for precise maneuvering and other control-based concerns, but also because the accurate knowledge of the vehicle trajectory is often the first step of other tasks, whether these are computed on-line (maneuvering, docking, marking interesting spots undersea, collecting samples...) or off-line (cartography, video mosaicking,...)[11]. The precision of position sensing is, however, limited, depending on many different factors like, for example, the kind and type of sensors used (Doppler-gyrocompas, Long BaseLine (LBL) and Ultra-Short BaseLine (USBL) acoustic systems, Inertial Measurement Unit (IMU), Global Positioning System (GPS), ...) as well as the events that are connected with sensors features or underwater environmental conditions such as noise, sensor misalignment, outliers and outages (see [13], [12], [8],[1]).

This paper extends previous work presented in Jouffroy and Opderbecke [6],[7] on the estimation of the horizontal position of an underwater vehicle using mainly gyro-Doppler measurements (speed measurements) and an acoustic positioning system (horizontal position). The approach relies on observers which process whole segments of a trajectory at a time, hence the terminology *trajectory observers*. The observers are described in a Partial Differential Equation (PDE) framework through a diffusion-reaction equation.

The paper elaborates on on-line versions of the observers that enable real-time applications. In section 2, we recall a few elements of underwater navigation in the horizontal plane. In section 3, we resume the main motivation of diffusion-based trajectory observers for off-line applications, detail implementation aspects, and discuss on-line versions with a theorem proving the convergence of the proposed observer. The sensor misalignment problem is also studied in this context in section 4. Simulation results are given to illustrate the approach.

II. Underwater vehicle navigation

In the following, we consider trajectory estimation of vehicles evolving on the horizontal plane, which seems to be the most relevant situation for vehicles such as the INFANTE AUV [10] for example (see figure 1) when they have to peform cartography tasks or video-mosaicking. In this context, the kinematics will be written as follows [4]

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \tag{1}$$

where the vector $(x, y)^T$ stands for the position of the vehicle in an earth-fixed frame, ψ the heading given by a gyrocompass, $(u, v)^T$ is the vector of body-fixed velocities, while ψ is the heading of the vehicle that is used to compute the time-varying rotation from the body-fixed or vehicle frame to the earth-fixed one.
By using a complex setting and defining $X \triangleq x + iy$ and $V \triangleq u + iv$, where i is the imaginary number, equation (1) is reduced to

$$\dot{X} = e^{i\psi} V \tag{2}$$

This complex notation will be used in the rest of the paper. A position given by the acoustic system (USBL or LBL) is denoted as X_{ac}. These measurements are typically corrupted by noise, and possibly suffer from a high percentage of outages and outliers, while the update rate of the positioning information lies between 0.1 and $1Hz$, depending on the system in use.

The body-fixed velocities are provided by a bottom-lock Doppler sonar consisting of four downward-looking beam transducers that measure the apparent velocity of the seafloor. These velocities, that we denote here V_{dop} are then transformed into the earth-fixed coordinates using the standard rotation

$$V_{geo} = e^{i\psi} V_{dop} \tag{3}$$

Fig. 1. The INFANTE AUV

where the heading ψ is measured by a gyrocompass, and V_{geo} is the speed vector in the earth-fixed frame. V_{dop} and V_{geo} are typically less noisy than the acoustic measurements X_{ac} while their update rates are higher (around $5Hz$). When the heading is measured by an optical gyrocompass, the precision and update rates of ψ are generally excellent. A dead-reckoning process, *i.e.* a time integration of the velocities might thus seem to be appropriate due to the good sensor performances. However, these are counterbalanced by other factors such as a misalignment of the gyrocompass, the problem of the initial position determination in the dead-reckoning, and accumulation of small noise due to the time-integration process that could lead to significant deviations of the estimated trajectory with respect to the real path of the vehicle [12].

III. TRAJECTORY OBSERVERS

Consider the system described by the following deterministic equation

$$\dot{X} = f(X, t) \tag{4}$$

where X represents the state of the system. By looking at the time evolution of the state in the state-space, X can be seen as a moving point, hence the often-seen terminology *particle* for a point initialized in X_0, i.e. $X(X_0, t)$. The path followed by this particle on a time interval is reffered to as *trajectory*. Using this terminology, Luenberger observers, also called *state* observers, may be regarded as dynamical systems that use the available measurements but also the time evolution as an iterative process to give an estimate $\hat{X}$ of the state that will eventually converge to the true particle X.

The point of view adopted in [6], [7] is to perform the deterministic estimation process considering a trajectory of an underwater vehicle as a whole. This leads to a quite unified view of an estimation process of a trajectory defined on a time interval, whether this estimation process is intended for off-line or on-line purposes. Let s describe the temporal position of the vehicle along its path or the trajectory, with s belonging to the trajectory time interval $[s_b, s_e]$ in seconds. In the following, s will therefore be referred to as the *trajectory time*.

Note the different notation that is used here compared to the usual t to stress the common point between off-line and on-line estimation which, in our opinion is better described in terms of a trajectory than of a particle.

A. Diffusion-based observers and off-line trajectory estimation

In order to consider the problem of combining the gyro-Doppler velocity measurements $V_{geo}(s)$ with acoustic positions $X_{ac}(s)$ on a *fixed* trajectory time interval, we introduce the following least-square-like criterion

$$J = \int_{s_b}^{s_e} (\nabla \hat{X} - V_{geo})^2 + K(\hat{X} - X_{ac})^2 ds \tag{5}$$

(where $\nabla \hat{X} \triangleq \partial \hat{X}/\partial s$) which, roughly speaking, states that the more the trajectory estimate $\hat{X}$ and its derivative $\nabla \hat{X}$ on $[s_b, s_e]$ will both ressemble $X_{ac}(s)$ and $V_{geo}(s)$, the lower the criterion J will be, with the perfect match being for the case where $V_{geo}(s) = \nabla X_{ac}(s)$. Here, K is a constant parameter balancing the priority of $X_{ac}(s)$ and $V_{geo}(s)$ over the final aspect on $\hat{X}$.

Minimizing J in an *iterative manner* corresponds to writing the gradient flow of the solution of the Euler-Lagrange equation of the integrand of (5), which is the following diffusion-reaction equation

$$\frac{\partial \hat{X}}{\partial t} = \nabla(\nabla \hat{X} - V_{geo}(s)) + K(X_{ac}(s) - \hat{X}) \tag{6}$$

with Neumann boundary conditions such that $\nabla \hat{X}(s_b, t) = V_{geo}(s_b)$ and $\nabla \hat{X}(s_e, t) = V_{geo}(s_e)$. Thus, t now represents the variable accounting for the evolution, or the improvement, of the iterative process. For this reason, we will refer to this variable as the *improvement time*.

To take into account of the fact that the acoustic measurements are not available at all trajectory time $s \in [s_b, s_e]$, replace the parameter K by the trajectory time dependent gain

$$k_X(s) = K \sum_{i=1}^{n_s} \delta(s - s_i) \tag{7}$$

where $\delta(\cdot)$ stands for the Dirac delta function, and $s_i \in [s_b, s_e]$ are the n_s trajectory times for which the acoustic signal is available.

Thus, it is guaranteed that the best estimate is finally obtained since

$$\frac{\partial \hat{X}}{\partial t} = \nabla(\nabla \hat{X} - V_{geo}(s)) + k_X(s)(X_{ac}(s) - \hat{X}) \tag{8}$$

converges to a unique solution independently of the initial guess $\hat{X}(0, t)$ [6].

One of the many possible ways of interpreting the role played by the diffusion term guided by V_{geo} in (8) is to consider a possible implementation of (8), where, for the sake of simplicity, we assume a discretization of $[s_b, s_e]$ in only three instants, each couple of instants being separated by one second. Hence, denoting the discrete version of $\hat{X}(s,t)$ by $\hat{X}_k^l$, where $l \in 1,2,3$ is the trajectory index, and $k \in \mathbb{N}$ is the improvement index, write

$$\begin{pmatrix} \hat{X}_{k+1}^1 \\ \hat{X}_{k+1}^2 \\ \hat{X}_{k+1}^3 \end{pmatrix} = \begin{pmatrix} \hat{X}_k^1 \\ \hat{X}_k^2 \\ \hat{X}_k^3 \end{pmatrix} + \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{X}_k^1 \\ \hat{X}_k^2 \\ \hat{X}_k^3 \end{pmatrix}$$
$$- \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} U^1 \\ U^2 \\ U^3 \end{pmatrix} + k_X(l)(X^l - \hat{X}_k^l) \tag{9}$$

where U^l is the approximation of $V_{geo}(s)$, and X^l is the discrete version of $X_{ac}(s)$.

The discrete Laplacian operator being a difference between two gradients, we can rewrite (9) as

$$\left\{ \begin{array}{l} \hat{X}_{k+1}^1 = \hat{X}_k^1 + 0 \\ \hat{X}_{k+1}^2 = \hat{X}_k^2 + [(\hat{X}_k^1 + U^1) - \hat{X}_k^2] \\ \hat{X}_{k+1}^3 = \hat{X}_k^3 + [(\hat{X}_k^2 + U^2) - \hat{X}_k^3] \end{array} \right.$$
$$\begin{array}{ll} +[\hat{X}_k^2 - (\hat{X}_k^1 + U^1)] & +k_X(1)(X^1 - \hat{X}_k^1) \\ +[\hat{X}_k^3 - (\hat{X}_k^2 + U^2)] & +k_X(2)(X^2 - \hat{X}_k^2) \\ +0 & +k_X(3)(X^3 - \hat{X}_k^2) \end{array} \tag{10}$$

where the first column after the integration part, *i.e.* the terms $[(\hat{X}_k^{l-1} + U^{l-1}) - \hat{X}_k^l]$, "influence the present with the past" by propagating the available information, say at $l = 1$ through the gain $k_X(1)$, to $\hat{X}_k^3$ through the states $\hat{X}_k^1$ and $\hat{X}_k^2$. The terms $[\hat{X}_k^{l+1} - (\hat{X}_k^l + U^l)]$ of the second column do exactly the opposite thing by using acoustic measurements that are available, say at $l = 3$ to influence past estimates up to $\hat{X}_k^1$.

Note the importance of U^l in these terms, since their role is to constrain the estimates towards the relation

$$\hat{X}_k^{l+1} = \hat{X}_k^l + U^l \tag{11}$$

which simply translates the effect that the measurements must have on the successive estimates. Also, the different information propagations induced by the Laplacian operator can be seen by noting that

$$\begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} + \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{12}$$

Finally, note that the system (10) could also have been obtained using the following discrete version of the criterion (5)

$$J_d = \sum_{l=s_b}^{s_e} L(l, \hat{X}^l, D\hat{X}^l) \tag{13}$$

where L is the Lagrangian function which, in our case will be written

$$L(l, \hat{X}^l, D\hat{X}^l) = (D\hat{X}^l - U^l)^2 + K(\hat{X}^l - X^l)^2 \tag{14}$$

where $D\hat{X}^l \triangleq \hat{X}^{l+1} - \hat{X}^l$ is the discrete differentiation operator. In this case, the minimum of (13) verifies the discrete Euler-Lagrange equation [3]

$$0 = \frac{\partial L}{\partial \hat{X}^l} - D\left(\frac{\partial L}{\partial D\hat{X}^l} - \frac{\partial L}{\partial \hat{X}^l}\right) \tag{15}$$

which, in the special case (14) gives

$$0 = [(\hat{X}_k^{l-1} + U^{l-1}) - \hat{X}_k^l] + [\hat{X}_k^{l+1} - (\hat{X}_k^l + U^l)] + K(X^l - \hat{X}_k^l) \tag{16}$$

This corresponds to the solution of the system (10) when $k \to \infty$ if we replace K by the more general gain $k_X(l)$.

In case the Doppler measurements are not available, and the objective is to obtain a smooth interpolation X_s of the acoustic data, one can use regularization techniques for interpolation as detailed in [2]. Roughly speaking, this consists in replacing the criterion (5) by

$$J_s = \int_{s_b}^{s_e} D(\nabla X_s)^2 + D_s(\nabla^2 X_s)^2 + k_X(s)(X_s - X_{ac})^2 ds \tag{17}$$

and computing the Euler-Lagrange equations to minimize this criterion, the gradient flow will now be written as

$$\frac{\partial X_s}{\partial t} = D\nabla^2 X_s - D_s \nabla^4 X_s + k_X(s)(X_{ac} - X_s) \tag{18}$$

where D and D_s are two positive constants.

B. On-line diffusion-based trajectory observers

Until now, we have only considered observers driven by informations which were fixed in time. For on-line purposes, however, we will consider informations which will be updated as the estimation process evolves.

In this perspective, the acoustic positions are now moving with the speed of time t in the trajectory time interval, and V_{geo} not only plays the role of defining the difference between different trajectory instants, but is also responsible for the evolution of the estimate $\hat{X}(s,t)$, as represented in the following equation

$$\frac{\partial \hat{X}}{\partial t} = V_{geo}(s,t) + \nabla(\nabla \hat{X} - V_{geo}(s,t)) + k_X(s,t)(X_{ac}(s,t) - \hat{X}) \tag{19}$$

In order to clarify the importance of the additional term V_{geo}, note that since we have $\dot{X}(t) = V(t)$, this also means that we have

$$\frac{\partial X}{\partial t} = V_{geo}(s,t) \tag{20}$$

for all $s \in [s_b, s_e]$ on the trajectory segment.

Hence, a possible 3-instant implementation of (19) would be

$$\begin{pmatrix} \hat{X}_{k+1}^1 \\ \hat{X}_{k+1}^2 \\ \hat{X}_{k+1}^3 \end{pmatrix} = \begin{pmatrix} \hat{X}_k^1 \\ \hat{X}_k^2 \\ \hat{X}_k^3 \end{pmatrix} + \begin{pmatrix} U_k^1 \\ U_k^2 \\ U_k^3 \end{pmatrix}$$
$$+ \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{X}_k^1 \\ \hat{X}_k^2 \\ \hat{X}_k^3 \end{pmatrix}$$
$$- \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} U_k^1 \\ U_k^2 \\ U_k^3 \end{pmatrix} + k_X(l,k)(X_k^l - \hat{X}_k^l) \tag{21}$$

where U_k^1 and X_k^l now depend on k since the measurements shift in the trajectory time interval during the estimation process (from $l = 3$ to 1) as k iterates. In a more general point-of-view, it can be proven under certain conditions that $\hat{X}(s,t)$ converges to $X(s,t)$ by using the following theorem.

Theorem 1: Let the system

$$\frac{\partial X}{\partial t} = \nabla X(s,t) \tag{22}$$

where $s \in [s_b, s_e]$, and the trajectory observer

$$\frac{\partial \hat{X}}{\partial t} = \nabla X(s,t) + \nabla(\nabla \hat{X} - \nabla X(s,t)) + k_X(s,t)(X(s,t) - \hat{X}) \tag{23}$$

with the same Neumann boundary conditions as in (22), and where

$$k_X(s,t) = K \sum_{i=1}^{n_s} \delta(s - (s_i + t)) \tag{24}$$

where n_s is a strictly positive integer and K is a strictly positive real constant. Then $\hat{X}$ converges exponentially to X. □

Proof: From (22) and (23), define the error system

$$\frac{\partial \tilde{X}}{\partial t} = \frac{\partial}{\partial s}\left(\frac{\partial \tilde{X}}{\partial s}\right) - k_X(s,t)\tilde{X} \tag{25}$$

where $\tilde{X} = \hat{X} - X$. Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\int_{s_b}^{s_e} \tilde{X}^2 ds \tag{26}$$

Then, the time-derivative of V is

$$\dot{V} = \frac{1}{2}\int_{s_b}^{s_e} \tilde{X}\dot{\tilde{X}} ds = \int_{s_b}^{s_e} \tilde{X}\left[\frac{\partial}{\partial s}\left(\frac{\partial \tilde{X}}{\partial s}\right) - k_X(s,t)\tilde{X}\right] ds \tag{27}$$

which, after integration by parts, gives

$$\dot{V} = \left[\tilde{X}\frac{\partial \tilde{X}}{\partial s}\right]_{s_b}^{s_e} - \int_{s_b}^{s_e}\left(\frac{\partial \tilde{X}}{\partial s}\right)^2 ds - \int_{s_b}^{s_e} k_X(s,t)\tilde{X}^2 ds \tag{28}$$

As a consequence of integration by parts and the Cauchy-Schwartz inequatity, we have

$$\tilde{X}(s,t) - \tilde{X}(s_i + t, t) \leq \left(H\int_{s_b}^{s_e}\left(\frac{\partial \tilde{X}}{\partial s}\right)^2 ds\right)^{\frac{1}{2}} \tag{29}$$

(where $H = s_e - s_b$), which implies

$$-\frac{1}{H^2}\int_{s_b}^{s_e}\left(\tilde{X}(s,t) - \tilde{X}(s_i + t, t)\right)^2 ds \geq -\int_{s_b}^{s_e}\left(\frac{\partial \tilde{X}}{\partial s}\right)^2 ds \tag{30}$$

Hence we have a bound on $\dot{V}$ as follows

$$\dot{V} \leq \left[\tilde{X}\frac{\partial \tilde{X}}{\partial s}\right]_{s_b}^{s_e} - \frac{1}{H^2}\int_{s_b}^{s_e}\left(\left[\tilde{X} - \tilde{X}_i\right]^2 + H^2 k_X(s,t)\tilde{X}^2\right) ds \tag{31}$$

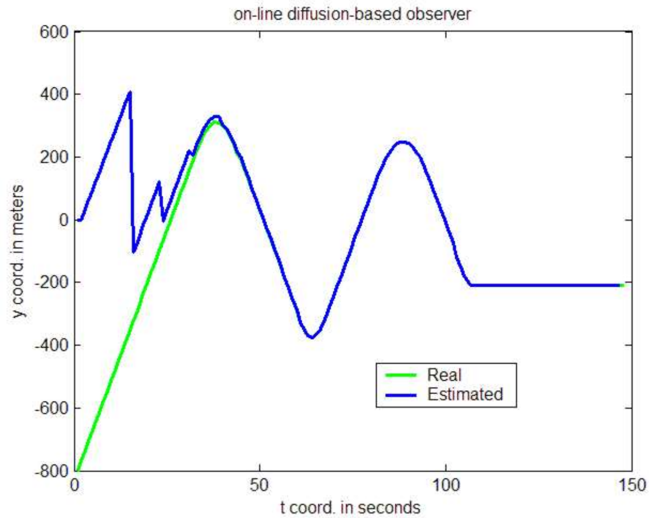

Fig. 2. Simulation of observer with sparse acoustic measurements (19) for a one-dimensional trajectory

Using the definition of the measurement feedback gain (24), we have

$$\dot{V} \leq \left[\tilde{X}\frac{\partial \tilde{X}}{\partial s}\right]_{s_b}^{s_e} - \frac{1}{H^2}\int_{s_b}^{s_e}\left(\begin{array}{c}\tilde{X}\\ \tilde{X}_i\end{array}\right)^T Q \left(\begin{array}{c}\tilde{X}\\ \tilde{X}_i\end{array}\right) ds - K\sum_{j=1, j\neq i}^{n_s} \tilde{X}^2(s_j + t, t) \tag{32}$$

where

$$Q = \left(\begin{array}{cc} 1 & -1 \\ -1 & 1 + KH^2 \end{array}\right) > 0 \tag{33}$$

for all $K > 0$. Thus, we have

$$\dot{V} \leq \left[\tilde{X}\frac{\partial \tilde{X}}{\partial s}\right]_{s_b}^{s_e} - \frac{1}{H^2}\lambda_{\min}(Q)\int_{s_b}^{s_e}\left\|\left(\begin{array}{c}\tilde{X}\\ \tilde{X}_i\end{array}\right)\right\|^2 ds \tag{34}$$

where $\lambda_{\min}(Q) > 0$ is the minimum eigenvalue of Q. Applying the boundary conditions and using the fact that

$$\left\|\left(\begin{array}{c}\tilde{X}\\ \tilde{X}_i\end{array}\right)\right\| \geq \left\|\tilde{X}\right\| \tag{35}$$

we have

$$\dot{V} \leq -\frac{1}{H^2}\lambda_{\min}(Q)\int_{s_b}^{s_e} \tilde{X}^2 ds = -\frac{2}{H^2}\lambda_{\min}(Q)V \tag{36}$$

Thus, we finally have

$$\dot{V} \leq -kV \tag{37}$$

where $k = \frac{2}{H^2}\lambda_{\min}(Q)$, which completes the proof of the theorem. ■

As an illustration, we applied observer (19) on a one-dimensional trajectory using a horizon $H = 20$ seconds and sparse position measurements that were available once every

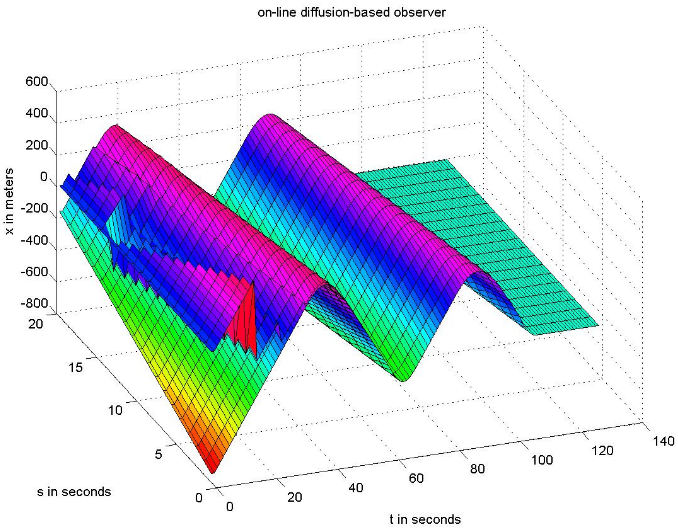

Fig. 3. 3D representation of the simulation of observer (19)

15 seconds. Figure 2 shows the behavior of the observer when initialized differently from the real trajectory. Figure 3 displays a more PDE-like representation of the same process, emphasizing the trajectory aspect of the approach.

IV. Sensor alignment and trajectory estimation

In the case where there is a misalignment in the way the gyrocompass is mounted on the vehicle, there is an undesirable constant bias in the heading measurement and the rotation (3) becomes

$$V_d = e^{i\psi_m} V_{dop} \tag{38}$$

where $\psi_m \triangleq \psi + \psi_b$ is the angle actually measured because of the constant bias ψ_b induced by the misalignment. Combining now (3) with (38), we have the relation between the bias-free velocity vector V_{geo} and the "disturbed" vector V_d

$$V_{geo} = bV_d \tag{39}$$

where $b \triangleq e^{-i\psi_b}$ is the constant unknown imaginary number standing for the rotation due to the gyro misalignment. Hence we would like to have an estimate of this unknown parameter in combination with the diffusion-based estimation process of the previous section.

Considering first the off-line problem, introduce the following adaptive observer

$$\frac{\partial \hat{X}}{\partial t} = \nabla(\nabla \hat{X} - \hat{b} V_d) + K(X_s - \hat{X}) \tag{40}$$

with the adaptation law

$$\dot{\hat{b}} = -\int_{s_b}^{s_e} \nabla V_d (X_s - \hat{X}) ds \tag{41}$$

where $\hat{b}(t)$ is the parameter estimate, and $X_s(s)$ is a smoothed version of the acoustic signal $X_{ac}(s)$. As detailed in [7], the structure of the observer (40)-(41) can be explained by noticing that an ODE or space approximation of (40)-(41) is

$$\dot{\hat{X}} = \mathbf{A}\hat{\mathbf{X}} - \mathbf{W}\hat{b} + K(\mathbf{X} - \hat{\mathbf{X}}) \tag{42}$$

$$\dot{\hat{b}} = -\mathbf{W}^T(\mathbf{X} - \hat{\mathbf{X}}) \tag{43}$$

where $\hat{\mathbf{X}} \in \mathbb{R}^n$ is the finite dimensional state approximation of $\hat{X}(s,t)$, $\mathbf{A}$ the Laplacian matrix and $\mathbf{W}(t)$ the approximation of $\nabla V_d(s)$. Then, combining with the model $\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} - \mathbf{W}b = 0$, we have the following error dynamics

$$\frac{d}{dt}\begin{pmatrix} \tilde{\mathbf{X}} \\ \tilde{b} \end{pmatrix} = \begin{pmatrix} \mathbf{A} + K\mathbf{I} & -\mathbf{W} \\ \mathbf{W}^T & 0 \end{pmatrix}\begin{pmatrix} \tilde{\mathbf{X}} \\ \tilde{b} \end{pmatrix} \tag{44}$$

(where $\tilde{\mathbf{X}} = \hat{\mathbf{X}} - \mathbf{X}$ and $\tilde{b} = \hat{b} - b$) which is very often encountered in adaptive control. Then, under appropriate assumptions on the persistence of excitation of $\mathbf{W}(t)$, one can assume the convergence of $\hat{\mathbf{X}}$ and $\hat{b}$ towards their true values.

In terms of implementation, the system (10) is now replaced with

$$\begin{cases} \hat{X}^1_{k+1} = \hat{X}^1_k + 0 \\ \hat{X}^2_{k+1} = \hat{X}^2_k + [(\hat{X}^1_k + \hat{b}_k U^1) - \hat{X}^2_k] \\ \hat{X}^3_{k+1} = \hat{X}^3_k + [(\hat{X}^2_k + \hat{b}_k U^2) - \hat{X}^3_k] \end{cases}$$
$$\begin{matrix} +[\hat{X}^2_k - (\hat{X}^1_k + \hat{b}_k U^1)] & +k_X(1)(X^1 - \hat{X}^1_k) \\ +[\hat{X}^3_k - (\hat{X}^2_k + \hat{b}_k U^2)] & +k_X(2)(X^2 - \hat{X}^2_k) \\ +0 & +k_X(3)(X^3 - \hat{X}^2_k) \end{matrix} \tag{45}$$

where $\hat{b}_k$ is updated according to

$$\hat{b}_{k+1} = \hat{b}_k - U^1(X^1 - \hat{X}^1_k) - (U^2 - U^1)(X^2 - \hat{X}^2_k) + U^2(X^3 - \hat{X}^2_k) \tag{46}$$

In figure 4, we took a trajectory $X(s)$, computed its derivative and created an artificial gyro misalignment of 5 degrees, which, after time integration gives the rotated trajectory $X_d(s)$. The blue line representing $\hat{X}$ shows the behavior of observer (40)-(41) which estimates $X(s)$ properly, while figure 5 represents the evolution of $\hat{b}$ reaching the bias value.

Because of the form of (39), an on-line version of (40)-(41) would be

$$\frac{\partial \hat{X}}{\partial t} = \hat{b}V_d + \nabla(\nabla \hat{X} - \hat{b}V_d) + K(X_s - \hat{X}) \tag{47}$$

$$\dot{\hat{b}} = -\int_{s_b}^{s_e} (\nabla V_d - V_d)(X_s - \hat{X}) ds \tag{48}$$

The stable behavior of such an observer can be demonstrated by computing the error dynamics

$$\frac{\partial \tilde{X}}{\partial t} = \nabla^2 \tilde{X} - (\nabla V_d - V_d)\tilde{b} - K\tilde{X} \tag{49}$$

$$\dot{\tilde{b}} = -\int_{s_b}^{s_e} (\nabla V_d - V_d)\tilde{X} ds \tag{50}$$

and using the following Lyapunov function candidate

$$V = \frac{1}{2}\int_{s_b}^{s_e} \tilde{X}^2 ds + \frac{1}{2}\tilde{b}^2 \tag{51}$$

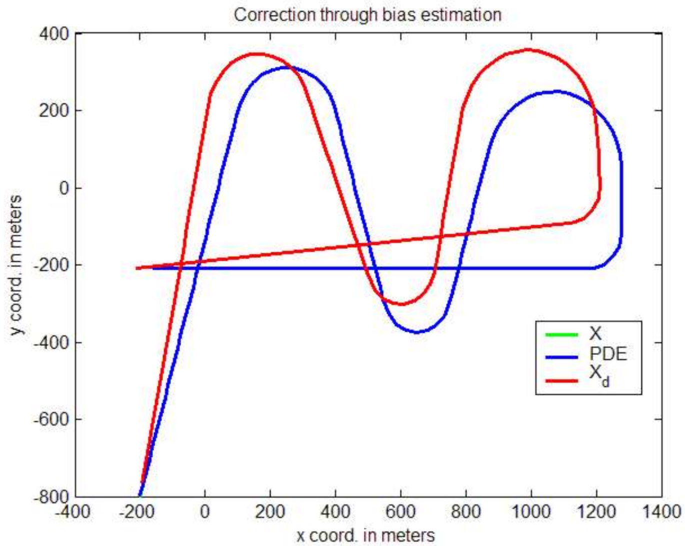

Fig. 4. Trajectory estimate with a gyro bias

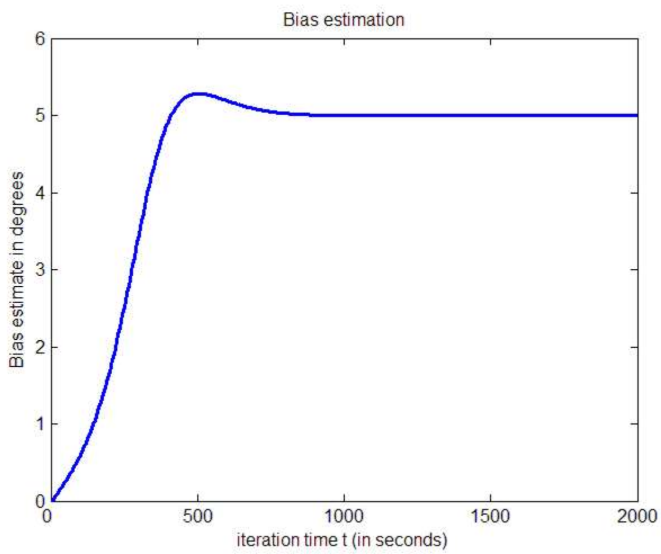

Fig. 5. Estimate $\hat{b}$ of the gyro bias

which time-derivative is

$$\dot{V} = -\int_{s_b}^{s_e} \left(\frac{\partial \tilde{X}}{\partial s} \right)^2 ds - K \int_{s_b}^{s_e} \tilde{X}^2 ds \leq 0 \qquad (52)$$

Then, following similar arguments as those used in [5], the convergence of the estimation error $\tilde{X}$ as well as $\tilde{b}$ towards 0 can be established.

V. Concluding remarks

In this paper, we extended the work in [6],[7] by studying the problem of designing diffusion-based trajectory observers for real-time problem related to underwater navigation issues. The convergence of the proposed observer was shown and simulation results were presented.

Current research focuses on studying in a quantative way the robustness issues of the observers as well as a more general discrete-time description of trajectory observers for underwater navigation for which the stable or convergent behavior would directly result from a discrete version such as (21) for example.

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