On the Performance of Turbo Coding for the Land Mobile Channel with Delay Constraints *

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Abstract

Turbo coding for a Rayleigh fading channel is considered in a scenario where an externally imposed delay constraint exists. The interactions between the delay constraint, the rate of variation of the fading channel, and the block size and complexity of the turbo codes are investigated by simulation. Furthermore, the choice of dimensions of the block interleaver is addressed.

1. Introduction

Turbo codes, introduced in [2], have been shown to achieve reliable transmission at signal-to-noise ratios (SNR) within 1dB of the Shannon limit on the additive white Gaussian noise (AWGN) channel. This powerful coding technique is also potentially attractive for application in land mobile communications. To break up the correlation of channel fades, a block interleaver is introduced for the turbo coding schemes in [7]. However, the size of the interleaver is typically determined by the end-to-end delay constraint and perfect interleaving is usually not achievable. In this work, we explore the roles of the delay constraint, the rate of variation of the fading channel, and the block size and memory length of the turbo codes.

The outline of the remaining parts of this paper is as follows. A brief overview of turbo codes is given in Section 2, followed by a discussion in Section 3 of the fading model, channel interleaving and the impact of the delay constraint. Simulation results are presented in Section 4 and conclusions are drawn in Section 5.

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2. Turbo coding

Turbo codes, introduced by Berrou, *et al.* [2], are parallel concatenated convolutional codes (PCCC). The turbo encoder consists of two or more recursive systematic convolutional (RSC) encoders, each separated by a permutation. Since the constituent encoders are systematic, they each produce an information stream and one or more parity streams. Only the information stream from the first constituent encoder is transmitted, resulting in an overall rate-$1/M$, for some M. A typical encoder consists of two identical rate-$1/2$ RSC encoders yielding a net code rate of $1/3$, as shown in Figure 1. High rate codes are obtained by puncturing the parity bits.

The encoders are assumed to start in the all-zero state. For the results presented in this paper, we assume that the first encoder is always driven back to the all-zero state through the transmission of tail-bits, whereas the remaining encoders are not terminated [10]. For the first encoder, the trellis termination scheme of [4] is used, where the encoder feedback bit is taken as the encoder input and both the feedback bits and the parity bits are transmitted when tailing off.

The main purpose of the permutation in the turbo codes is reducing the multiplicity of error events at the minimum distances (called "spectral thinning" [10]). It has been shown that the turbo codes using random permutation dramatically outperform turbo codes using block permutations [10]. The permutation adopted for the simulation in this paper is so-called "S-Random" permutation [4], which prohibits the mapping of a bit-position to another within a distance $\pm S$ of a bit-position already chosen in any of the S previous selections. As a rule of thumb, for a permutation with block size N, $S < \sqrt{N/2}$ is chosen [4].

Iterative decoding is used to decode turbo codes. The decoder consists of two or more concatenated decoding elements matched to each encoder. Each decoding element receives as input the received systematic code symbols and parity code symbols corresponding to its particular RSC encoder. The output of the decoding elements are the soft *a posteriori* probabilities of the information sequence, and these are passed to other decoding elements to produce more reliable decisions. As the iterative procedures go on, the performance converges to a maximum-likelihood decoder at moderate to high SNR. In our simulation, the LOG-BCJR algorithm [1] is used in the decoding elements and 20 iterations are used to assure convergence.

3. Fading Model and Channel Interleaving

We are considering coherent BPSK signaling over a flat fading channel. The channel model suggested by Jakes [8] is used, in which the envelope is assumed to be Rayleigh distributed. The Doppler spectrum is given by

$$S(f) = \frac{1}{\sqrt{(1-(f/f_D)^2}} \quad (1)$$

where f_D is the Doppler frequency, given by $f_D = vf_c/c$, and where v is the speed of the mobile, f_c is the carrier frequency and c is the speed of light.

The covariance function of the fading process, α, for this channel model can be shown to be given by the first order Bessel function, namely

$$r_\alpha(\tau) = J_0(2\pi f_D|\tau|) \quad (2)$$

The product $f_D\tau$ is usually called the *normalized Doppler frequency*, and it determines the correlation between two symbols with separation τ. Note that the first zero of this function is at $f_D\tau = 0.38$.

Generally, error correcting coding works well when the code symbols used in the decoding procedure are affected by independent channel conditions. In order to break up the memory of the channel, a channel interleaver is usually employed. A block interleaver formats the encoded data in a rectangular array of m rows and n columns. The code symbols are written in row-by-row and read out column-by-column. The received symbols are first de-interleaved before they enter the decoder. As a result of this reordering, the fading samples of two consecutive symbols entering the decoder are actually mT_s apart in time. Thus, m is often called the depth of the interleaver.

The amount of interleaving required is a function of the rate of variation of the fading. When there is no limit on the size of the interleaver, perfect interleaving can be achieved, which assures the fading envelopes are uncorrelated. In a practical situation, the size is typically determined by the end-to-end delay constraint of the service, and the channel is usually partially interleaved.

In this paper, the carrier frequency is assumed to be 900MHz and rate-1/3 codes are used. Three typical mobile speeds are chosen: 4 mph for pedestrian traffic, 30 mph for local traffic and 70 mph for freeway traffic. For 9.6 kbps information bit rate, the normalized Doppler frequencies f_DT_s are listed in Table 1. Note that the f_DT_s are so small that the fading will remain correlated for interleaver depths of up to one hundred. For higher information rates, the normalized Doppler frequencies will be even smaller.

Mobile speed (mph)	f_D(Hz)	f_DT_s
4	5.33	1.85×10^{-4}
30	40	1.39×10^{-3}
70	93.3	3.24×10^{-3}

Table 1. Normalized Doppler frequency and corresponding speed, carrier frequency=900 MHz, code rate = 1/3, source rate 9.6 kbps

It has been shown in [7] that block interleaving achieves better performance than systematic interleaving for turbo coding over fading channels. In our system, illustrated in Figure 1, the turbo codewords are formed by traditional serialization, and block interleaved before transmission over a fading channel. The purpose of the channel interleaver is to break up the correlation of the fading channel, and the dimensions of the interleaver influence the performance.

For a convolutional coding system, the dimensions are chosen to maximize the interleaving depth while avoiding the wrap-around effect [11]. The wrap-around effect means that the length of an error event exceeds the number of columns in the interleaver. This results in more than one symbol in an error event being affected by virtually the same channel conditions, and thus degraded performance. As a rule of thumb, the number of columns is chosen slightly larger than the length of the shortest error event of the convolutional code. Since turbo codes are the concatenation of convolutional codes, we expect that the same rule exists for turbo codes, as will be verified by our simulation.

The delay constraint is a fundamental limit for reliable communications over a fading channel. For a block coding scheme like either turbo coding or terminated convolutional coding, the available diversity order is determined by the block duration and the Doppler frequency. In fact, the number of degrees of freedom of the fading process during a time interval of $[0, T_B]$ is around $L = \lceil 2f_DT_B + 1\rceil$ and the pairwise error probability that the coding scheme can achieve is given [9] by

$$P_2 \approx A(\frac{N_0}{rE_b})^L \quad (3)$$

where r is the code rate and A is a constant related to the eigenvaules of the covariance matrix of fading samples. To fully benefit from this available diversity order, the free distance of a convolutional code should be greater than L and the channel interleaver should spread the weight evenly throughout the block. This is because the error events of smallest distances dominate the error probability of convolutional codes. For turbo codes, the error events of free distance become dominant only in the “error floor” region where the BER is usually below 10^{-5}. For the BER range from 10^{-3} to 10^{-5}, which is our main interest, the errors are caused mostly by codewords of larger weights. With the aid of the channel interleaver, the weights of these codewords can be assumed to be spread evenly over the block, which implies that the turbo coding schemes are able to achieve higher diversity in this region. However, the available diversity is constrained by the delay requirement. For three different values of block durations, T_B, Table 2 shows the $f_D T_B$ product for the three typical mobile speeds.

	26.7ms	106.7ms	426.7ms
4 mph	0.14	0.57	2.27
30 mph	1.07	4.28	17.12
70 mph	2.48	9.92	39.68

Table 2. $f_D T_B$ for different delay constraints and mobile speeds, carrier frequency = 900MHz

The delay constraint not only limits the available diversity order, but also determines the effectiveness of the turbo codes [5]. For a fixed information bit rate, the block size of the turbo code, N, is proportional to the block duration T_B, which is depicted in Table 3.

block duration(ms)	26.7	106.7	426.7
block size(N)	256	1024	4096

Table 3. The block size, N, of turbo codes for different delay constraints, source bit rate = 9.6 kbps

4. Simulation Results

Upper bounds have been developed for turbo codes [3, 6]. Although they are accurate in the “error floor” region, they become loose at lower SNRs, where our interest lies. For this reason, all of the following results are obtained by simulations. Perfect channel state information is assumed to be available, and we employ the LOG-BCJR algorithm. The rate-1/3 turbo code used in the simulation consists of two 16-state RSC encoders with generator $(33/31)_8$.

For a perfectly interleaved channel, performance in terms of bit error rate for different block sizes is shown in Figure 2. The effect of increasing the block size of turbo codes is obvious, as the $N = 1024$ scheme is about 1.0dB more efficient than the $N = 256$ scheme at a BER of 10^{-5}. Compared to the performance in an AWGN channel, the degradation caused by uncorrelated fading is no more than 1.5dB for BER $\geq 10^{-5}$.

In Figure 3, the performance of turbo coding over correlated fading is shown. The block duration T_B is assumed to be 106.7 ms, which implies $N = 1024$ for a 9.6 kbps source bit rate. For BER $= 10^{-4}$, the scheme suffers a loss of 4.0 dB, 7.1 dB and 16 dB, respectively, from perfect interleaving for freeway, local and pedestrian speeds. The degradation in the pedestrian case is huge, where there are insufficient degrees of freedom of the fading process during the time span of one block. Other diversity schemes should be considered to compensate in this situation.

Figure 4 shows how the performance for the turbo encoder with constituent RSC encoders of memory length 4 is affected by varying the number of columns of the channel interleaver, n. For this code, the lengths of the shortest error event in both the systematic and parity fragments are 5. As can be seen from this figure, provided that n is slightly larger than this number, the exact choice of interleaver dimensions has very little impact on the performance.

In Figures 5 and 6, the BER and frame error rate (FER), respectively, are plotted versus the normalized Doppler frequency $f_D T_s$ for different values of E_b/N_0. For instance, if $f_D T_s$ is increased from 3.24×10^{-3} to 5.7×10^{-3}, the BER achieved at $E_b/N_0 = 5.0$dB decreases dramatically from 8.5×10^{-4} to 1.9×10^{-5}. However, for a source bit rate of 9.6 kbps, this change of $f_D T_s$ corresponds to increasing the mobile speed from 70 mph to 123 mph, which is unrealistic.

When we are given a fixed delay constraint and fixed mobile speed, the available diversity order in one block is $L = \lceil 2 f_D T_B + 1 \rceil$. In this case, we might try to use stronger coding by increasing the block size, N, of the turbo code. This can be done by either increasing the source bit rate or multiplexing information from multiple sources. Figures 7 and 8 show the BER and FER, respectively, vs. different block sizes in this situation. However, expected improvement only occurs when there is a sufficient amount of available diversity. If the available diversity order is smaller than $L = \lceil 2 f_D T_B + 1 \rceil |_{f_D T_B = 3.2} = 8$, increasing the block size of the turbo code does not bring any additional coding gain, and the FER may even increase.

If the delay constraint is loosened, more time diversity could be exploited and stronger turbo codes can be used.

Thus, dramatic improvement is expected. Figures 9 and 10 show the BER and FER, respectively, vs. E_b/N_0 for different delay constraints, with a fixed data rate of 9.6 kbps. Clearly, the delay constraint puts a fundamental limit for turbo coded transmission over a fading channel.

Finally, in Figures 11 and 12, the turbo code with two 16-state RSC encoders is compared to a convolutional code with equivalent hardware complexity, as well as to another turbo code of lower complexity. The simpler rate-1/3 turbo code consists of two 4-state RSC encoders with generator $(5/7)_8$. Although the simpler turbo code offers worse performance than does the more complex turbo code at large values of normalized Doppler frequency, $f_D T_s$, the BER performance becomes slightly better when $f_D T_s \leq 3 \times 10^{-3}$. It is claimed in [12] that the processing load of the LOG-MAP decoder is no more than four times that of a conventional Viterbi decoder, for the same convolutional code. So the Viterbi decoder of the 128-state convolutional code is of comparable complexity to the iterative decoder for the turbo code with 16-state RSC encoders. The BER performance of the convolutional code is virtually the same as that of the turbo code of equivalent complexity for small values of $f_D T_s$. But at large values of $f_D T_s$, even the simpler turbo code works significantly better than the convolutional code. The FER performance of the more complex turbo code is the best of the three, even at small values of normalized Doppler frequency. Also, note that at large values of $f_D T_s$, whereas the BER of the more complex turbo code is only a factor of two better than that of the simpler turbo code, the improvement of FER is more than an order of magnitude. The bit error patterns of the more complex codes appear to be more clustered than those of simpler codes. This can be explained by the fact that for the more complex turbo code the error events are more often caused by codewords of larger weights than for simpler turbo code under the same condition, while the error events of convolutional codes are dominated by the codewords of smallest distances.

5. Conclusions

Turbo coding for the land mobile channel was considered with an externally imposed delay constraint. For even moderate delay requirements, the performance degradation was seen to be severe at low speeds of the mobile, and significant at moderate and high speeds. The degradation is caused mainly by the limited diversity order that turbo codes can achieve from the fading process during one block. The delay constraint also limits the block length of the coding scheme, which is a major factor in determining the effectiveness of the turbo codes. It was found that, whereas a turbo code with a larger block size and more complexity can give a large gain for a rapidly varying channel, for low and moderate speeds of the mobile this gain is greatly reduced.

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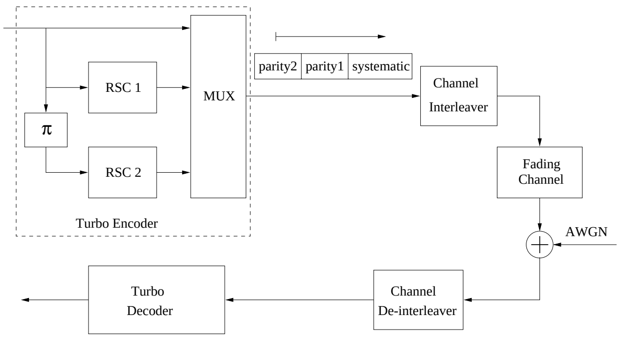

Figure 1. Block diagram for the system model

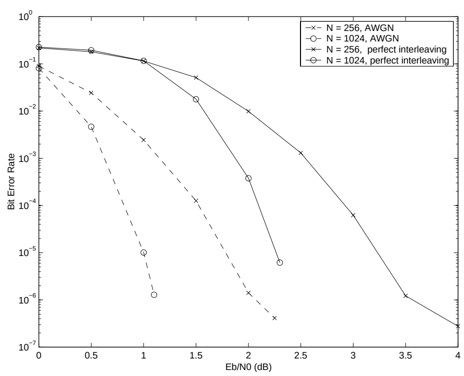

Figure 2. Simulation of the rate-1/3, (1, 33/31, 33/31) turbo code on AWGN and perfect interleaving fading channel

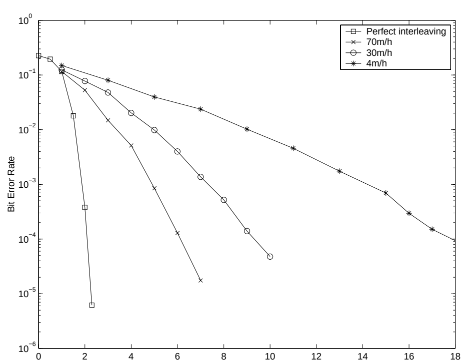

Figure 3. Simulation of the rate-1/3 turbo code over partially interleaved fading channel, $N = 1024$, interleaver size=3072, delay=106.7ms

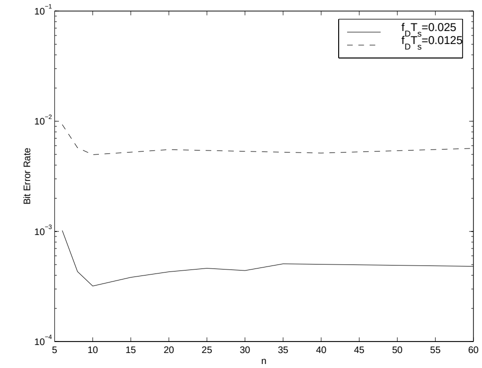

Figure 4. BER vs. the number of columns of the channel interleaver of size 768, the rate-1/3 turbo code, $N = 256$, E_b/N_0 = 4.0dB

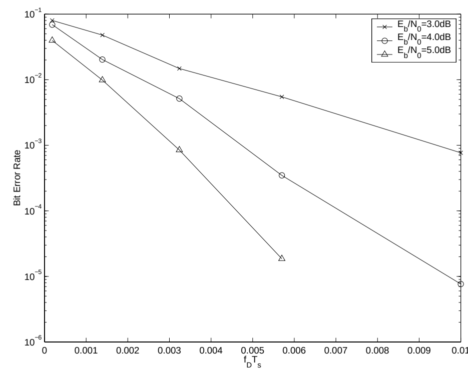

Figure 5. BER vs. the normalized Doppler frequency for the rate-1/3 turbo code, $N = 1024$, interleaver size=3072

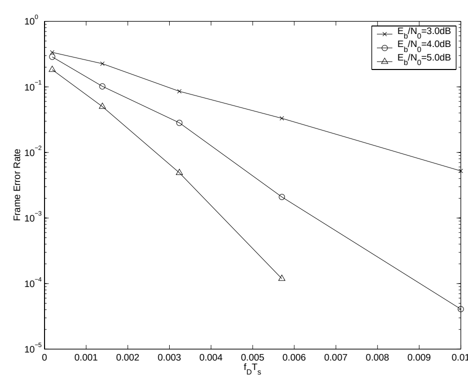

Figure 6. FER vs. the normalized Doppler frequency for the rate-1/3 turbo code, $N = 1024$, interleaver size=3072

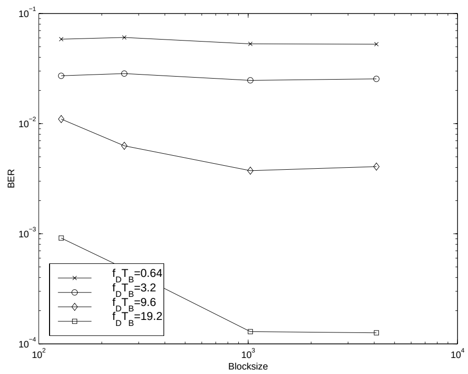

Figure 7. BER vs. blocksize for the rate-1/3 turbo code with fixed diversity order, E_b/N_0 = 4.0dB

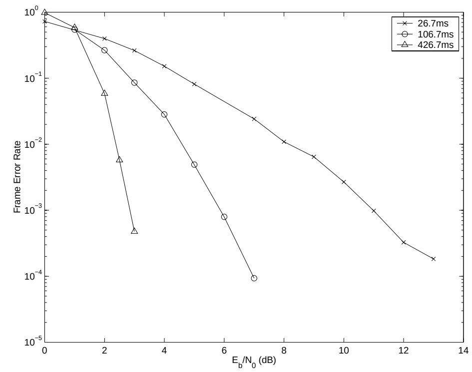

Figure 10. FER vs. E_b/N_0 for different delay constraints, the rate-1/3 turbo code, 70mph

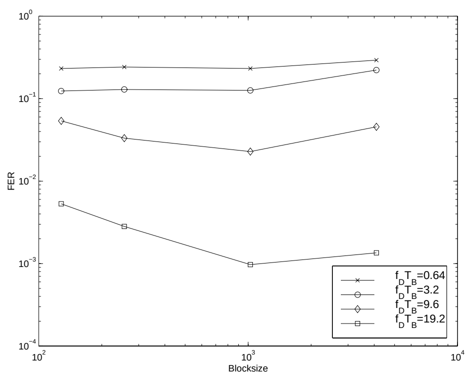

Figure 8. FER vs. blocksize for the rate-1/3 turbo code with fixed diversity order, E_b/N_0 = 4.0dB

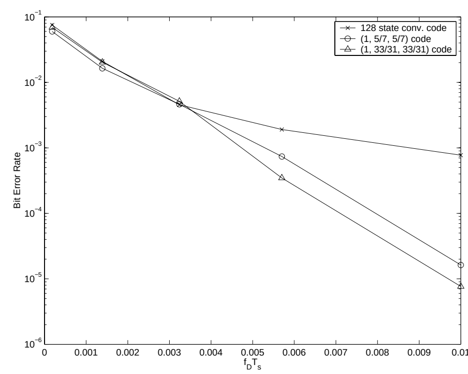

Figure 11. BER vs. the normalized Doppler frequency for turbo codes and convolutional codes, N=1024, rate 1/3, E_b/N_0 = 4.0dB

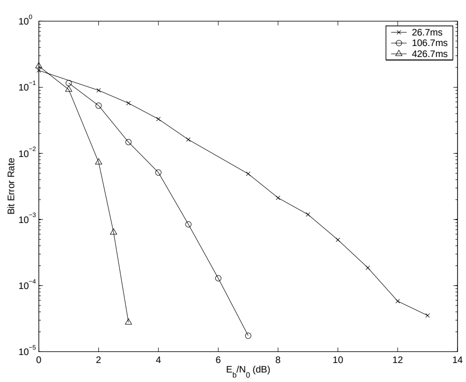

Figure 9. BER vs. E_b/N_0 for different delay constraints, the rate-1/3 turbo code, 70mph

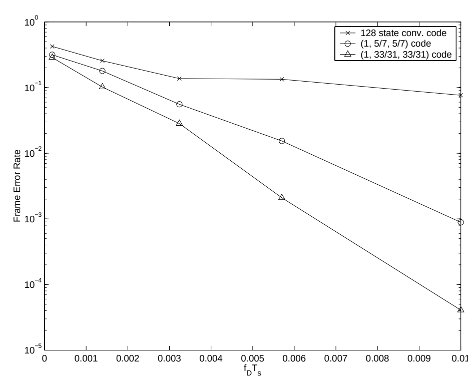

Figure 12. FER vs. the normalized Doppler frequency for turbo codes and convolutional codes, N=1024, rate 1/3, E_b/N_0 = 4.0dB