Reduced Complexity MIMO Detectors for LDPC Coded Systems

Adina Matache, Christopher Jones, and Richard Wesel
Electrical Engineering Department, University of California, Los Angeles, CA

ABSTRACT

We consider the performance and complexity analysis of low-density parity-check (LDPC) coded multiple-input multiple-output (MIMO) systems with reduced-complexity detectors in fast Rayleigh fading channels. We use a turbo iterative detection-and-decoding receiver which combines a soft MIMO demodulator and a message-passing LDPC decoder. We examine three low complexity detectors: minimum mean-square error (MMSE) with soft interference cancellation (SIC), MMSE suppression, and MMSE with hard decision interference cancellation (MMSE-HIC). We compare the performance and complexity of these detectors with the standard soft maximum a posteriori (MAP) detector and show that they provide a lower computational complexity with only a small penalty in performance (~ 0.5 *dB). We also provide comparisons with the Shannon capacity limits for ergodic multiple-antenna channels and show that an LDPC coded system using a length* $15,000$ *code and QPSK modulation with reduced-complexity detection operates* 2 *and* 3 *dB from the ergodic capacity of the Rayleigh MIMO channel.*

I. INTRODUCTION AND MODEL

The application of LDPC codes to spectrally efficient multiple-input multiple-output signaling has been proposed in Wireless Local Area Network (WLAN) and Unshielded Twisted Pair (UTP) (10 Gigabit Ethernet over copper) working groups within the IEEE 802.11 and 802.3 standards bodies. Spatially multiplexed coded modulations that are decoded using iterative detection-and-decoding techniques can offer capacity-approaching performance at complexities within the reach of advanced process technology. Work to mitigate complexity in the detection process has been conducted by the authors in [1] and [2]. These authors introduce a suboptimal receiver based on a minimum mean-square error (MMSE) soft interference cancellation detector. In [3] a “list” sphere decoder is used to iteratively detect and decode either simple convolutional or more powerful turbo codes. In [4] the authors present an iterative greedy demodulation-decoding technique for turbo codes based on a greedy detection method for multiuser communications. More recently in [5] the authors present a low-density parity-check (LDPC) coded MIMO OFDM system using either the optimal soft maximum *a posteriori* (MAP) demodulator or the low complexity minimum-mean square-error soft interference cancellation (MMSE-SIC) demodulator.

Lacking in the literature is a performance comparison of detectors that have varying degrees of complexity. The intent of such a comparison is to aid systems engineers in the selection of appropriate detection techniques for deployment in next-generation devices. The work of the present paper provides performance and complexity measures of the optimal MAP detector and of three lower complexity MMSE based detection schemes: MMSE-SIC, MMSE suppression and MMSE with *hard* decision interference cancellation (MMSE-HIC). Specifically, algorithmic descriptions and operation counts per decoding pass are given and the performance of all four detectors in uncorrelated (fast) Rayleigh fading 2×2, 4×4, and 8×8 MIMO signaling is provided.

The primary result of the paper is that the selection of an appropriate MMSE-based detector yields several orders of magnitude in complexity reduction while sacrificing no more than 0.5 dB of SNR in Rayleigh fast fading. We also provide comparisons with the Shannon capacity limits for ergodic multiple-antenna channels and show that an LDPC coded system using a length $15,000$ code, QPSK modulation, and reduced-complexity detectors, performs between 2 and 3 dB of the theoretical capacity of the Rayleigh MIMO channel.

II. ITERATIVE DETECTION AND LDPC DECODING

The system model under consideration is an LDPC-coded MIMO system with n_t transmitter antennas and n_r receiver antennas, signaling through frequency-nonselective fading. We use a high-performance irregular LDPC code designed in our previous work [6], [7]. The transmitter structure is illustrated in Fig. 1. The information

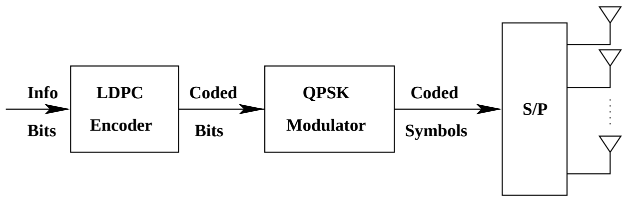

Fig. 1. Transmitter structure of an LDPC coded MIMO system.

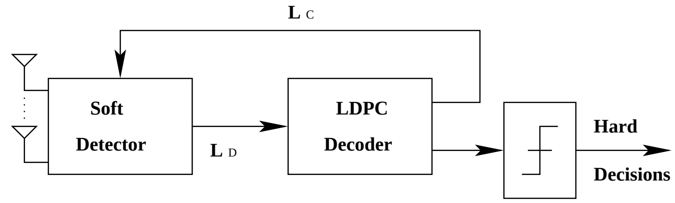

Fig. 2. Turbo iterative detection-and-decoding receiver for an LDPC coded MIMO system.

data is first encoded by the rate-R LDPC code, modulated by a complex constellation with 2^{M_c} possible signal points and average energy equal to E_x, and then distributed among the n_t antennas. Let $\mathbf{x}$ be an $n_t \times 1$ vector of transmitted symbols with components $x_1, x_2, \ldots, x_{n_t}$ and $\mathbf{y}$ an $n_r \times 1$ vector of received signals with components $y_1, y_2, \ldots, y_{n_r}$, related by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} = [\mathbf{h}_1 \mathbf{h}_2 \ldots \mathbf{h}_{n_t}]$ is the $n_r \times n_t$ complex channel matrix known perfectly by the receiver, and $\mathbf{n}$ is a vector of independent zero-mean, complex Gaussian noise entries with variance $\sigma^2 = N_0/2$ per real component. We assume that the average signal-to-noise ratio (SNR) at each receiver antenna, denoted by ρ, is independent of the number of transmitter antennas n_t. The fading model we assume is a fast Rayleigh fading model in which the channel characteristics are changing every vector constellation symbol.

Iterative detection-and-decoding is used to approach the maximum-likelihood (ML) performance of joint MIMO detection and LDPC decoding. Fig. 2 gives a flowchart of the turbo iterative receiver structure. In this structure, the soft MIMO detector incorporates extrinsic information provided by the LDPC decoder, and the LDPC decoder incorporates soft information provided by the MIMO detector. Extrinsic information between the detector and decoder is then exchanged in an iterative fashion until an LDPC codeword is found or a maximum number of iterations is performed. With LDPC codes, convergence to a codeword is easy to detect since we need only verify that the parity checks are satisfied. The message-passing (also known as belief-propagation) decoding algorithm used to decode the LDPC code is described in detail in [8].

In the following we detail the soft MAP MIMO detector and the reduced-complexity MMSE-SIC, MMSE suppression and MMSE-HIC detectors. We examine the complexity of each scheme and select parameters for modulation cardinality and transmit-receive antenna multiplicity to facilitate a numerical comparison of complexity.

A. MAP Detector

In the soft MAP detector, the received vector $\mathbf{y}$ is demapped by a log-likelihood ratio (LLR) calculation for each bit of the $n_t M_c$ coded bits included in the transmit vector $\mathbf{x}$. The *extrinsic* information provided by the MAP detector is the difference of the soft-input and soft-output LLR values on the coded bits. For the ith code bit b_i ($i \in \{1, \ldots, n_t M_c\}$) of the transmit vector $\mathbf{x}$, the extrinsic LLR value of the estimated bit is computed as

$$\begin{aligned} L_D(b_i) &= \log \frac{P(b_i = +1|\mathbf{y}, \mathbf{H})}{P(b_i = -1|\mathbf{y}, \mathbf{H})} - \log \frac{P(b_i = +1)}{P(b_i = -1)} \\ &= \log \frac{\sum_{\mathbf{x} \in \mathcal{X}_i^{+1}} P(\mathbf{y}|\mathbf{x}, \mathbf{H}) P(\mathbf{x})}{\sum_{\mathbf{x} \in \mathcal{X}_i^{-1}} P(\mathbf{y}|\mathbf{x}, \mathbf{H}) P(\mathbf{x})} - L_C(b_i) \end{aligned} \tag{2}$$

where $L_C(b_i)$ is the extrinsic information of the bit b_i computed by the LDPC decoder in the previous turbo iteration ($L_C(b_i) = 0$ at the first iteration) and $\mathcal{X}_i^{+1}$ is the set of $2^{n_t M_c - 1}$ vector hypotheses $\mathbf{x}$ having $b_i = +1$, ($\mathcal{X}_i^{-1}$ is similarly defined). Assuming the bits within $\mathbf{x}$ are statistically independent of one another, the *a priori* probability $P(\mathbf{x})$ can be written as

$$P(\mathbf{x}) = \prod_{j=1}^{n_t M_c} P(b_j) = \prod_{j=1}^{n_t M_c} \left[1 + \exp(-\mathbf{x}^{b_j} L_C(b_j))\right]^{-1} \tag{3}$$

where $\mathbf{x}^{b_j}$ corresponds to the value $(+1, -1)$ of the jth bit in the vector $\mathbf{x}$. In the above LLR value calculation, the likelihood function $P(\mathbf{y}|\mathbf{x}, \mathbf{H})$ is specified by a multivariate Gaussian density function.

Since the cardinality of the vector sets $\mathcal{X}_i^{+1}$ and $\mathcal{X}_i^{-1}$ in (2) equals $2^{n_t M_c - 1}$, the complexity of the soft MAP detector is exponential in the number of transmitter antennas and the number of bits per constellation symbol. At each iteration, the MAP detector has to compute the LLRs for $n_t M_c$ bits in each transmit symbol vector. With an equal number of transmit and receive antennas, $n_t = n_r = n$, evaluating (2) involves the following steps:

for $i = 1$ to 2^{nM_c}

1) Compute the *a priori* probability $P(\mathbf{x})$ in (3): $2nM_c$ flops.

end

for $i = 1$ to nM_c

2) Compute the LLR value in (2): $2 \cdot 2^{nM_c}$ flops.

end

Our complexity analysis uses an 'operand-counting' approach where the total number of operations performed on elements fetched from memory to produce a given output is accrued. For instance the cost of evaluating (3) is found by counting the cost of the form $\frac{1}{1+x}$ as a single op (additions with constants are not included since '1' is not a fetched operand and table lookups to exp() and log() function are omitted from the aggregate cost as they don't involve additional operand fetches). This form is evaluated nM_c times and $nM_c - 1$ multiplications produce the final product. To simplify the expression, the multiplication cost is rounded up to nM_c and the total cost of evaluating (3) is said to be $2nM_c$.

The likelihood function $P(\mathbf{y}|\mathbf{x}, \mathbf{H})$ is precomputed for all 2^{nM_c} hypotheses at the beginning of the iterative process and has a cost of $2n^2 + 3n$ flops per vector hypothesis. Then, the (approximate) cost of the MAP detector per turbo iteration is $(4nM_c) \cdot 2^{nM_c}$ flops and the initial cost of precomputing the likelihood functions for all hypotheses is $(2n^2 + 3n) \cdot 2^{nM_c}$ flops.

B. MMSE-SIC Detector

The suboptimal demodulator based on a minimum mean-square error soft-interference cancellation (MMSE-SIC) criterion consists of a parallel interference canceler followed by an MMSE filter. It is analogous to a multiuser detector proposed in [9]. It is also described in [2] and [10]. Below we review this algorithm and comment on its complexity.

The MMSE-SIC detector first forms soft estimates of the transmitted symbols by computing the symbol mean $\bar{x}_j$ based on the available *a-priori* information:

$$\bar{x}_j = \sum_{x \in \mathcal{A}} x P(x_j = x) \tag{4}$$

where $\mathcal{A}$ is the complex constellation set. The *a-priori* probabilities are calculated from the extrinsic LLRs provided by the LDPC decoder and assuming the bits within a symbol are statistically independent of one another:

$$P(x_j = x) = \prod_{l=1}^{M_c} [1 + \exp(-x^{b_l} L_C(b_{(j-1)M_c+l}))]^{-1}, \tag{5}$$

where x^{b_l} indicates the value of the lth bit of symbol x. At the beginning of the iterative process, all symbols are equally likely and their probability is 2^{-M_c}. It follows that for a complex symmetric constellation like QPSK, the soft estimates are equal to zero and in effect no cancellation is performed in the first iteration. In subsequent iterations, as the bit reliabilities provided by the LDPC decoder improve, the soft estimates become closer to their true value.

For the kth transmit antenna, the soft interference from the other $n_t - 1$ antennas is canceled to obtain

$$\mathbf{y}_k = \mathbf{y} - \sum_{j=1, j\neq k}^{n_t} \bar{x}_j \mathbf{h}_j \tag{6}$$

$$= x_k \mathbf{h}_k + \sum_{j=1, j\neq k}^{n_t} (x_j - \bar{x}_j)\mathbf{h}_j + \mathbf{n}. \tag{7}$$

A detection estimate $\hat{x}_k$ of the transmitted symbol on the kth antenna is obtained by applying a linear filter $\mathbf{w}_k$ to $\mathbf{y}_k$:

$$\begin{aligned} \hat{x}_k &= \mathbf{w}_k^\dagger \mathbf{y}_k \\ &= (\mathbf{w}_k^\dagger \mathbf{h}_k) x_k + \sum_{j=1, j\neq k}^{n_t} (\mathbf{w}_j^\dagger \mathbf{h}_j)(x_j - \bar{x}_j) + \mathbf{w}_k^\dagger \mathbf{n}, \end{aligned} \tag{8}$$

where † represents the conjugate transpose operator. In the above equation, the first term represents the desired term, the second term is the residual interference from the other transmitter antennas, and the last term is a phase rotated noise term.

The filter $\mathbf{w}_k$ is chosen to minimize the mean-square error between the transmit symbol x_k and the filter output $\hat{x}_k$ and depends on the variance of the symbols used in the cancellation step. It can be shown that the MMSE-SIC solution is given by

$$\mathbf{w}_k = \left(\frac{N_0}{E_x} \mathbf{I}_{n_r} + \mathbf{H} \mathbf{\Delta}_k \mathbf{H}^\dagger \right)^{-1} \mathbf{h}_k, \tag{9}$$

where the covariance matrix $\mathbf{\Delta}_k$ is

$$\mathbf{\Delta}_k = \text{diag}\left[\frac{\sigma_{x_1}^2}{E_x}, \ldots, \frac{\sigma_{x_{k-1}}^2}{E_x}, 1, \frac{\sigma_{x_{k+1}}^2}{E_x}, \ldots, \frac{\sigma_{x_{n_t}}^2}{E_x} \right], \tag{10}$$

and $\sigma_{x_i}^2$ is the variance of the ith antenna symbol computed as:

$$\sigma_{x_i}^2 = \sum_{x \in \mathcal{A}} |x - \bar{x}_i|^2 P(x_i = x). \tag{11}$$

Note that the MMSE-SIC filter adjusts its weights according to the quality of the soft-canceled symbols through the covariance matrix $\mathbf{\Delta}_k$. In the two extreme cases, the MMSE-SIC filter reduces to a simple suppression filter or a maximum-ratio combining filter. The first case occurs when the canceled symbols are all zero (i.e. no cancellations) and the symbol variances $\sigma^2_{x_i}$ are all equal to E_x. It follows that the covariance matrix $\mathbf{\Delta}_k$ becomes an identity matrix and the filter reduces to the well-known MMSE suppression filter:

$$\mathbf{w}_k = \left(\frac{N_0}{E_x}\mathbf{I}_{n_r} + \mathbf{H}\mathbf{H}^\dagger\right)^{-1}\mathbf{h}_k. \tag{12}$$

The second case occurs when the canceled symbols are the true symbols (i.e. perfect cancellations) and the symbol variances $\sigma^2_{x_i}$ are all zero. It can be shown that the MMSE-SIC filter reduces to a filter of the form

$$\mathbf{w}_k = \left(\frac{N_0}{E_x} + \mathbf{h}_k^\dagger\mathbf{h}_k\right)^{-1}\mathbf{h}_k, \tag{13}$$

which in effect forms a maximum-ratio combining with the corresponding column vector of the channel matrix.

Therefore, in general, the MMSE-SIC detector performs a combination of suppressions and cancellations. The amount of suppression done by the detector is determined by the quality of the canceled symbols, which ultimately dictates the performance of the MMSE-SIC detector.

As we saw before, the output of the MMSE-SIC filter includes the desired symbol, residual co-antenna interference, and noise. Note that under a Gaussian input, the output of the filter is also Gaussian. However, under a constrained input scenario in which the symbols belong to a complex constellation like QPSK, the filter output is neither Gaussian nor *i.i.d.* Despite this fact, following [9], we approximate $\hat{x}_k$ by the output of an equivalent AWGN channel with $\hat{x}_k = \mu_k x_k + z_k$, where

$$\mu_k = \mathbf{w}_k^\dagger\mathbf{h}_k, \tag{14}$$

and z_k is a zero-mean complex Gaussian variable with variance η_k^2 given by

$$\eta_k^2 = E_x(\mu_k - \mu_k^2). \tag{15}$$

Then, the extrinsic log-likelihood ratio computed by the MMSE-SIC detector for the lth bit ($l \in \{1, \ldots, M_c\}$) of the symbol x_k transmitted by the kth antenna is

$$\begin{aligned} &L_D(b_{(k-1)M_c+l}) = \\ &\log\frac{P(b_{(k-1)M_c+l}=+1|\hat{x}_k)}{P(b_{(k-1)M_c+l}=-1|\hat{x}_k)} - \log\frac{P(b_{(k-1)M_c+l}=+1)}{P(b_{(k-1)M_c+l}=-1)} \\ &= \log\frac{\sum_{x\in\mathcal{A}_l^{+1}} P(\hat{x}_k|x)P(x)}{\sum_{x\in\mathcal{A}_l^{-1}} P(\hat{x}_k|x)P(x)} - L_C(b_{(k-1)M_c+l}) \end{aligned} \tag{16}$$

where $\mathcal{A}_l^{+1}$ is the set of 2^{M_c-1} hypotheses x for which the lth bit is $+1$ ($\mathcal{A}_l^{-1}$ is similarly defined). In the above calculation of the extrinsic LLR value, the *a-priori* probability $P(x)$ is given by (5) and the likelihood function $P(\hat{x}_k|x)$ is approximated by

$$P(\hat{x}_k|x) \simeq \frac{1}{\pi\eta_k^2}\exp\left(-\frac{1}{\eta_k^2}|\hat{x}_k - \mu_k x|^2\right). \tag{17}$$

Note that the MMSE-SIC detector has a lower complexity than the MAP detector. This can be seen from (16) where the extrinsic LLR is computed from the *scalar* output $\hat{x}_k$ of the MMSE filter, in contrast with (2) where the extrinsic LLR is computed from the received vector $\mathbf{y}$. With n transmit and n receive antennas, evaluating (16) involves the following steps:

for $k = 1$ to n
  for $i = 1$ to 2^{M_c}
    1) Compute the *a-priori* probability $P(x)$ in (5): $2M_c$ flops
  end
  2) Evaluate the symbol mean $\bar{x}_i$ and variance $\sigma^2_{x_i}$: $5 \cdot 2^{M_c}$ flops
  3) Cancel the soft estimates in (7): $2n^2$ flops.
  4) Evaluate the matrix $\mathbf{G} = \left(\frac{N_0}{E_x}\mathbf{I}_{n_r} + \mathbf{H}\mathbf{\Delta}_k\mathbf{H}^\dagger\right)$: $2n^3 + n^2 + 3n$ flops
  5) Solve $\mathbf{G}\mathbf{w}_k = \mathbf{h}_k$ for $\mathbf{w}_k$: $n^3/3$ flops using Choleski factorization [11]
  6) Compute the detection estimate $\hat{x}_k$: $2n$ flops
  7) Compute μ_k and η_k^2: $2n$ flops
  for $l = 1$ to 2^{M_c}
    8) Evaluate the likelihood function $P(\hat{x}_k|x)$ in (17): 4 flops
  end
  for $l = 1$ to M_c
    9) Compute the LLR value in (16): $2 \cdot 2^{M_c}$ flops
  end
end

Then, the (approximate) computational complexity of the MMSE-SIC detector is $7n^4/3 + 3n^3 + 7n^2 + (4nM_c) \cdot 2^{M_c} + (9n) \cdot 2^{M_c}$ flops per turbo iteration.

C. MMSE Suppression Detector

An even further reduction in complexity is obtained with a simple MMSE suppression detector. Since no interference cancellation is performed, the MMSE suppression filter $\mathbf{w}_k$ from (12) is evaluated only once, at the beginning of the turbo iterative process, and applied to the received vector $\mathbf{y}$ to obtain a detection estimate $\hat{x}_k$ for the kth antenna.

Assuming an equal number of transmit and receive antennas, evaluating the LLR values involves the following steps:

for $k = 1$ to n
  for $i = 1$ to 2^{M_c}
    1) Compute the *a-priori* probability $P(x)$ in (5): $2M_c$ flops
  end
  for $l = 1$ to M_c
    2) Compute the LLR value in (16): $2 \cdot 2^{M_c}$ flops
  end
end

Then, the complexity per turbo iteration is $(4nM_c) \cdot 2^{M_c}$ flops. Moreover, the initial cost of evaluating the MMSE suppression filters $\mathbf{w}_k$, the detection estimates $\hat{x}_k$, and the likelihood functions $P(\hat{x}_k|x)$ is $7n^4/3 + 5n^2 + (4n) \cdot 2^{M_c}$ flops. Note that the MMSE suppression detector reduces the complexity per turbo iteration from $\mathcal{O}(n^4)$ (for the MMSE-SIC detector) to $\mathcal{O}(n)$. Of course, the overall complexity for the MMSE suppression scheme remains $\mathcal{O}(n^4)$ because of required initial processing. However, a more flexible allocation of computational resources becomes possible given the need to solve a system of equations to determine $\mathbf{w}_k$ only once rather than on a per iteration basis.

D. MMSE-HIC Detector

Here we introduce another reduced-complexity detector based on the simple MMSE filter with hard-decision interference cancellations. The main idea is to improve the performance of the MMSE suppression detector by canceling hard decision estimates of the interfering symbols while also maintaining the reduced computational complexity. At the beginning of the iterative process, cancellations are not possible since no reliability information is yet available from the decoder. Therefore, in the first iteration, we use the MMSE suppression filter $\mathbf{w}_k$ from (12) in the same manner as above to obtain the detection estimate $\hat{x}_k$ for the kth antenna. However, in subsequent iterations, as soft information from the decoder becomes available, hard decision estimates on the LDPC code bits b_i ($i \in \{1, \ldots, n_t M_c\}$) can be obtained from

$$\hat{b}_i = \text{sign}\left[L_D(b_i) + L_C(b_i)\right], \tag{18}$$

and a hard decision estimate on the kth antenna symbol can be formed as

$$\tilde{x}_k = f\left(\hat{b}_{(k-1)M_c+1}\hat{b}_{(k-1)M_c+2}\ldots\hat{b}_{kM_c}\right) \tag{19}$$

where f is a function that maps an input bit vector to a complex constellation point. Assuming these hard decision symbol estimates are correct, their cancellation would provide a better detection estimate for the antenna of interest. More specifically, for the kth transmit antenna, the hard decision estimates of the other $n_t - 1$ interfering antennas can be canceled and then the maximum-ratio combining filter $\mathbf{w}_k$ in (13) can be used to obtain the detection estimate $\hat{x}_k$.

The assumption of correct hard decisions does not hold very well in the early stages of the iterative process. In order to avoid error propagation due to incorrect hard decisions, it is very important that cancellations be performed only when the reliability of the canceled symbols is high according to some cancellation criterion. We experimented with different criteria for interference cancellation and found that the following two methods give good results:

- **Average of LLRs**
  With this criterion, first the following average is computed and then compared to a predetermined threshold value

  $$\Phi_a = \frac{1}{n_t M_c}\sum_{i=1}^{n_t M_c} |L_C(b_i)| \geq \Theta_a. \tag{20}$$

  The threshold Θ_a is found experimentally as the threshold that yields the best bit-error rate (BER) performance. Note that a too low threshold value would introduce undesirable error propagation due to incorrect cancellations, while a too high threshold value would give the same performance as the MMSE suppression detector since no cancellations are performed in this case.
- **Probability of bit vector**
  With this criterion, the probability of a bit vector is first computed and then compared to a threshold value

  $$\begin{aligned}\Phi_p =& P(b_1 b_2 \ldots b_{n_t M_c}) \\ =& \prod_{i=1}^{n_t M_c} \left[1 + \exp(-|L_C(b_i)|)\right]^{-1} \geq \Theta_p.\end{aligned} \tag{21}$$

  As before, the threshold Θ_p is found experimentally to optimize the BER performance.

The computational complexity of the MMSE-HIC detector is that of the MMSE suppression detector plus the additional cost of checking the cancellation criterion every turbo iteration and performing the hard decision interference cancellation whenever the criterion is satisfied.

In Table I we give a complexity comparison example based on a flop count for the MAP, MMSE-SIC, and MMSE

TABLE I

COST (IN FLOPS) OF COMPUTING THE LLRS FOR DIFFERENT MIMO DETECTORS.

	$n = 2$	$n = 4$	$n = 8$
MAP initial	224	$1.13 \cdot 10^4$	$9.96 \cdot 10^6$
MAP per it	256	$8.19 \cdot 10^3$	$4.19 \cdot 10^6$
MAP total (30 iterations)	$7.90 \cdot 10^3$	$2.57 \cdot 10^5$	$1.36 \cdot 10^8$
MMSE-SIC initial	0	0	0
MMSE-SIC per it	225	$1.17 \cdot 10^3$	$1.21 \cdot 10^4$
MMSE-SIC total (30 iterations)	$6.76 \cdot 10^3$	$3.52 \cdot 10^4$	$3.63 \cdot 10^5$
MMSE initial	89	741	$1.0 \cdot 10^4$
MMSE per it	64	128	256
MMSE total (30 iterations)	$2.0 \cdot 10^3$	$4.58 \cdot 10^3$	$1.77 \cdot 10^4$

suppression detectors for $n \times n$ antenna configurations with $n = 2, 4, 8$, using QPSK modulation. For each detector, we provide the initial cost (which is the cost of computations that must be done only once for all of the iterations), the cost per turbo iteration, and the total computational cost assuming 30 iterations.

III. RESULTS

In this section we examine the performance of LDPC coded BLAST systems using the soft MIMO detectors introduced in the previous section. In our study, we assume that the number of receive antennas is the same as the number of transmit antennas. The LDPC code used in the simulations is a rate-$1/3$, length 15000 code that was realized from the ensemble of codes with variable (left) node distribution $\lambda(x) = 0.27603x + 0.11195x^2 + 0.17229x^3 + 0.01712x^4 + 0.42261x^{14}$, and check (right) node distribution $\rho(x) = x^5$. The density evolution threshold for this code in AWGN is -4.92 dB. This degree sequence was found via a linear program that sought the highest rate ensemble under a given threshold and maximum left and right node degree constraints [12]. In order to acquire a given rate goal, the density evolution initial mean was adjusted to achieve the desired rate. The mapping in all our simulations is a Gray-labeled QPSK constellation. The resulting spectral efficiency is $2/3$ bits/antenna/channel use.

We assume a fast Rayleigh fading scenario, where the channel matrix is realized independently from one transmission time to the next. We compare our bit-error rate results with the theoretical channel capacity limit. Under

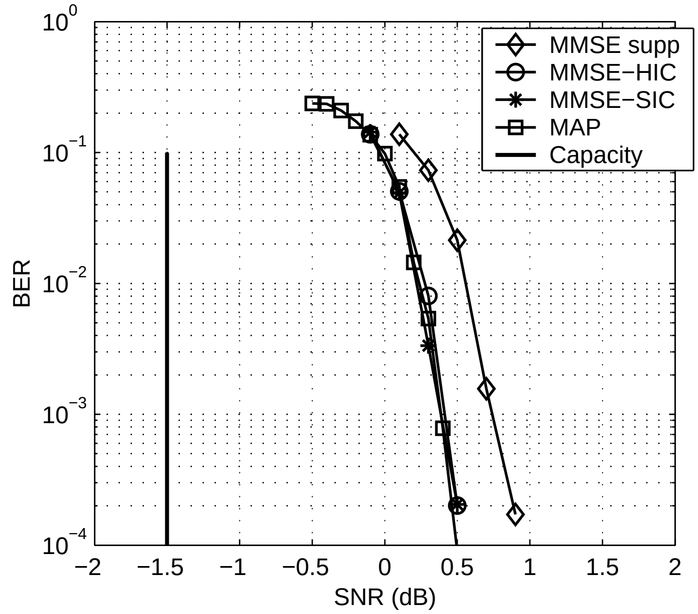

Fig. 3. Performance of LDPC ($n = 15,000$ rate = $1/3$) coded 2×2 MIMO system with MAP, MMSE-SIC, MMSE-HIC and MMSE suppression detectors.

the fast fading assumption, the theoretical capacity limit is the ergodic channel capacity given by [13]:

$$C = E\left[\log_2 \det\left(\mathbf{I}_{n_r} + \frac{\rho}{n_t}\mathbf{H}\mathbf{H}^\dagger\right)\right] \quad (22)$$

where the expectation is over the entries of $\mathbf{H}$.

Figs. 3, 4, and 5 show the BER performance versus average SNR per receiver antenna on 2×2, 4×4, and 8×8 fast fading Rayleigh channels. On these plots we also show the channel capacity at the corresponding transmission bit rate for these systems. As expected, the MAP detector yields the best performance, which is 2 dB away from the theoretical capacity at BER $= 10^{-4}$ on the 2×2 channel. We note that the MMSE-SIC detector has essentially the same performance as the MAP detector on the 2×2, and 4×4 systems. For the 8×8 channel the computational complexity associated with the MAP detector is prohibitive and this result was not simulated.

Our results differ from the results reported in [5] where a system based on the MMSE-SIC detector has a performance degradation (less than 1 dB) compared to a system based on the MAP detector. We attribute this difference to the fact that in [5] a detector iteration is performed only after a number of decoder iterations, whereas in our work every decoder iteration is followed by a detector iteration.

Moreover, our simulation results show that the very low complexity MMSE suppression detector has a performance loss of only 0.5 dB or less. This loss is approximately cut in

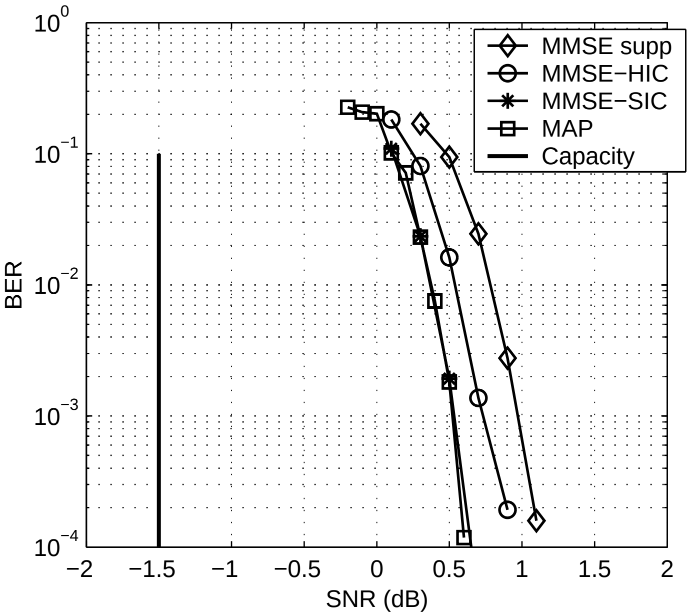

Fig. 4. Performance of LDPC ($n = 15,000$ rate = $1/3$) coded 4×4 MIMO system with MAP, MMSE-SIC, MMSE-HIC and MMSE suppression detectors.

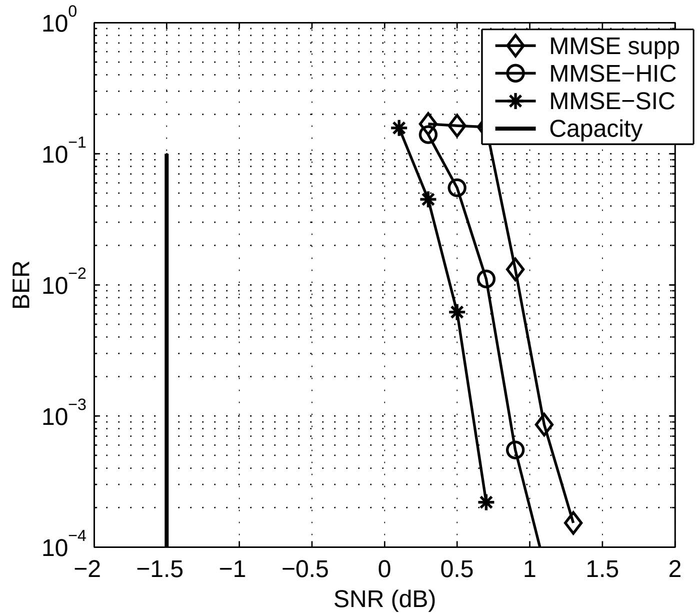

Fig. 5. Performance of LDPC ($n = 15,000$ rate = $1/3$) coded 8×8 MIMO system with MMSE-SIC, MMSE-HIC and MMSE suppression detectors.

half if the MMSE-HIC detector with an optimized cancellation threshold is used instead. In fact, in the 2×2 system, the MMSE-HIC detector performance is very close to the MAP and MMSE-SIC detectors.

IV. Conclusion

In this paper we show that LDPC coded MIMO systems using a length $15,000$ code and QPSK modulation with reduced-complexity detectors perform within 2 to 3 dB of the capacity of the Rayleigh MIMO channel. The soft MAP detector is optimal, however the MMSE-SIC detector exhibits similar performance and has a lower overall computational complexity. The MMSE suppression and MMSE-HIC detectors offer yet lower complexities, for relatively small performance penalties. A performance loss of around 0.5 dB is observed for the MMSE suppression detector. This loss is cut approximately in half by an MMSE-HIC detector using a carefully chosen cancellation threshold.

References

[1] M. Sellathurai and S. Haykin, "TURBO-BLAST for high speed wireless communications," in *Proceedings of WCNC*, vol. 1, Sept 2000, pp. 315–320.

[2] ——, "Turbo-BLAST for wireless communications: theory and experiments," *IEEE Trans. Signal Proc.*, vol. 50, no. 10, pp. 2538–2546, Oct 2002.

[3] S. Ten Brink and B. Hochwald, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. on Comm.*, vol. 51, no. 3, pp. 389–399, Mar. 2003.

[4] A. A. AlRustamani, A. Stefanov, and B. R. Vojcic, "Turbo-greedy coding for multiple antenna systems," in *Proceedings of ICC*, vol. 6, June 2001, pp. 1684 –1689.

[5] B. Lu, G. Yue, and X. Wang, "Performance analysis and design optimization of LDPC coded MIMO OFDM systems," *to appear in IEEE Trans. Signal Proc., Special Issue on MIMO Wireless*, vol. 51, no. 11, Nov. 2003.

[6] C. Jones, R. Tian, A. Matache, R. Wesel, and J. Villasenor, "Robustness of LDPC codes on periodic fading channels," in *Proceedings of GlobeCom*, Nov 2002.

[7] C. Jones, A. Matache, T. Tian, J. Villasenor, and R. Wesel, "The universality of LDPC codes on wireless channels," in *Proceedings of MilCom*, Oct 2003.

[8] T. Richardson and R. Urbanke, "The capacity of low-density parity check codes under message-passing decoding," *IEEE Trans. on Inform. Theory*, vol. 47, pp. 599–618, Feb 2001.

[9] X. Wang and H. V. Poor, "Iterative (Turbo) soft interference cancellation and decoding for coded CDMA ," *IEEE Trans. on Comm.*, vol. 47, pp. 1046–1061, July 1999.

[10] B. Lu, G. Yue, and X. Wang, "Performance analysis and design optimization of LDPC coded MIMO OFDM systems," *to appear in EURASIP JASP*, vol. 51, no. 11.

[11] G. H. Golub and C. F. van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD: Johns Hopkins Univ. Press, 1996.

[12] S. Chung, T. Richardson, and R. Urbanke, "Analysis of sum-product decoding of low-density parity-check codes using a Gaussian approximation," *IEEE Trans. on Inform. Theory*, vol. 47, pp. 657–670, Feb 2001.

[13] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov.-Dec. 1999, available: http://mars.bell-labs.com/cm/ms/what/mars/papers/proof/.