A VALUE-AT-RISK CALCULATION OF REQUIRED RESERVES FOR CREDIT RISK IN CORPORATE LENDING PORTFOLIOS

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ABSTRACT

This paper demonstrates that the building blocks of the insurance process, under similar assumptions, produce identical results to the option-pricing approach in the case of pricing individual loans. We examine the performance of a collective of such building blocks, using portfolio historical performance to endogenously parameterize the default cost function unique to a particular portfolio. Having estimated the appropriate default cost function, we can then specify the reserve requirement for a bank operating such a portfolio. In this respect, the additivity of the Poisson parameter is a powerful feature, allowing one to decompose portfolio performance over time and homogeneous portfolio subsections. Portfolios with greater and lesser risk and profitability respectively are hypothesized, and a capital adequacy framework which equates risk across such portfolios is examined. Finally, we simulate the operation of the proposed capital adequacy model. Observed insolvencies are fewer than those observable under present regulation, and specific problems may be identified earlier than at present.

1. INTRODUCTION

Until the late 1980s, banking regulation was considered a matter for national supervisory authorities. Each country specified required excess assets—often referred to as capital adequacy—for banks operating within its jurisdiction. The increasing internationalization of the banking business was perceived to permit regulatory arbitrage and to produce differential capital requirements between jurisdictions. The Basle Committee of the Bank for International Settlements attempted to eliminate these differentials by proposing a uniform capital adequacy calculation, which was set out in a consultative document in March 1988. The proposal was accepted, and since 1992 all banks have calculated their required capital adequacy by reference to the uniform calculation proposed by the Basle Committee.

Briefly described, the Basle Committee's proposal was that lending be weighted by category. Lending to government normally carries a zero weighting, interbank lending and lending to local authorities a 20% weighting, residential mortgage lending a 50% weighting, and all other private-sector lending a 100% weighting. When loan volumes outstanding have been multiplied by their respective weightings and summed, a weighted exposure results. Banks are required to maintain a minimum capital of 8% of weighted exposure. This minimum capital requirement must comprise at least 4% (that is, 50% of 8%) share capital and reserves, while the balance may consist of subordinated debt. The share capital and reserves are referred to as Tier 1 capital, while subordinated debt is referred to as Tier 2 capital.

The uniform calculation is often referred to as risk-based capital (RBC), because it attaches different weightings to differing forms of lending in the calculation of required capital. However, the weightings proposed by the Basle Committee are necessarily broad, and do not adequately reflect the differing risk

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of loans which may be contained within portfolios. RBC implies that risk is directly incorporated into the calculation of capital, which strictly speaking is not the case. This paper illustrates that RBC as presently calculated is insufficiently sensitive to produce a level risk playing field worldwide, and that more sensitive bank-specific measures may be required in order to achieve this objective.

The dataset used to illustrate the principles contained in this paper is the medium/large corporate loan portfolio of a major UK bank, representing a 20% share of its relevant market category. The number of loans outstanding averaged 59,000 over the 5¼-year period (January 1, 1990 through April 1, 1995), while the volume of credit outstanding varied between £19.7 billion and £22.4 billion.

Over the period for which detailed individual loan default data were available (that is, July 1, 1993 through April 1, 1995), a total number of 3,887 loans were observed to default, of which 910 resulted in positive cost to the bank.

Given that the dataset represents private-sector lending, its RBC requirement is 8% of loan volumes outstanding. This paper explores the insolvency probability associated with this RBC requirement, and proposes an alternative portfolio-specific risk calculation that would produce differential capital requirements for portfolios with varying risk profiles.

2. BACKGROUND DEVELOPMENT

Santomero (1984) states that "If one accepts [the] view that bank liabilities are essentially 100% insured, then the entire issue of bank capital and risk-taking should be recast in terms of a discussion of insurance pricing." He continues, "Literature on optimal bank capital is a bit vague and very model-specific."

Current regulatory requirements for lending banks stipulate capital adequacy (asset excess) of 8% of risk-weighted assets; our dataset is entirely composed of private sector lending, with a risk weighting of 100%. If this minimum capital-adequacy requirement appropriate to lending of the type included within our dataset is taken as indicative of required reserves, then this reserve level appears to be amply sufficient (based on our dataset bank's empirical experience) to ensure the *de facto* 100% insurance of bank liabilities, even without deposit insurance, since the probability of insolvency (that is, liability excess) is extremely small for the chosen portfolio, and also for riskier portfolios on any short-term time horizon. It is possible to hypothesize the existence of portfolios that would have significant insolvency probability at a reserve level of 8%, but average provisions for these portfolios would be at 3 to 5 times the level observed in the portfolio under examination.

We propose to examine risk in a specific credit portfolio, and hypothesize differing risk levels, in order to illustrate that a uniform capital adequacy requirement does not sufficiently discriminate between credit portfolios and that portfolio-specific risk models, based in a general insurance framework, may be an improvement on present regulation.

3. DEVELOPMENT OF MODEL

A lending bank may be considered a credit insurer, with net-interest income forming a flow of premium and costs of loans defaulting forming a flow of claims. We are interested in applying risk-theoretic calculations to the above flows in order to quantify required reserves. We propose to use the standard general-insurance modeling framework, that is,

$$\text{COST} = \text{FREQUENCY} \times \text{SEVERITY}, \qquad (3.1)$$

and to investigate frequency and severity separately, applying convolution theory to quantify the distribution of the cost function and thus to specify, at appropriate insolvency probabilities, reserve requirements for specific credit portfolios.

Frequency of default is assumed to be Poisson distributed, with the underlying Poisson parameter itself having an incomplete gamma distribution. Both parameters are to be fitted using the method of moments.

Severity is assumed to be independent and identically distributed with respect to each default. Moments of the distribution of cost of one default are again calculated using the method of moments. The alternative approach of fitting a parametric distribution using maximum likelihood was attempted, but failed to produce any acceptable fit.

Using the "Best Fit"® software package, a total of 20 parametric distributions were fitted to severity data. The severity data are effectively asymptotic to both axes, with a high proportion of defaults resulting in no cost, and a small number of defaults resulting in large costs of £5 million to £15 million each. The empirical data is heavily influenced by its asymptotes, with a paucity of data in the important mid-region of the distribution. Thus maximum likelihood was rejected as failing to produce an acceptable Chi Square goodness-of-fit score for any standard parametric distribution. It may be possible to fit subsections of the distribution by using splines to produce an acceptable overall fit, but the software necessary for this approach was not available to the present researchers.

All subsequent probability measures were calculated by solving the resulting risk-theoretic equation for underlying probabilities, using a normal power approximation to produce standard normal ordinates. The relevant risk-theoretic equation is as follows:[1]

$$\mu = Y_\varepsilon P\sqrt{r_2/n + \sigma_q^2} - \lambda P + \frac{\frac{1}{6}P\left(Y_\varepsilon^2 - 1\right)\left(\frac{r_3}{n^2} + \frac{3r_2\sigma_q^2}{n} + \gamma_q\sigma_q^3\right)}{\left(\frac{r_2}{n} + \sigma_q^2\right)} \quad (3.2)$$

μ = required reserves

ε = ruin probability

Y_ε = standard normal ordinate, evaluated at ε

λ = profitability

σ_q = standard deviation of structure function

γ_q = skewness of structure function

n = number of defaults

P = expected total default costs

r_2, r_3 = risk indices, as calculated in Table 2.

4. THE GENERAL SOLVENCY MODEL

4.1 The Underlying Dataset

The dataset used comprises the corporate loan portfolio of a major United Kingdom bank. The average number of loans exposed to risk of default was approximately 59,000. Information on loan defaults was available only in crude form from January 1, 1990 to June 30, 1993, while detailed loan-specific information was available for July 1, 1993 to April 1, 1995.

Using the assumption that portfolio composition from January 1, 1990 to June 30, 1993 was not markedly different to that observed from July 1, 1993 to April 1, 1995 enabled estimated quarterly default frequencies to be calculated for the entire 5¼-year period.

The application of insurance mathematics contained in Formula 3.2 assumes a portfolio of independent risks. It has been observed by many commentators that loan performance has positive covariance, and thus that individual loans are not independent with respect to their risk of default. We assume that movements in risk intensity are 100% correlated (for example, a doubling of risk intensity with respect to the portfolio implies a doubling of risk intensity with respect to each individual loan within the portfolio).

Risk intensity itself is assumed constant only over 3-month intervals, during which loans are regarded as independent of each other with respect to their risk of default. The combination of 100% correlation with respect to movements in risk intensity and independence within periods during which risk intensity is assumed constant allows covariance to be empirically fitted. Effectively, the more risk intensity varies the higher the portfolio covariance, while the longer the period for which risk intensity is assumed constant, the lower the portfolio covariance. Observation of portfolio covariance leads directly to the assumption that risk intensity is constant over 3-month intervals but varies between 3-month intervals.

Table 1 sets out these estimates, the actual observed data, and the variance calculation for the estimation of the Poisson parameter n and the incomplete gamma variable h.

4.2 Cost Per Default

Using in the general solvency model the approach suggested by Daykin, Pentikainen, and Pesonen (1994), the first three moments of the cost per default may be directly estimated from empirical data. Underlying assumptions are that cost of default is crystallised at the instant of default, and that after indexing for inflation the cost per default remains relatively stable over time, implying a slow changing or constant mixing of underlying loans. (See Table 2.)

5. ESTIMATION OF F(X) FOR THE DATASET

5.1 Recursive Estimation of F(x)

We can now proceed to the recursive estimation of $F(x)$, the cumulative distribution of total default. This estimation procedure could involve up to 5×10^9 individual arithmetic operations, and therefore a

[1]Equation 3.2 is taken directly from Daykin et al. (1994) and represents the reserves required (L.H.S.) for an insurance collective in which the underlying risk is compound Poisson distributed, the Poisson parameter itself having the incomplete gamma distribution, and the normal power approximation having been applied. This equation is directly applied to our loan collective to produce estimates of required reserves at given insolvency probabilities, or to estimate insolvency probabilities implied by given reserves.

Table 1

Estimated Historical
and Actual Current Default Frequencies
and Implied Parameter Variance

Quarter	Frequency	Contribution to Variance Sum
1 90-13	740	497
2 90-12	850	7,691
3 90-11	970	43,139
4 90-10	1,100	114,041
1 91-9	1,210	200,435
2 91-8	1,320	311,029
3 91-7	1,440	459,277
4 91-6	1,340	333,737
1 92-5	1,240	228,192
2 92-4	1,150	150,311
3 92-3	1,050	82,771
4 92-2	840	6,037
1 93-1	640	14,957
2-93-0	430	110,423
3 93-1	353	167,526
4 93-2	278	234,546
1 94-3	277	235,516
2 94-4	245	267,599
3 94-5	152	372,466
4 94-6	219	295,175
1 95-7	164	357,963
	AVERAGE: 762.3	TOTAL: 3,993,333

Source: Empirical

Notes: a) i) ${}_n = 762.3$; ii) h derived from $\sigma_n^2 = 199667 = {}_n + \frac{n^2}{h} \Rightarrow h = 2.9$; iii) σ_q^2, y_q derived from $\sigma_q = \frac{1}{\sqrt{h}}$ $'y_q = \frac{2}{\sqrt{h}} \Rightarrow \alpha_q^2 = 0.342$ $y_q = 1.170$; Methodology from Daykin, Pentikainen and Pesonen (1994, pp. 45–54).

b) An h value of 2.9 implies wide dispersion of the underlying parameter, that is, highly variable default rates through time.

c) A y_q value of 1.17 implies that the distribution of the default rate parameter is highly positively skewed.

Table 2

Moments and Risk Indices
of the Distribution of Cost
of One Default

(1)	MEAN COST	= m =	£2.9966 × 10⁴
(2)	$\sum_{1}^{\infty} (\text{COST})^2$	$= a_2 =$	£5.59 × 10^{10}
(3)	$\sum_{0}^{\infty} (\text{COST})^3$	$= a_3 =$	2.9316 × 10^{17}
(4)	$\frac{\Sigma(\text{COST})^2}{(\text{MEAN COST})^2}$	$= \frac{(2)}{(1)^2} r_2$	63.36
(5)	$\frac{\Sigma(\text{COST})^3}{(\text{MEAN COST})^3}$	$= \frac{(3)}{(1)^3} r_3$	10920.8

Source: Empirical

Notes: a) Based on 3,887 resolved defaults of which 910 resulted in provisions being raised.

b) This is a ''risky'' distribution, high risk indices (r_2, r_3) being occasioned by several large (£5–£15 million) individual provisions.

reasonable approximation is sought. Daykin, Pentikainen, and Pesonen (1994, p. 129) suggest the approximation formula

$$F(x) \approx N\left(-\frac{3}{\gamma_x} + \sqrt{\frac{9}{\gamma_x^2} + 1 + \frac{6}{\gamma_x}\left(\frac{x - \mu_x}{\sigma_x}\right)} \right) \quad 5.1$$

which is valid where the coefficient of skewness does not exceed 1.2. Note that the coefficient of skewness γ_q at 1.17 is close to the upper feasible limit for this approximation, so that relative error may be large. However, note also that small proportions of the total function in reserve estimation are being dealt with, so that even though relative error may be large, absolute error in relation to size of reserves or required loan pricing is likely to remain small.

This approximation formula (5.1) allows one to specify the upper tail of the distribution function $F(x)$ as a transformed quadratic standard normal distribution and directly produces Equation (3.2). The following assumptions underly the given results:

- Insolvency probability is set at two different levels, one in approximately 40 ($y_\varepsilon = 2.0$) and one in 800 [$(y_\varepsilon) = 3.09$, allowing for a relative error of 25% in our approximation of $F(x)$].
- Interest earnings are ignored. Alternatively, an equivalent assumption would be that a dividend equal to the risk-free rate of interest earned on reserves is paid each year.
- The portfolio earns returns of 0.77% of loans outstanding (historical experience).
- Administrative expenses total 0.50% of loans outstanding.

Table 3 illustrates required reserves as a percentage of loans outstanding for given ruin probabilities.

In terms of interpretation, the portfolio requires excess reserves of 0.86% of loans outstanding in order to avoid regulatory constraints on its asset choice at one-in-40 probability on a one-year view. At one-in-800 probability it would require excess reserves of 2.22%, again on a one-year view. This would imply a total reserve of 8.86% or 10.22%, respectively.

On a five-year view, excess reserves of 4.10% of loans outstanding (1 in 40) and 10.78% (1 in 800), respectively, would be required to avoid regulatory constraints. This would imply a total reserve of 12.10% or

Table 3
Required Reserves and Ruin Probabilities for Dataset / Portfolio Reserves Percentage Loans Outstanding

Y_ε	Time Period	
	1 Year	5 Years
2.0	0.86	4.10
3.09	2.22	10.78
2.75	1.68	8.00
6.63	8.00	N.R.*

Source: Empirical
Notes: a) Ruin is defined as loss of more than stated reserves.
b) Required reserves increase with time.

18.78%, respectively. The insight provided by the five-year view is that regulators may expect banks that become asset-constrained to remain so, and wish to provide them with resources sufficient for a medium-term workout financed by asset recomposition.

In this context, the minimum capital adequacy requirement of 8% would imply $Y_\varepsilon = 2.75$ corresponding to approximately 3.0×10^{-3} ruin probability (on the artificial assumption of no asset recomposition). Asset recomposition would commence when the ratio dips below 8% and would continue until regulatory capital is exhausted, the bank at that stage possessing an asset portfolio comprising only government securities.

In practice, observation of current capital adequacy ratios of UK clearing banks would imply adherence to the first interpretation above, so that sufficient asset excess is held by these banks to ensure that regulatory constraints on asset choice have less than a 1% probability of occurrence. Quite small excess reserves thus ensure unconstrained asset choice for these banks, at the margin and in the short term.

5.2 Comparative Static Analysis

Given the uniform 8% minimum capital adequacy requirement, it is interesting to explore the variability of mathematical reserves in response to changing provisions, loan margins, and size. We hypothesize 20% greater and lesser provisions, 20% higher and lower average loan prices, and a bank 10% the size of this portfolio and examine the effect of these on reserves.

This is equivalent to assuming that there exist within the uniform regulatory environment banks which are 20% more or less risky (See Table 1), 20% more or less expensive in terms of loan pricing, and 90% smaller, and examining the effect of these on reserves.

Table 4 shows reserves required as a percentage of loans outstanding for specified ruin probabilities.

Clearly, mathematical reserves are influenced by both provisions and loan pricing, with provisions having a nonlinear impact on reserves and loan pricing a linear impact. Profitability appears as a direct reduction in required reserves. A small size effect is also apparent.

Table 5 illustrates the Y_ε levels associated with Table 4 banks, in this case assuming a uniform capital adequacy requirement of 8%.

Evaluating the unit normal variate in the five-year case produces insolvency ''probabilities'' of 1.13%, 0.169%, 0.298%, 0.071%, 1.02%, and 0.321%, respectively. These insolvency ''probabilities'' differ substantially and indicate the potential difficulties associated with uniform capital adequacy requirements.

6. TOWARD A VAR SYSTEM

The preceding calculations (simulations) have been on an artificial basis, since they do not take into account the risk-reduction effects of asset recomposition. Asset-recomposition risk may be defined as the risk of a bank's asset choice becoming constrained by capital adequacy. Under present regulation this is a real risk for safer banks, whose true mathematical capital requirement is likely to be lower than the 8% minimum [Banks (b) and (d), Table 4]. Bank (b) in particular may have decided to make low-risk loans, and may have accepted returns on those loans commensurate with the lower risk on its portfolio. It could thus find itself unable to remunerate its regulatory minimum capital at a competitive rate. Of course, if

Table 4
Reserves Required:
Unchanged Policies at Given Y_ε

Y_ε	(a)	(b)	(c)	(d)	(e)	(f)
BANK						
1 year time period. Reserves % loans outstanding						
2.0	1.39	0.33	0.86	0.36	0.30	1.34
3.09	3.03	1.44	2.22	1.72	2.73	3.08
5 year time period. Reserves % loans outstanding						
2.0	6.74	1.78	4.10	1.88	6.32	4.36
3.09	14.7	6.83	10.78	8.56	13.00	11.16

Source: Empirical
Notes: Columns (a) and (b) represent 20% higher and lower provisions respectively; column (c) gives the actual observed portfolio; columns (d) and (e) represent 20% higher and lower loan prices respectively; while column (f) gives 10% of portfolio size, permitting greater random variation.

Table 5

Y_ε Values Associated With 8% Uniform Capital Adequacy

Time Period	BANK					
	(a)	(b)	(c)	(d)	(e)	(f)
1 Year	5.84	7.67	6.63	6.87	6.39	5.53
5 Years	2.28	3.42	2.75	3.19	2.32	2.68

Source: Empirical

information asymmetries did not exist, its shareholders would accept lower and safer expected returns. However, if profit is the only observable output, investors may not recognize Bank (b)'s relatively safer strategy. Banks (a) and (e) might also find it difficult to remunerate their regulatory capital, but for entirely understandable reasons. Bank (a) is relatively risky, while Bank (e) is relatively less profitable.

What is required is a system capable of discriminating between Banks (a) through (f) with respect to the specific nature of their credit portfolios. Required reserves should be a function of a portfolio-specific VaR calculation, sensitive to provisions and profitability.

6.1 Investor Choice and CAPM Inputs

The existing regulatory system does not recognize any difference between Banks (a) through (f). Table 6 sets out the capital asset pricing model (CAPM) inputs generated by each of these banks and presents the opportunity set available to investors.

If CAPM can so easily sort these hypothetical banks by value, why can't the regulator? The regulator may have a different opportunity set from the investor, but should still be capable of ranking these banks by relative insolvency risk.

Table 6

Set of Available Investments

Bank	Percentage Expected Return on Equity	Standard Deviation of Equity Investment Percentage
(a)	12.0	16.25
(b)	19.0	10.9
(c)	15.5	13.6
(d)	23.8	13.6
(e)	7.2	13.6
(f)	15.5	17.3

Source: Empirical

Notes: The assumptions underlying the above entries are as follows: a) Each bank has capital adequacy of 10% (6% equity, 4% debt); b) debt yields 5% in excess of risk free rate; c) expected excess return = average of last 5 years excess return; d) Risk free rate = 6% per annum; e) Standard deviation calculated as {0.5 (1 year reserves (Y_ε = 2.0) – Profit)}/0.06. Thus CAPM would value these banks from a shareholder viewpoint as follows: (d) ≥ (b) > (c) > (f) > (a) > (e).

6.2 A Specific VaR Computation

We require a VaR formula that is directly proportionate to risk, responds to movements in risk, has a standard calculation framework, calculates for the ''whole book,'' and represents a justifiable improvement on present practice.

The formula is required to incorporate provisioning and profitability measures. Taxation and dividends both reduce the extent to which reserves may be financed out of internal resources. Taxation effects are included in the computation, and a generous dividend constraint is imposed. To ease comparison, and also to assuage regulators, the formula is constrained to produce an identical capital adequacy requirement to that operative at present in the case of the original dataset.

The proposed computation formula is as below:

$$\text{VaR Capital \%} = \left[2 \times \sum_{1}^{5} \text{Provisions \%} - (1 - t)\sum_{1}^{5} \text{Margins \%}\right] \times \text{Loans Currently Outstanding.} \qquad (6.1)$$

Subject to a minimum value of 2% (to avoid infinitesimal or negative values) where:

- VaR CAPITAL % is the computed value at risk capital requirement, expressed as a percentage of loans currently outstanding,
- $\sum_{1}^{5}$ provisions % is simply the sum of the past five years' provisions expressed as a percentage of loans then outstanding,
- $\sum_{1}^{5}$ margins % is the sum of the past five years' margins, expressed as a percentage of loans then outstanding and,
- t is the operative corporate profit taxation rate, currently assumed to equal 38%.

The margin calculation would take the form

$$\text{Margin \%} = \frac{\text{Net Interest Income} - \text{Administration Expenses} - \text{Change in Provisions}}{\text{Average Total Amount of Loans Outstanding}}. \qquad (6.2)$$

The provision calculation would take the form[2]

$$\text{Provisions \%} = \frac{\text{Change in Provisions}}{\text{Average Total Amount of Loans Outstanding}} \qquad (6.3)$$

The rationale behind Formula (6.1) is as follows:

- The formula should be simple to understand and to apply
- Riskier loans require greater provisions
- Profitable lending should require less capital backing than unprofitable lending
- The multiplier of 2 × provisions % reflects the relatively greater weight of provisioning in risk calculations as opposed to profitability
- The starting point for VaR capital in the case of the empirical dataset has been constrained to equal present regulatory capital, that is, 8% of loans outstanding.

This computation produces results as outlined in Table 7.

The effect of the VaR computation has been to substantially reduce the observed variation in the probability measure. Variation in this measure has been reduced by 98% from its pre-VaR spread (Table 5).

In declaring dividend payments, if banks were limited in their declarations by directly attributable streams of income as follows:

$$0.005 \times (1 - t)\left[\sum_{1}^{5} \frac{\text{margins \%}}{\text{VaR capital \%}} + \sum_{1}^{5} \frac{\text{average interbank rate \% per annum}}{100}\right] \times \text{Loans Currently Outstanding}, \qquad (6.4)$$

this would have the effect of reducing the observed variation in the probability measure by constraining loss-making banks, while allowing profitable banks to distribute a high proportion of their net earnings.

The reasoning behind this dividend formula is that reserves earn both margin as computed plus an opportunity cost equal to the interbank rate. At least theoretically, some small proportion of reporting banks could distribute more than their annualized net return, thus reducing their capital base by excess distribution. This would apply only in the case of banks with little or no annual provisioning requirement.

The multiplier of .005 was deduced through simulation exercises examining the effect of greater and lesser dividend constraints. The former was observed to produce an unacceptably high number of insolvencies, while the latter produced too few insolvencies to allow comparison between the proposed VaR capital and current regulatory requirements. The actual dividend constraint remains open to choice; the specific multiplier used here is chosen only to facilitate comparison.

Table 8 illustrates the probability measures resulting from this dividend restriction.

The probability measure has increased, because permissible dividends are, on average, larger than those assumed in the earlier computations. However, the variation in this measure has reduced by a further 65%.

One is now very close to a ''level playing field'' in terms of probability measures, the variation in these measures being only 0.7% of its level as calculated using conventional capital adequacy. Thus a VaR capital-adequacy fixed premium deposit insurance scheme seems feasible. Whether this proposed approach actually represents a viable improvement on present practice awaits a full simulation approach and comparative evaluation ''tests,'' developed in Section 7.

The simulation of a number of differing portfolios is introduced in Section 7, with identical operating conditions under both existing regulation and the VaR methodology proposed.

[2]Both these formulas relate to the year in question.

Table 7

VaR Capital and 5-Year Probability Measures

Bank	(a)	(b)	(c)	(d)	(e)	(f)
VaR Reserves %	10.82	5.28	8.00	6.45	9.55	8.00
Insolvency Y_ε	2.75	2.91	2.75	2.92	2.63	2.68
Probability Measure %	0.298	0.181	0.298	0.175	0.427	0.368

Source: Empirical
Note: The specific provisions and margins underlying these computations remain confidential to the author and the bank concerned.

Table 8
5-Year Probability Measures for VaR Computation with Restricted Dividends

Bank	Insolvency Y_ε	Probability Measure Percentage
a	2.45	0.71
b	2.53	0.59
c	2.42	0.78
d	2.50	0.62
e	2.41	0.80
f	2.36	0.91

Source: Empirical

7. DISCUSSION OF SIMULATION MODELS AND STATEMENT OF COMMON FEATURES

The volatility of underlying loan losses requires any simulation to prossess significant feedback in order to constrain outcomes into a reasonable stochastic bundle. One significant area of difference in the credit process, compared with simulation models in insurance, is the relatively simple nature of investments undertaken by banks, so that reduced variability of investment returns may partly compensate for the volatility of default losses.

Daykin and Hey's (1989) paper is taken as the starting point due to its expositional simplicity. Modifications are required and are discussed while proceeding. Viewing the credit portfolio in isolation from the bank's other activities, we have:

Inflow: Loan Margin Income
Short Term Interest Income on Reserves

Outflow: Loan Provisions
Administrative Expenses
Taxation
Dividends

Thus a simple model, with only primary variable input, would be

$$A(t) = A(t-1) + R(t)(A(t-1)) + L(t) - P(t) - E(t) - T(t) - D(t), \qquad (7.1)$$

where:
$A(t)$ is the amount of assets at end year t
$R(t)$ is the average short-term interest rate in year t
$L(t)$ is the loan margin income in year t
$P(t)$ is the provision requirement in year t
$E(t)$ is the administration expense in year t
$T(t)$ is the taxation charge in respect of year $t-1$
$D(t)$ is the dividend payable in respect of year $t-1$.

Reserves are assumed to be held in short-term deposits, and taxation and dividends are assumed paid after a one-year lag. Loan provisions are a slow-moving average, with an average two-year run off. Thus, on average, current loan provisions are assumed to relate to loans made two years ago. In an inflationary environment this can make a significant difference to the simulation. Most of the flow variables would be expected to bear some relationship to inflation over time, so that a model for future inflation is necessary. Daykin and Hey's model is

$$LN(1+i(t)) - U_q = \alpha_q[LN(1+i(t-1)) - U_q] + \sigma_q Z_q(t), \qquad (7.2)$$

where
$i(t)$ is the rate of inflation
U_q is the average inflation rate
α_q is the serial correlation of last year's inflation rate with this year's
σ_q is the amplitude of random ''white noise''
$Z_q(t)$ is a sequence of independent identically distributed unit normal variables.

Note: This corrects Daykin and Hey's (1989) misstatement of the underlying equation.

Values suggested by Wilkie for the above on the basis of 60 years data are $U_q = .05$, $\alpha_q = 0.6$, and $\sigma_q = .05$. These have been criticized by Daykin and Hey as producing too many negative inflation observations. For more recent experience they suggest $U_q = .07$, $\alpha_q = 0.6$, and $\sigma_q = .03$. The author suggests (with five low-inflation years added to the experience!): $U_q = .05$, $\alpha_q = 0.6$, and $\sigma_q = .03$.

The average short-term interest rate in any year is assumed to be normally distributed around a mean 2% above current inflation, with standard deviation of 1.5%, making negative interest rates possible but extremely unlikely.

Loan-margin income growth is assumed to be normally distributed around a mean of current inflation plus 2.5% with standard deviation of 5% per annum. Administration expenses are assumed to be normally distributed around inflation with standard deviation of 2% per annum.

Provisions are assumed to occur as a normally distributed process around a mean value of 1% of advances outstanding two years previously with standard deviation 0.5%.

It is assumed that negative taxation is possible and that taxation is at annual rate of 38%. The negative taxation possibility assumes that the overall bank does not operate at a loss over prolonged periods, and thus either tax can immediately be reclaimed by offset against other currently profitable areas, or tax losses can be offset against taxable income, after a short time lag. Taxation may be expressed as

$$T(t) = .38[L(t-1) - P(t-1) - E(t-1) + R(t-1)A(t-2)] \quad (7.3)$$

assuming interest income on reserves subject to tax.

Dividends are assumed payable in arrears, based on (6.4). The basic simulation equation thus becomes

$$A(t) = A(t-1) + R(t)A(t-1) + L(t) - P(t) - D(t-1) - E(t) - 0.38[L(t-1) - P(t-1) - E(t-1) + R(t-1)A(t-2)]. \quad (7.4)$$

7.1 Differences in Model Operation

In the previous section, the common features of the proposed simulation model were set out. They differ from current and proposed VaR regulation only with respect to starting reserve values and definition of insolvency. In other words, exactly identical operating conditions are assumed to apply, regardless of the regulatory framework.

In the case of the existing regulatory framework (Model 1), the starting points for relevant values are as given below:

Reserves:	11% of loan volumes outstanding
Loan margins:	Bank dependent
Provisions:	Bank dependent
Expenses:	0.5% of loan volumes outstanding
Short-term interest rate:	6.0%
Dividend:	Bank dependent

Insolvency in this case is defined as reserves at any point in time falling below 8% of loan volumes outstanding.

In the case of the proposed VaR regulatory framework (Model 2), the starting points for relevant values are again as given below:

Reserves:	$1.375 \times \left[2 \sum_1^5 \text{Provisions \%} - (1-t) \sum_1^5 \text{Margins \%}\right] \times$ Loans currently outstanding
Loan margins:	Bank dependent
Provisions:	Bank dependent
Expenses:	0.5% of loan volumes outstanding
Short-term interest rate:	6.0%
Dividend:	Bank dependent

Insolvency in this case is defined as reserves at any point in time falling below

$$\left[2 \sum_1^5 \text{Provisions \%} - (1-t) \sum_1^5 \text{Margins \%}\right] \times \text{Loans currently outstanding.} \quad (7.5)$$

We now proceed to set out the simulation results for the chosen range of banks under both models.

7.2 Presentation and Discussion of Results

A total of 2500 simulations were run, comprising 500 for each of Banks (a) through (e) (see Table 9). Bank (f) was not included because random variation caused by small size does not lend itself to simulation. Overall, substantially fewer insolvencies were observed to occur in total (122 under the VaR approach as opposed to 323 under the present 8% regulation). The overall 20-year observed insolvency rate was 4.9% for the VaR methodology, versus 12.9% for present regulation. There is a bias present in the simulation that may involve banks paying dividends under the simulation payment formula, which causes them to become insolvent under existing regulation but not under the VaR approach. This bias is estimated at 40% from consideration of the outturn for the Bank (c) simulation, in which identical starting capital and operating conditions produce correspondingly greater percentage insolvencies. Even allowing for this bias, it appears that the VaR approach produces fewer insolvencies overall then present regulation.

In the earlier years, however, the VaR approach produces higher numbers of insolvencies compared with present regulation. This may be explained by the observation that banks which are under financial strain, either because of high provisions, low margins, or a

Table 9

Presentation of Results from Simulation of the Credit Process

Bank	Simulation	Model	Number of Insolvencies Observed After:				Percentage of Insolvencies Observed After:			
			5 yrs	10 yrs	15 yrs	20 yrs	5 yrs	10 yrs	15 yrs	20 yrs
Overall	2500	1	—	10	100	323	—	0.4	4.0	12.9
		2	1	31	86	122	—	1.2	3.4	4.9
(a)	500	1	—	2	17	68	—	0.4	3.4	13.6
		2	—	5	8	14	—	1.0	1.6	2.8
(b)	500	1	—	4	50	119	—	0.8	10.0	23.8
		2	—	7	33	47	—	1.4	6.6	9.4
(c)	500	1	—	1	8	28	—	0.2	1.6	5.6
		2	—	4	13	20	—	0.8	2.6	4.0
(d)	500	1	—	1	7	28	—	0.2	1.4	4.6
		2	1	11	21	28	0.2	2.2	4.1	4.6
(e)	500	1	—	2	18	85	—	0.4	3.6	17.0
		2	—	4	11	13	—	0.8	2.2	2.6

Source: Empirical

Note: Bank (a) refers to 20% higher provisions, Bank (b) to 20% lower provisions, Bank (c) to the original dataset, and Bank (d) to 20% higher margins, while Bank (e) refers to 20% lower margins.

combination of both, are likely to find themselves facing a VaR solvency requirement greater than 8% of loans outstanding as is the case under present regulation. Thus, the need for corrective action may be signaled sooner under the VaR methodology than under present regulation. However, a rising capital requirement as credit conditions worsen may encourage pro-cyclical behavior by regulated banks, causing lending to contract as provisions rise and vice versa. The extent to which this might occur would be a function of the number of operationally constrained banks within a regulatory system at any point in time and could only be quantified by a regulator.

Examining the individual bank simulations in turn results in the following:

1. Bank (a) (20% higher provisioning rates than standard)

   Under the VaR (Model 2) approach, Bank (a) is equipped with higher initial capital than under the Model 1 approach. Notwithstanding this higher capital, the VaR requirement is more onerous than 8%, producing 5 as opposed to 2 insolvencies after 10 years. Subsequently, lack of profitability causes more Model 1 banks to fail as time progresses, so that after 20 years 13.6% of Model 1 simulations are observed as insolvencies versus 2.8% of Model 2 simulations.

2. Bank (b) (20% lower provisioning rates than standard)

   Intuitively, one would expect fewer failures than the number observed, because under both models, Bank (b) is more profitable than Bank (a). However, this increased profitability permits the payment of substantially higher dividends than under the previous scenario. High dividend payments associated with a VaR requirement lower than 8% precipitates a large number of Model 1 insolvencies, while the overdistribution of excess profits earned also increases Model 2 insolvencies.

   It should be pointed out that the dividend formula represented in 6.4 is intended to represent a maximum distribution, rather than an automatic entitlement. Prudent management in Bank (b) would restrict dividend payouts when solvency is under threat. A minor adjustment to the maximum permissible dividend could sharply reduce observed insolvencies for Bank (b), but if applied generally would see no insolvencies in other bank types. As the purpose of these simulations is to observe and compare insolvency experience in all bank types, this result has been allowed to stand, even though it is most unlikely to occur in practice.

3. Bank (c) (Parameters as in original dataset)

   Here we see the evidence of the bias referred to in the earlier discussion. Insolvency rates are some 40% higher for Model 1 than for Model 2, notwithstanding identical starting capital and operating assumptions. This is caused by the relationship of dividends to VaR capital, and again may lead Model 1 banks to overdistribute inadvertently. Once again, changing the distribution rules by model type invalidates the observation of identical sample paths so that this bias cannot be eliminated.

4. Bank (d) (20% higher margins than standard)

   As excess earnings can be paid away as dividends, approximately the same number of insolvencies as under the previous scenario is evident. This is the only scenario in which Model 2 does not outperform Model 1. The explanation for this is that lower initial capital under Model 2 is not recouped from retained earnings, because these earnings are fully distributed. Starting with capital of 9.0% of loans outstanding, versus 11.0% for Model 1, Model 2 banks remain at higher risk of insolvency throughout. However, total insolvencies in banks of type (d) are of lower-than-average proportion, and dividend policy is available to reduce insolvency risk.

5. Bank (e) (20% lower margins than standard)

   This shows the widest discrepancy in observed insolvencies of any bank type (85 versus 13). Once again, lack of profitability is to blame, compensated in the case of Model 2 by higher initial capital.

7.3 Interpretation of Findings

The VaR methodology suggested in this paper has been observed to produce fewer overall insolvencies than present regulation on a 20-year time horizon. It may be argued, thus, that it represents an improvement on present regulation. If, however, the simulation were to have run for only 10 years, the VaR methodology would have produced three times as many insolvencies as present regulation, and the opposite argument might hold sway. The argument in favor of VaR contains more subtleties than simply when the simulation clock stops; after all, regulation does not affect banking operations in this specific model, so we cannot simply claim victory for VaR after 20 years or defeat after 10 years.

Viewed overall and in a practical setting, the VaR methodology does appear to identify weak banks earlier than does present regulation. The differential initial capital associated with VaR permits a larger number of banks to survive the simulation process. From a regulatory standpoint, having fewer weak banks and identifying them earlier than otherwise has two main advantages: the ability to concentrate remedial regulatory effort on fewer banks at any point in time, and the knowledge that relatively high residual capital will be available within banks which require regulatory attention due to high provisions, lack of profits, or both. The main disadvantages are the possibility of procyclical action by operationally constrained banks and the over-optimistic reporting of provisions and earnings by the industry, which reduces its calculated VaR capital.

The former disadvantage has been discussed. The latter has been addressed by the requirement that provisions and margins are calculated over five years. The scope for management of reported earnings (Rajan, 1994) is substantially reduced the longer the timescale over which such management is required. The choice of five years is not accidental; it corresponds to the peak-to-trough duration of the banking cycle.

From the individual bank viewpoint, Banks (a) through (e) have much less variable insolvency probability under VaR than under current regulation (Table 9). This may mean that a level premium-deposit insurance scheme becomes more feasible. Further, VaR allows banks to specialize without requiring substantial regulatory capital that cannot be compensated without increasing risk if such specialization is at the lower end of the risk-return spectrum of private sector banking. Banks would be free to price loans in the knowledge of a closer correspondence between economic and regulatory capital than exists at present. Efficiencies might be generated, and informational asymmetries reduced.

Banks are not passive victims in the regulatory process. If the advantages of VaR outweigh its disadvantages in terms of computation, reporting, and reduction of scope for adverse selection, then banks can canvass the regulators for such regulation. The regulator similarly cannot impose regulation by fiat. Thus, VaR is likely to emerge only if both the banking industry and its regulators are in favor. This section has outlined the advantages and disadvantages from the perspective of each. These advantages and disadvantages are summarized below:

Advantages

Regulator
- Problem banks are fewer and identified earlier
- Concentration of remedial effort
- Higher asset excess in problem banks than under current regulations.

Regulated Banks
- Reduction in adverse selection
- No capital ''dead-weight''
- Ability to specialize
- Direct link to economic capital.

Disadvantages

Regulator
- Greater sophistication required
- Risk of pro-cycle behavior by regulated banks
- Resulting possible increase in systemic risk.

Regulated Banks

- Expense of configuring systems
- Greater exposure to ''shocks,'' that is, sudden changes in VaR adequacy.

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