GLOBAL OPTIMIZATION OF SIGNAL SETTINGS SUBJECT TO EITHER SYSTEM OPTIMAL OR USER EQUILIBRIUM TRAFFIC ASSIGNMENT

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1 BACKGROUND

In the last three decades various researchers had been facing the problem of maximizing global network performances by optimizing traffic signals. The main difficulty of the problem arises from the existing interaction between network performance and users' route choice. In fact, drivers make their travel choices in order to minimize their own travel time. Simultaneous choices performed by different drivers lead to the so-called descriptive User Equilibrium (UE). As traffic controller sets signals on the basis of the observed traffic patterns, its action changes the network performance and so stimulates drivers to adjust their route choice in order to find their new minimal paths. A wide review of the scientific literature on this matter can be found in Cipriani and Gori (2000) or in Tale and Van Zuylen (2000). In the following, we deal with only some fundamentals steps, which are necessary to introduce the object of the paper.

Allsop (1974) and Gartner (1974) proposed independently each other an iterative approach that integrates signal settings and traffic assignment, in order to simulate the actual process where signal control and route choice are adjusted reciprocally. Smith (1979) pointed out that it is not guaranteed that such an iterative method converges even to a local optimum. Indeed, it may also lead to increase the total travel time. Moreover, it has been observed that also the actual process of adjusting signal settings depending on current traffic conditions, as performed by flow-responsive signals, may worsen the total travel time on the network even by 30% (Smith, 1980). Thus, Smith and Van Vuren (1993) proposed a new signal policy (called Po) that ensures consistency with user equilibrium. Cantarella and Sforza (1995) showed that flow responsive policies that set signals independently each other are sub-optimal with respect to global policies where all signals are set to minimize one objective function describing the global network performance, according to user equilibrium conditions. The former approach is referred here as Local Optimization Signal Settings and User Equilibrium traffic assignment Problem (LOSS-UE Problem), the latter as Global Optimization Signal Settings and User Equilibrium traffic assignment Problem (GOSS-UE Problem).

Several formulations and different solution procedures have been proposed for each of the two approaches. Readers can refer, among others, to Cipriani and Gori (2000) for LOSS-UE and Cipriani and Fusco (2002) for GOSS-UE problem.

It is well known that both problems can have multiple solutions, and there is no method to ensure *a priori* neither reaching the global optimum nor avoiding a poor solution. Hence, the main interest of researchers has been focused on developing effective heuristic procedures to improve the quality of the solution found. In a former work, we investigated mathematical properties of the problem by performing a systematic investigation of the objective function of GOSS-UE problem (Cipriani and Fusco, 2000). This allowed us to compare different solving methods and locate the solutions of different algorithms (as well as by the same algorithm applied from different starting points) on cross-sections of the objective function. Results obtained on a test network showed that in most cases a local search algorithm succeeded in finding a solution belonging to a large quasi-convex region of the objective function. It is clear that it is a good solution, but it is not known how good is it (except verify the reduction of total travel time with respect to the current network).

However, the best network performances are achieved by System Optimum (SO) traffic assignment, when also drivers’ route choices are set in order to minimize the total travel time on the network. In general, system optimal traffic patterns are not consistent to user equilibrium. In this way, they are an unfeasible solution of the real problem, where some user can improve his or her own travel time by autonomously changing his or her route.

Thus, the fundamental question is whether the signal settings optimized according to GOSS-UE problem can lead user equilibrium toward the system optimum traffic flow configuration.

Along these lines, the solution of SO problem can be seen as a ‘benchmark’ for the usual GOSS-UE problem.

2 OBJECT OF THE PAPER

In this paper we introduce a suitable mathematical formulation of the Global Optimization of Signal Settings and System Optimal traffic assignment Problem (GOSS-SO problem). The problem seeks to determine both the signal settings and the assignment matrix that optimize the global network performance. It is formulated as a bi-level problem, where the total cost of the network is optimized with respect to green splits and traffic flows are determined by solving the SO assignment problem. The solution procedure applies the Projected Gradient Algorithm with respect to green splits (Sheffi and Powell, 1983) where

SO condition is imposed to traffic flows. Analytical expressions of marginal link costs have been obtained by deriving the well-known 2-term linearized Webster delay formula for signalized junctions. Thus, SO problem can be simply solved by applying standard Frank-Wolfe algorithm with marginal link costs. At each step of the gradient algorithm, a SO assignment problem is solved and the corresponding flow pattern on the network is found. Thus, the total cost of the network is computed by multiplying the flow on each link by the corresponding value of the average link cost function. It is easy to show that the problem admits multiple solutions, as marginal link costs are functions of the green splits, which depend on the traffic flow pattern; consequently, their Jacobian matrix is asymmetrical. Thus, comparing GOSS-UE and GOSS-SO problems can provide useful indications about the effectiveness of both solving procedures and, more important, it is addressed to verify if GOSS-UE is an effective method to make feasible (or even to approximate) GOSS-SO configuration.

3 PRELIMINARY RESULTS

A preliminary analysis conducted on a 4-centroid 9-node test network has provided very encouraging results. They are briefly summarized in the figure below. The two big dots are solutions of GOSS-UE that have been obtained by starting the projected gradient algorithm from two different initial points. The two curves are sections of the objective functions of both GOSS-UE and GOSS-SO, which have been obtained by varying the green split vector linearly along the segment connecting the two solutions. For any green split configuration, the corresponding values of the two objective functions have been computed by assigning the OD matrix to the network following either UE or SO criterion, respectively.

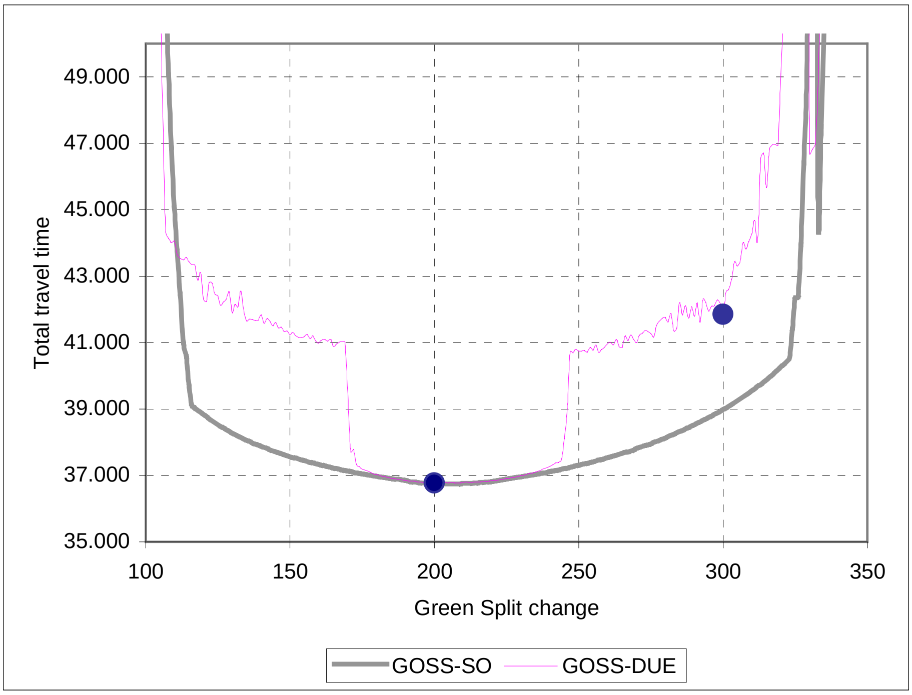

Figure1 . Scanning of the objective functions of GOSS-UE and GOSS-SO between two solutions of GOSS-UE.

The following remarks can be drawn from the figure.

- The better solution of GOSS-UE is coincident to the solution of GOSS-SO problem.
- There is an interval of the green split vector that makes UE coincident to SO, while out of it drivers can find routes that are more convenient from their own point of view, but lead to increase the total travel time on network.
- In this optimal region, the objective functions of both problems are convex and the Projected Gradient Algorithm can easily find their minimum.
- The objective function of GOSS-SO problem can be considered globally convex in the whole interval examined (small irregularities depicted for higher values of the total travel time are due to very slow convergence rate of Frank-Wolfe algorithm when using 2-term linearized Webster delay functions).

It is expected that more general indications about mathematical properties of the problems and about the actual capability of GOSS-UE to approximate GOSS-SO can be obtained in the full paper, where a systematic analysis of both problems will be conducted to a real-size network.

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