On the use of contraction theory for the design of nonlinear observers for ocean vehicles

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Abstract

Guaranteeing that traditional concepts of stability like uniform global exponential or asymptotic stability (UGES or UGAS) are verified when using design tools based on new concepts of stability may be of significant importance. It is especially so when attempting to bridge the gap between theory and practice. This paper addresses the question of the applicability of contraction theory to the design of UGES observers for ocean vehicles. A relation between the concept of exponential convergence of a contracting system and uniform global exponential stability (UGES) is first given. Then two contraction-based GES observers, respectively for unmanned underwater vehicles (UUV) and a class of ships, are constructed, and simulation results are provided.

1 Introduction

Contraction theory is a recent tool enabling to study the stability of nonlinear systems trajectories with respect to one another, and therefore belongs to the class of incremental stability methods (see (Lohmiller and Slotine, 1998; Lohmiller and Slotine, 2000) and references therein). Among the articles in which contraction analysis is quoted (Angeli, 2002; Fossen and Blanke, 2000; Johansen, *et al.*, 2000; Sontag, 2000), questions seem to arise about the applicability of the theory and its relations with more traditional tools.

In particular, one may wonder about the relations between the differential convergence that a contracting system exhibits and more classical concepts of stability (as for example asymptotic stability, global exponential stability and their time-varying counterparts (Khalil, 1996)). Indeed, in (Fossen and Blanke, 2000) where contraction theory was mentioned, it was also said that the focus of their study is "Lyapunov-based output feedback control and global exponential stability properties which are important from a robust point of view. This gives a different observer structure than the one obtained from contraction theory".

Following this interesting remark, this article addresses the question of the relation between contraction and exponential stability, and then shows that it can be used as an alternative to Lyapunov-based control design as it can lead to exactly the same control structures. This idea is illustrated by demonstrating that the Fossen's observer for UUV is contracting whereas its equilibrium point is GES, and that by using contraction combination properties on the design of an observer for ships, robustness issues can also be addressed.

After briefly reviewing the concepts of contraction theory that will be used throughout this paper, the relation between a contraction system and UGES is examined in Section 2. In section 3, this relation is then used to prove that the observer in (Fossen and Blanke, 2000) is UGES and contracting. Finally, for the sake of comparing with other methods, a complete procedure leading to a robust UGES observer for ships is given in section 4, and simulation results are provided. Brief concluding remarks end the paper.

2 Theoretical preliminaries

2.1 A few contraction theory principles

The problem considered in contraction theory is to analyze the behavior of a system, possibly subject to control, for which a nonlinear model is known of the following form

$$\dot{x} = f(x, t) \qquad (1)$$

where $x \in \mathbb{R}^n$ stands for the state whereas f is a nonlinear function. By this equation, it can be noticed that the control may easily be expressed implicitly for it is merely a function of state and time. Contracting behavior is determined upon the exact differential relation

$$\delta \dot{x} = \frac{\partial f}{\partial x}(x,t)\delta x \qquad (2)$$

where δx is a virtual displacement, *i.e.* an infinitesimal displacement at fixed time.
For the sake of clarity, the main definition and theorem of contraction taken from (Lohmiller and Slotine, 1998) are reproduced hereafter.

Definition 2.1 *A region of the state space is called a contraction region with respect to a uniformly positive definite metric $M(x,t) = \Theta^T(x,t)\Theta(x,t)$ where Θ stands for a differential coordinate transformation matrix, if equivalently $F = (\dot{\Theta} + \Theta\frac{\partial f}{\partial x})\Theta^{-1}$ or $\frac{\partial f}{\partial x}^T M + \dot{M} + M\frac{\partial f}{\partial x}$ are uniformly negative definite.*

which leads to the following convergence result:

Theorem 2.1 *Any trajectory, which starts in a ball of constant radius with respect to the metric $M(x,t)$, centered at a given trajectory and contained at all times in a contraction region, remains in that ball and converges exponentially to this trajectory.*

In the following, we will only consider *global* convergence, *i.e.* the contraction region corresponds to the whole state space.

The next combination property will also be useful in section 4.

When two systems, contracting under possibly different metrics, are connected in feedback form as

$$\frac{d}{dt}\begin{pmatrix} \delta z_1 \\ \delta z_2 \end{pmatrix} = \begin{pmatrix} F_1 & G \\ -G^T & F_2 \end{pmatrix}\begin{pmatrix} \delta z_1 \\ \delta z_2 \end{pmatrix} \qquad (3)$$

where $\delta z_i = \Theta_i \delta x_i$, then the resulting global system is in turn contracting. The above statement can of course be related to the well known passivity concepts and the feedback interconnection of passive systems, which is passive as well. This result also holds with a pre- and post-multiplication by a matrix as seen above (see (Sepulchre, *et al.*, 1997, p. 34)).

2.2 Links with UGES

One of the main features of contraction theory is the use of virtual and therefore infinitesimal displacements when describing the stability behavior of the dynamic flow. Let us now see how this is linked to finite and relative displacements.

Theorem 2.2 *If the system (1) is globally contracting with respect to a constant Θ, then it is also UGES, i.e. $\exists a, b > 0$ such that $\forall t \geq 0, \forall x_{r0}, x_{p0} \in \mathbb{R}^n$,*

$$\|x_r - x_p\| \leq a\|x_{r0} - x_{p0}\|e^{-bt}$$

where $x_r = x_r(x_{r0}, t)$ (resp. $x_p = x_p(x_{p0}, t)$) and $\| \bullet \|$ denotes the Euclidean norm.

Several proofs may easily be found, in particular by integrating the virtual displacements δx (hints may be found in (Lohmiller and Slotine, 1997)), but it is of interest to relate contraction theory to other incremental approaches like the one developed by Fromion and coworkers ((Fromion, *et al.*, 1996; Fromion, *et al.*, 1999; Fromion, 2000)). The two advantages in doing this are first to be able to transfer the design methods from one approach to the other, and second to use other incremental paradigms as "shortcuts" to traditional forms of stability when the search for such "bridges" has been undertaken.
Since this is the case for Fromion's work, we will use the concept of *quadratic incremental stability* defined in (Fromion, *et al.*, 1999) and (Fromion, 2000) as follows.

Definition 2.2 *Given the system equation*

$$\dot{x} = f(x, v, t) \qquad (4)$$

where $v \in \mathbb{R}^p$. If it satisfies the two conditions

$$P\frac{\partial f}{\partial x} + \frac{\partial f}{\partial x}^T P \leq -\epsilon I_n \qquad (5)$$

$$\left(\frac{\partial f}{\partial v}\right)^T \left(\frac{\partial f}{\partial v}\right) \leq \sigma_{f_v} I_p \qquad (6)$$

with P a positive definite matrix and ϵ, σ_{f_v} two positive constants, then the system (4) is said to be quadratically incrementally stable.

Considering definition 2.1, and replacing P in (5) with M, it is clear that a system, contracting with respect to a constant Θ will satisfy condition (5). Condition (6) will easily be verified as $\frac{\partial f}{\partial v} = 0$ for equation (1), hence the quadratic incremental stability of the considered contracting system.
Moreover, it has also been proven that every quadratically incrementally stable system is also UGES (Fromion, 2000), which completes the proof of theorem 2.2.

Additionally, other results regarding contracting systems may be found (for more details, see (Jouffroy and Lottin, 2002b)).

3 Unmanned Underwater Vehicle observer

This section is dedicated to the establishment of the proof that the observer for UUV of (Fossen and

Blanke, 2000) is both contracting and UGES (see also (Jouffroy and Lottin, 2001) where additional remarks are made). For more details —in particular regarding the precise nomenclature of the UUV model or the Lyapunov-based observer design procedure, interested readers are referred to the above-mentioned paper.

Recall that the UUV model in state-space form can be written as

$$H\dot{x} + D_0 x + D_1(x,n,y)x + |n|Ex = f(n) \qquad (7)$$

with $x = (u, u_p)^T$ the state of the system, $y = u$ its measured output, and n its time-varying input.
In addition, the parameters are such that the matrices H and D_0 are positive definite, whereas

$$E = \begin{pmatrix} 0 & (1-t)T^0_{|n|u_a} \\ 0 & T^0_{|n|u_a} \end{pmatrix} \qquad (8)$$

with $0 < t < 1$[1] and $T^0_{|n|u_a} > 0$.
The nonlinear term D_1 is given by

$$D_1(x,n,y) = \begin{pmatrix} -X_{u|u|}|u| & 0 \\ 0 & d_f|u_p| - d_f(1-w)sign(n)y \end{pmatrix} \qquad (9)$$

in which $-X_{u|u|} > 0$ and $d_f > 0$ are two constant parameters.
The observer structure simply copies the model dynamics as

$$H\dot{\hat{x}} + D_0\hat{x} + D_1(\hat{x},n,y)\hat{x} + |n|E\hat{x} = f(n) + k(n)(y-\hat{y}) \qquad (10)$$

where

$$k(n) = \begin{pmatrix} K_{10} \\ K_{20} \end{pmatrix} + |n| \begin{pmatrix} K_{11} \\ K_{21} \end{pmatrix} \qquad (11)$$

stands for the observer gain vector.

Checking that x is a solution of (10) (i.e. if $\hat{x}$ in (10) is replaced with x, equation (7) is obtained), it just has to be shown that the estimate will converge to the actual trajectory.
Indeed, noting that (10) transforms into

$$\dot{\hat{x}} = -H^{-1}D_0\hat{x} - H^{-1}D_1\hat{x} - |n|H^{-1}E\hat{x} + H^{-1}f(n) + H^{-1}k(n)y - H^{-1}k(n)h^T\hat{x} \qquad (12)$$

where $h^T = (1,0)$. The jacobian can be computed as

$$\frac{\partial\dot{\hat{x}}}{\partial\hat{x}} = -H^{-1}D_0 - H^{-1}\frac{\partial(D_1\hat{x})}{\partial\hat{x}} - |n|H^{-1}E - |n|H^{-1}\begin{pmatrix} K_{11} \\ K_{21} \end{pmatrix}h^T - H^{-1}\begin{pmatrix} K_{10} \\ K_{20} \end{pmatrix}h^T \qquad (13)$$

which must be made uniformly negative definite (u.n.d.) for the system to be contracting, with the metric M being the identity matrix I.
Under assumption A1 of the referred paper, the nonlinear estimation error can be written as

$$D_1(x,n,y)x - D_1(\hat{x},n,y)\hat{x} = \begin{pmatrix} -X_{u|u|}|u|u + X_{u|u|}|\hat{u}|\hat{u} \\ d_f|u_p|u_p - d_f|\hat{u}_p|\hat{u}_p \end{pmatrix} \qquad (14)$$

which is equivalent to

$$\int_{\hat{x}}^{x} \frac{\partial(D_1(\zeta,n,y)\zeta)}{\partial\zeta}d\zeta \qquad (15)$$

Hence the jacobian

$$\frac{\partial(D_1\hat{x})}{\partial\hat{x}} = \begin{pmatrix} -2X_{u|u|}|\hat{u}| & 0 \\ 0 & 2d_f|\hat{u}_p| \end{pmatrix} \geq 0 \qquad (16)$$

Remembering that $D_0 > 0$ and choosing $K_{10} = K_{20} = 0$, K_{11} and K_{21} have to be set such that $E + \begin{pmatrix} K_{11} \\ K_{21} \end{pmatrix} h^T \geq 0$ to conclude that $\frac{\partial\dot{\hat{x}}}{\partial\hat{x}} < 0$ and therefore that the observer is contracting.
It can be checked that with values $K_{11} > 1$ and $K_{21} = -(1-t)T^0_{|n|u_a}$ the condition is verified.

The reader will notice that at any moment, no use of the error term $\tilde{x}$ —classical in Lyapunov-based approaches— was made throughout the procedure. Hence, additional nonlinear terms could easily have been chosen to increase observer complexity without adding too much difficulty in the stability analysis, as long as the u.n.d property holds for the new jacobian.
Using the results of the former section, the global exponential stability of the equilibrium point $\tilde{x} = 0$ can finally be concluded. As a matter of fact, this last statement is nothing but another way of stating that estimated and actual trajectories converge to one another in an exponential manner.
It should now be clear that the observer above could have been designed directly from contraction theory and that the latter may be regarded as another method yielding the same results. This can be partly explained by the mere fact that under specific but mild conditions, a Lyapunov function can be found from the jacobian matrix (see (Lohmiller, 1999, p. 11 and 27)).

4 Ship observer

The design of GES observers for marine systems using Lyapunov-based methods is one of Fossen's main contributions (see (Fossen and Strand, 1999; Fossen and Grovlen, 1998) and references therein). In this section, the complete design procedure of an observer estimating the body-fixed velocities of a ship is given.

[1] In this study, t refers to a constant parameter, and not to the time variable.

It is also shown that by addressing the question of robustification of the first contracting design we obtain the same observer as the one in (Fossen and Grøvlen, 1998). Some qualitative advantages of the method over Lyapunov-based designs are also given.

The state-space model of the ship is given as

$$\left\{ \begin{array}{l} \dot{\eta} = J(\eta)\nu \\ \dot{\nu} = A_1\eta + A_2\nu + B\tau \end{array} \right. \tag{17}$$

with $A_1 = -M^{-1}K$, $A_2 = -M^{-1}D$ and $B = M^{-1}$. $\nu \in \mathbb{R}^3$ represents the vector of body-fixed velocities and $\eta \in \mathbb{R}^3$ the earth-fixed positions. The positive definite matrices M, K, and D correspond respectively to the inertia matrix, the mooring forces matrix and the damping matrix, while τ stands for the thruster system control force vector. $J(\nu)$ is a state-dependent rotation matrix. Earth-fixed positions are measured thanks to a DGPS.

Using contraction theory, let us choose an observer that copies the model dynamics and take the earth-fixed measure as observer feedback.

$$\left\{ \begin{array}{l} \dot{\hat{\eta}} = J(\eta)\hat{\nu} + K_1(\eta - \hat{\eta}) \\ \dot{\hat{\nu}} = A_1\hat{\eta} + A_2\hat{\nu} + B\tau + K_2(\eta - \hat{\eta}) \end{array} \right. \tag{18}$$

where K_1 and K_2 are the observer gain matrices to be tuned.
Isolating the two subsystems, respectively $\dot{\eta} = f_1(\eta)$ and $\dot{\nu} = f_2(\nu)$ and calculating their jacobians, we see that

$$\delta\dot{\hat{\eta}} = -K_1\delta\hat{\eta} \tag{19}$$

and

$$\delta\dot{\hat{\nu}} = A_2\delta\hat{\nu} \tag{20}$$

(20) is contracting because $A_2 < 0$ by definition. Equation (19) shows that K_1 must be selected positive if this subsystem is also to be contracting. Assuming it is done so, the jacobian of the overall system is now studied.

$$\begin{pmatrix} \delta\dot{\hat{\eta}} \\ \delta\dot{\hat{\nu}} \end{pmatrix} = \begin{pmatrix} -K_1 & J(\eta) \\ A_1 - K_2 & A_2 \end{pmatrix} \begin{pmatrix} \delta\hat{\eta} \\ \delta\hat{\nu} \end{pmatrix} \tag{21}$$

Using the combination property of section (2) and considering that (19) and (20) are in feedback connection, we see that if K_2 is tuned so that $A_1 - K_2 = -J^T(\eta)$ then the observer is contracting. To ensure that the estimate will converge to the actual trajectory, the next step is to verify that the ship model is indeed contained in the observer flow field, which is the case.
Resuming the gain matrices constraints, we have

$$\left\{ \begin{array}{l} K_1 > 0 \\ K_2 = A_1 + J^T(\eta) \end{array} \right. \tag{22}$$

This design is somewhat restrictive because only K_1 can be tuned arbitrarily. Also, it will be shown experimentally that it may lack robustness.
To prevent this limitation, let us now make use of appropriate constant metrics and transformations. Defining $\delta\hat{z}_1$ and $\delta\hat{z}_2$ as

$$\left\{ \begin{array}{l} \delta\hat{z}_1 = \Theta_1\delta\hat{\eta} \\ \delta\hat{z}_2 = \Theta_2\delta\hat{\nu} \end{array} \right. \tag{23}$$

The new jacobian

$$\begin{pmatrix} \delta\dot{\hat{z}}_1 \\ \delta\dot{\hat{z}}_2 \end{pmatrix} = \begin{pmatrix} \Theta_1(-K_1)\Theta_1^{-1} & \Theta_1 J(\eta)\Theta_2^{-1} \\ \Theta_2(A_1 - K_2)\Theta_1^{-1} & \Theta_2 A_2\Theta_2^{-1} \end{pmatrix} \begin{pmatrix} \delta\hat{z}_1 \\ \delta\hat{z}_2 \end{pmatrix} \tag{24}$$

is obtained, implying that the conditions for the system to be contracting are therefore

$$\left\{ \begin{array}{l} \Theta_1(-K_1)\Theta_1^{-1} < 0 \\ \Theta_2 A_2\Theta_2^{-1} < 0 \\ \Theta_1 J(\eta)\Theta_2^{-1} = -(\Theta_2(A_1 - K_2)\Theta_1^{-1})^T \end{array} \right. \tag{25}$$

After straightforward manipulations, the above conditions simplify into

$$\left\{ \begin{array}{l} K_1^T M_1 + M_1 K_1 > 0 \\ A_2^T M_2 + M_2 A_2 < 0 \\ K_2(\eta) = M_2^{-1} J^T(\eta) M_1 + A_1 \end{array} \right. \tag{26}$$

where obviously $M_i = \Theta_i^T\Theta_i$.

Not surprisingly, the conditions obtained here turn out to be equivalent to those of (Fossen and Grøvlen, 1998). However, using this kind of method brings several qualitative advantages over Lyapunov-based techniques. To begin with, these latter procedures often start with "define a Lyapunov function candidate..." without any further information regarding the interpretability of the chosen function. Our claim is that by making use of transformation matrices as seen above, the designer gets a clearer insight of what is done (what is the coordinate frame under which the system will be *seen* stable). Furthermore, this also gives a smooth transition from linear state-space control designs to nonlinear ones, and thus may be of interest for engineering and educating purposes. In our opinion, Lyapunov functions lack these properties, mainly because of the underlying energy concept, which is not too frequently encountered in linear control theory.

Simulation of the above observers with different tunings was undertaken to illustrate the ideas developed above. Model parameters are those of the supply vessel in (Fossen and Grøvlen, 1998).

Recall that

$$M = \begin{pmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.0744 \\ 0 & -0.0744 & 0.1278 \end{pmatrix} \tag{27}$$

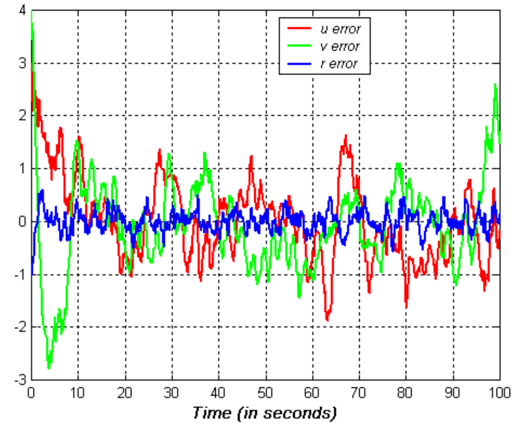

Figure 1: $\hat{\nu}$ for the contracting observer

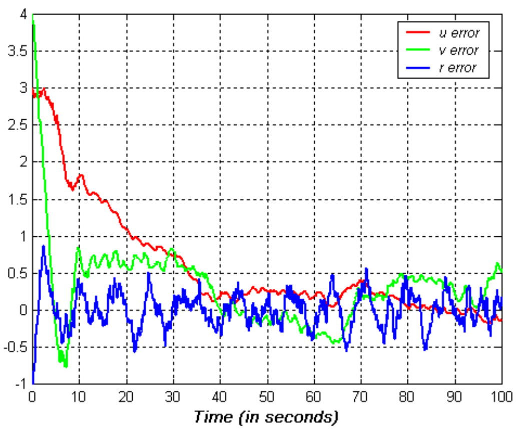

Figure 2: $\hat{\nu}$ with the Lyapunov-based observer

and

$$D = \begin{pmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0041 & 0.0308 \end{pmatrix} \quad (28)$$

and $K = 0$. The control inputs are chosen as

$$\tau = \begin{pmatrix} sin(0.5t) \\ 0 \\ 0.1sin(0.1t) \end{pmatrix} \quad (29)$$

In figure 1, the evolution of the estimation error of the three components of the velocity vector $\tilde{\nu} = \nu - \hat{\nu}$ (namely surge u, sway v and yaw r) corresponding to gain matrices verifying conditions in (22) is represented. K_1 is set to I_3. The observer has zero initial conditions while initial ship vectors are chosen as $\eta(0) = \nu(0) = (3, 4, -1)^T$. Though noise-free testing has shown good convergence properties, this particular tuning exhibits a rather important "unrobust" behavior when measurement noise is added (in the simulation, power spectral density of noise was set to 0.1). For comparison, in figure 2 the same simulation

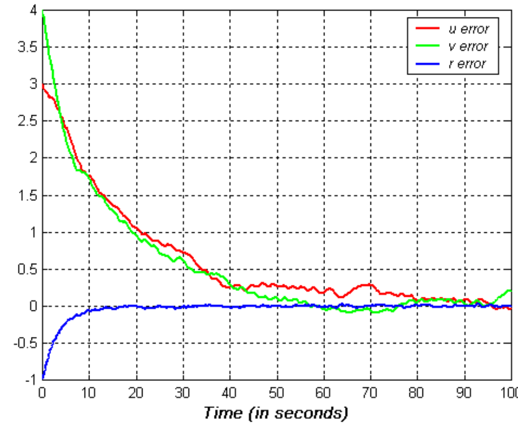

Figure 3: $\hat{\nu}$ for the contracting observer with robustification

results are given this time with the parameter values that came along with the Lyapunov-based procedure[2] in (Fossen and Grøvlen, 1998), somehow confirming Fossen's remark mentioned in the introduction.

However, as has been demonstrated in this study, contraction-based designs can lead to the same design as by using Lyapunov functions. Furthermore, by choosing appropriate values of parameters ($K_1 = I_3$, $M_1 = 3I_3$ and $M_2^{-1} = 0.02I_3$), a less noise-sensitive tuning can be found (see figure 3). These robustness issues can be related to the concept of *asymptotic and bounded observers* developed in (Fromion and Ferreres, 1997) where the question of designing an observer for systems with a disturbance on the measurement was addressed. The application of contraction principles to such problems is treated in a separate publication.

5 Concluding remarks

Using the fact that a contracting system can yield, under some specific conditions, to global exponential stability in the traditional sense, contraction theory-based methods for the design of observers for two ocean vehicles (UUV and ships) were presented in this paper. To this end, equivalence between systems which are contracting under a constant metric and UGES was first demonstrated. Then, the two observer design procedures were fully described along with simulation results, making the comparison with Lyapunov-based methods possible.
Quantitative characterization of the observer sensitivity to output measurement noise and robustification

[2] $K_1 = 0.33I_3$, $K_2 = A_1 + P_2 J^T(\eta) P_1$ with $P_1 = 3I_3$ and $P_2 = \begin{pmatrix} 31.4916 & 0 & 0 \\ 0 & 15.9487 & -0.2164 \\ 0 & -0.2164 & 4.1222 \end{pmatrix}$

of the observer design using additional nonlinear terms are currently under research. The construction of a complete contraction-based observer-controller structure for ships using the results in (Jouffroy and Lottin, 2002a) is also studied.

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References

Angeli, D. (2002). A Lyapunov approach to incremental stability properties. *IEEE Transactions on Automatic Control*, (To appear).

Fossen, T. I. and M. Blanke (2000). Nonlinear output feedback control of underwater vehicle propellers using feedback form estimated axial flow velocity. *IEEE Journal of Oceanic Engineering*, **25**(2), 241–255.

Fossen, T. I. and Å. Grøvlen (1998). Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping. *IEEE Transactions on Control Systems Technology*, **6**(1), 121–128.

Fossen, T. I. and J. P. Strand (1999). Passive nonlinear observer design for ships using Lyapunov methods: full-scale experiments with a supply vessel. *Automatica*, **35**(1), 3–16.

Fromion, V. (2000). The behavior of Lipschitz continuous systems on $\mathcal{L}_p$. LASB-Internal report, INRA Montpellier, France.

Fromion, V. and G. Ferreres (1997). An incremental framework for the extended Kalman filter. *European Control Conference*, Brussels, Belgium.

Fromion, V., S. Monaco and D. Normand-Cyrot (1996). Asymptotic properties of incrementally stable systems. *IEEE Transactions on Automatic Control*, **41**, 721–723.

Fromion, V., G. Scorletti and G. Ferreres (1999). Nonlinear performance of a PI controlled missile: an explanation. *International Journal of Robust and Nonlinear Control*, **9**(8), 485–518.

Johansen, T. A., R. Shorten and R. Murray-Smith (2000). On the interpretation and identification of dynamic Takagi-Sugeno fuzzy models. *IEEE Transactions on Fuzzy Systems*, **8**, 297–313.

Jouffroy, J. and J. Lottin (2001). Remarks on "Nonlinear output feedback control of underwater vehicle propellers using feedback form estimated axial flow velocity". *IEEE Journal of Oceanic Engineering*, (submitted for publication).

Jouffroy, J. and J. Lottin (2002a). Integrator backstepping using contraction theory: a brief methodological note. *IFAC World Congress 2002*, Barcelona, Spain.

Jouffroy, J. and J. Lottin (2002b). Some links between contraction theory and incremental stability. (In preparation).

Khalil, H. (1996). *Nonlinear systems (2nd ed.)*. Prentice-Hall.

Lohmiller, W. (1999). *Contraction analysis of nonlinear systems*. Ph.D. Dissertation, Dep. Mechanical Eng., M.I.T., 90 pages.

Lohmiller, W. and J.J.E. Slotine (1997). Simple observers for Hamiltonian systems. *IEEE American Control Conf.*, Albuquerque, New Mexico.

Lohmiller, W. and J.J.E. Slotine (1998). On contraction analysis for nonlinear systems. *Automatica*, **34**(6), 683–696.

Lohmiller, W. and J.J.E. Slotine (2000). Nonlinear process control using contraction theory. *A.I.Ch.E. Journal*, **46**(3), 588–596.

Sepulchre, R., M. Janković and P. Kokotović (1997). *Constructive nonlinear control*. Springer-Verlag.

Sontag, E. D. (2000). The ISS philosophy as a unifying framework for stability-like behavior. *Nonlinear Control in the Year 2000 (Volume 2)*. (Lecture Notes in Control and Information Sciences, A. Isidori, F. Lamnabhi-Lagarrigue, and W. Respondek, eds.), Springer-Verlag, 443–468.