Scenario Reoptimisation under Data Uncertainty

Antonio Manca [a]***, Giovanni M. Sechi*** [a] ***and <u>Paola Zuddas</u>*** [a]

[a] *Research and Educational Centre for Network Optimisation, Department of Land Engineering*

University of Cagliari, Italy. (zuddas@unica.it)

Abstract: Many dynamic planning and management problems are typically characterised by a level of uncertainty regarding the value of data input such as supply and demand patterns. Assigning inaccurate values to them could invalidate the results of the study. Consequently, deterministic models are inadequate for the representation of these problems where the most crucial parameters are either unknown or are based on an uncertain future. In these cases, the scenario analysis technique could be an alternative approach. Scenario analysis can model many real problems in which decisions are based on an uncertain future, whose uncertainty is described by means of a set of possible future outcomes, called "scenarios". In this paper we present a scenario analysis approach to dynamic multi-period systems by integrating scenario optimisation and subsequent deterministic reoptimisation. In the scenario optimisation phase we represent data uncertainty by a robust chance optimisation model obtaining a so-called barycentric value with respect to selected decision variables. The successive reoptimisation model based on this barycentric solution allows planning a part of the risk of a wrong decision, reducing the negative consequences deriving from it.

Keywords: Scenario analysis; Optimisation under uncertainty; Dynamic problems; Reoptimisation.

1. INTRODUCTION

A system is dynamic if each component is associated with a time *t* and represents a decision in time. A dynamic system can be generated by replicating a static system over time with inter-period connections. Multiperiod systems are defined in a dynamic planning horizon in which management decisions have to be made sequentially in time or decided globally as a decision strategy referring to a predefined set of data and time horizon. Many dynamic planning and management problems are typically characterised by a level of uncertainty regarding the value of data input such as supply and demand patterns. (Glockner and Nemhauser, [2000]. Assigning inaccurate values to them could invalidate the results of the study. Consequently, deterministic models are inadequate for the representation of these problems where the most crucial parameters are either unknown or are based on an uncertain future.

The traditional stochastic approach gives a probabilistic description of the unknown parameters on the basis of historical data. This is a very efficient approach when a substantial statistical base is available and reliable probabilistic laws can adequately describe parameters' uncertainty and their possible outcomes (Infanger[1994]; Kall and Wallace [1994]; Ruszczynski[1997]). It is well known that stochastic optimisation approaches cannot be used when there is insufficient statistical information on data estimation to support the model, when probabilistic rules are not available, and/or when it is necessary to take into account information not derived from historical data.

In these cases, the scenario analysis technique could be an alternative approach (Dembo[1991]; Rockafellar and Wets[1991]). Scenario analysis can model many real problems where decisions are based on an uncertain future, whose uncertainty is described by means of a set of possible future outcomes, called "scenarios". Therefore, a scenario represents a possible realisation of some sets of uncertain data in the time horizon examined (Onnis et al., [1999]).

The scenario analysis approach considers a set of statistically independent scenarios, and exploits the inner structure of their temporal evolution in order to obtain a "robust" decision policy, in the sense that the risk of wrong decisions is minimised.

Some examples are given in Pallottino et al. [2003] for water resources management, in Mulvey and Vladimirou[1989] for investment and production planning, in Glockner[1996] for air traffic management and in Hoyland and Wallace [2001] for insurance policy and production planning.

The aim of this paper is to generalize the effectiveness of scenario analysis when evaluating the risk of wrong decisions in order to reduce the negative consequences.

In Pallottino et al. [2004] the authors analysed the scenario approach for water resources management offering some general rules for making a scenario tree from a predefined set of scenarios and for identifying a complete set of decision variables relative to all the scenarios under investigation. In this paper we extend that approach to general dynamic systems and propose a reoptimisation procedure, which facilitates reaching a robust solution and planning a part of the risk of wrong decisions caused by wrong assumptions on adopted parameters.

2. DETERMINISTIC DYNAMIC CHANGE DYNAMIC MODEL OPTIMISATION MODEL

In a deterministic dynamic framework we extend the analysis to a sufficiently wide time horizon and assume a time step (time-period), *t*. The scale and number of time-steps must be adequate to reach a significant representation of the variability the system components.

A dynamic multi-period system is then generated by replicating the static basic system over time, for each time-period *t*, having previous knowledge of the time sequence of historical data. We then connect the corresponding copies for different consecutive periods by additional components carrying the information (decision) stored at the end of each period in such a way that the whole multi-period system is connected. We call these components inter-period components.

A dynamic mathematical model is a mathematical model associated with a dynamic system. The data and the decision variables of the dynamic mathematical model are associated to each component of the dynamic system for each time-period *t*.

In a deterministic approach, the database is derived from available historical data submitted to statistical validation on the basis of a forecast and adopted as a reference scenario. In the deterministic optimisation model, we assume that the manager has previous knowledge of the time sequence of input data to the system. As a consequence, the solution obtained is strictly connected to the adopted scenario. We can formalize a model ($\boldsymbol{P_g}$) for a specific scenario g, as an optimisation model:

$$(\boldsymbol{P_g}) \qquad \min\ f_g(\boldsymbol{x}_g) \quad \text{s.t.} \quad \boldsymbol{x}_g \in \boldsymbol{X}_g$$

Once scenario g is adopted, where $\boldsymbol{x_g}$ represents the vector comprehensive of all management and planning variables for all time-periods t, $f_g(\boldsymbol{x}_g)$ represents the objective function of the problem and, $\boldsymbol{x}_g \in \boldsymbol{X}_g$, represents the set of all constraints (technical, physical, social, etc.) that are peculiar to the examined problem (standard constraints). The solution $\boldsymbol{x}_g$ of problem ($\boldsymbol{P_g}$) represents the set of decisions that should be adopted if scenario g takes place.

3. CHANGE DYNAMIC OPTIMISATION MODEL

Deterministic models are not adequate to describe the variability of some crucial parameters and small differences in data in two different scenarios can produce significantly different solutions. Typically, most of the data in model ($\boldsymbol{P_g}$) can be affected by a high level of uncertainty. In an uncertain environment the stochastic optimisation approach cannot be adopted since it is unreliable to match a valid occurrence probability to each scenario.

The simulation approach studies a number of outcomes obtained by solving a number of optimisation problems ($\boldsymbol{P_g}$) for each scenario g. During the optimisation process, different scenarios, corresponding to different dynamic multi-period models, proceed independently obtaining a different management policy for each scenario. Simulation verifies the performance of all policies selecting one for future decisions. Usually, to reach a viable management policy, a large number of scenarios must be considered. The simulation approach can prove very demanding from a computational point of view, especially if continuously replicated when the hydrological events occurring are very different from those foreseen in the selected scenario.

The scenario analysis approach attempts to face the uncertainty factor by taking into account a set, $\boldsymbol{G}$, of different supposed scenarios corresponding to the different possible time evolution of crucial data. Unlike simulation, the different scenarios are considered together to obtain a global set of decision variables on the whole set of scenarios. More precisely, two scenarios sharing a common initial portion of data must be considered together and partially aggregated with the same decision variables for the aggregated part, in order to take into account the two possible evolutions in the subsequent diverse parts. In this way, the set of parallel scenarios is aggregated by producing a tree structure, called *scenario-tree*. The aggregation rules guarantee that the solution in any given period is independent of the information not yet available. This result can be obtained by inserting *congruity constraints* which require that the subsets of decision variables, corresponding to the indistinguishable part of different scenarios, must be equal among themselves In other words, model evolution is only based on the information available at the moment. (Rockafellar and Wets, [1991]).

The problem supported by the *scenario tree*, is described by a mathematical model that includes all single-scenario problems ($\boldsymbol{P_g}$), $\forall g \in \boldsymbol{G}$, plus some inter-scenario linking constraints representing the requirement that if two scenarios *g1* and *g2* are identical up to time *t* on the basis of information available at that time, then the corresponding set of decision variables, $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$, must be identical up to time *t*. These constraints represent the congruity requirement that the subsets of decision variables corresponding to the indistinguishable part of different scenarios must be equal among themselves. Moreover, a weight can be assigned to each scenario representing the "importance" assigned by the manager to the running configuration. At times the weights can be viewed as the probability of occurrence of the examined scenario. More often they are determined on the basis of background knowledge about the system.

The resulting mathematical model is named *chance-model* to indicate that it is not stochastically based but, due to the impossibility of adopting probabilistic rules and/or to the necessity of inserting information that cannot be deduced from historical data, it attempts to represent the set of possible performances of the system, as uncertain parameters vary.

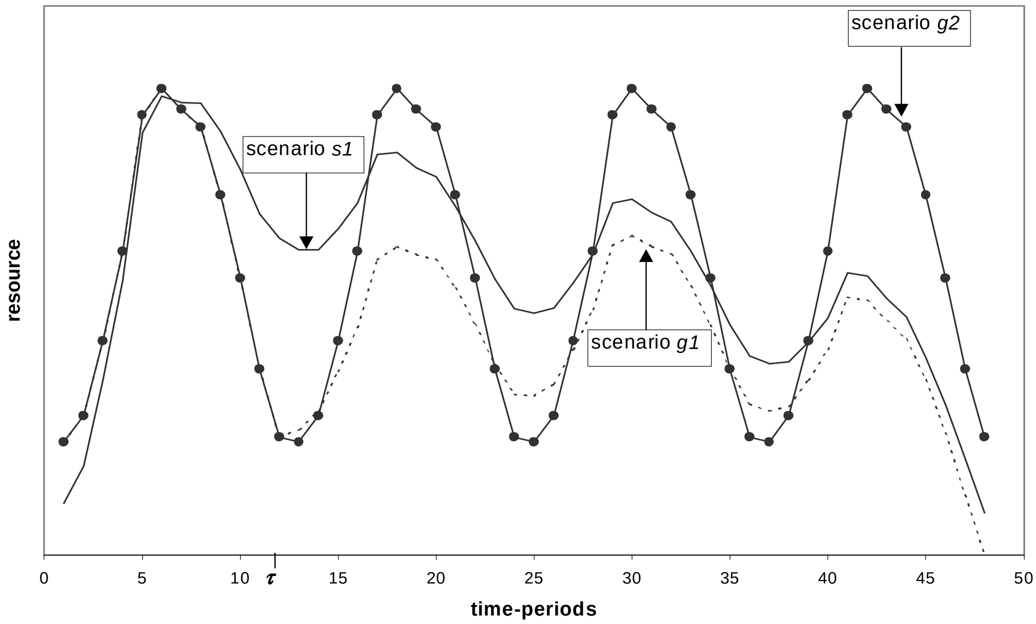

Figure 1. Stored resources in scenario and deterministic optimisation.

The chance model ($\boldsymbol{P_C}$) can have the following structure:

$$(\boldsymbol{P_C}) \qquad \min \Sigma_g \, w_g \, f_g(\boldsymbol{x}_g)$$
$$s.t.$$
$$\boldsymbol{x}_g \in \boldsymbol{X}_g \qquad \forall g \in \boldsymbol{G}$$
$$\mathbf{x}^* \in \mathbf{S}$$

Where:

w_g represents the weight assigned to a scenario $g \in \boldsymbol{G}$; $\boldsymbol{x}^*$ represents the vector of variables submitted to congruity constraints; $\mathbf{x}_g \in \mathbf{X}_g$ represents the set of standard constraints for each scenario g ; $\boldsymbol{x}^* \in \boldsymbol{S}$, represents the set of congruity constraints.

The objective function is the weighted sum of the objective functions of problems ($\boldsymbol{P_g}$) and all standard constraints are included.

Congruity constraints require that the decision variables in those scenarios that are indistinguishable up to a specific time $\boldsymbol{\tau}$ (branching-time) are the same up to τ. Specifically, the decisions at the end of the time τ, must be the same of those at the beginning of period $\tau+1$.

To generate the set $\boldsymbol{G}$ of scenarios, different approaches such as Monte Carlo generation scheme, Neural network techniques or ARMA models can be performed. The aim of this paper is not to detail these procedures and we assume that the set $\boldsymbol{G}$ is available.

Regarding weight definitions, if the manager were able to evaluate the weight w_g as the probability that scenario g will occur, he could estimate it by some stochastic technique or statistical test. More often the manager has few, if any, possibilities to do this due to the difficulty in deriving a probabilistic rule from conceptual considerations. Instead, in scenario analysis, a weight w_g assigned to a scenario g can be interpreted as the "relative importance" of that scenario in the uncertain environment. In other words, in scenario analysis, weights are interpreted as subjective parameters assigned on the basis of the experience of the water management board.

3.1 A sample system

To illustrate the scenario analysis approach we refer to a sample dynamic supply-demand system with a resource supply and a demand centre. The supply centre can deliver a resource or store it to deliver in a successive time-period. We assume that the dimensions of the supply and demand centres are known, and that the system is operational. We want to determine the resource management policy over a time horizon such that the known resource demand is satisfied (as much as possible) and the total cost is minimized. Objective function and constraints will be analytically expressed on the basis of the feature of the examined system. Variables of the optimisation problem, for each scenario g at time-period t, are referred to stored resource y^t_g, delivered resource from supply centre to demand centre z^t_g. Resource demand p is assigned and we suppose that historical data are available. Deficits u^t_g can be then calculated as the difference between demand p and delivered resources z^t_g, in each time-period t. We then generate two scenarios, *g1* and *g2*, assuming that uncertain parameters correspond to resource supplies in supply centre in period t in scenario g.
The two scenarios are both identical to the historical data up to branching time τ. We suppose that scenario *g2* follows the historical data from $\tau+1$ to the last time-period, while scenario *g1* has the resource supplies reduced by 50% with respect to it ("scarce" scenario). This means that two different possible resource supply configurations can occur. Finally the two scenarios run until they

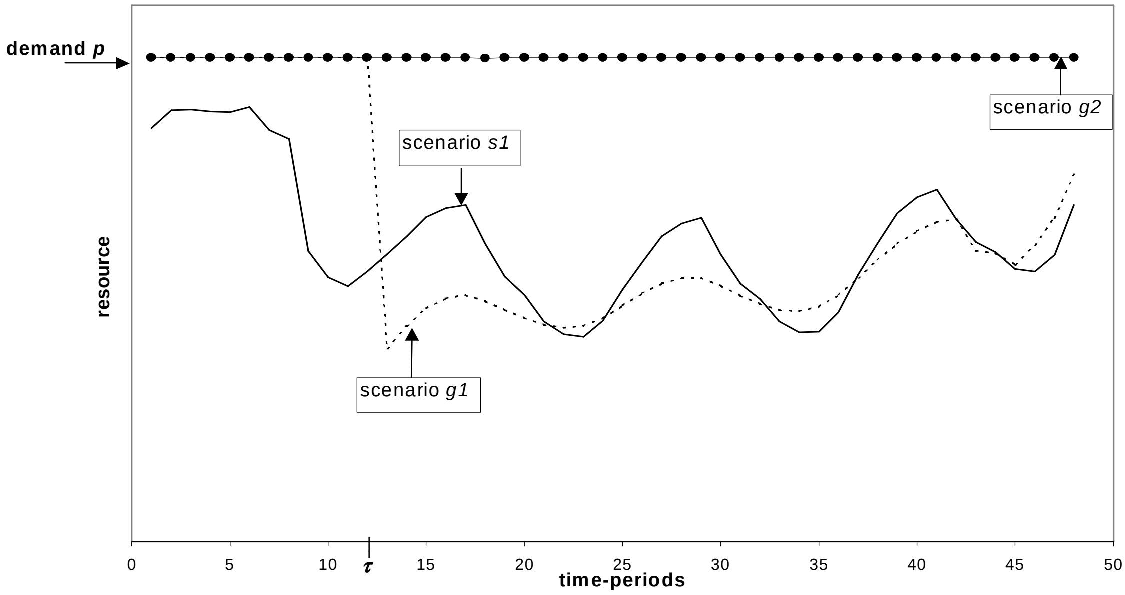

Figure 2. Resources delivered to demand centre in scenario and deterministic optimisation.

reach the end of the time-horizon. The optimisation model requires minimizing a function representing the total weighted cost $\Sigma_g \Sigma_t w_g f_g (y^t_g, z^t_g, u^t_g)$ subject to standard and congruity constraints.
To illustrate, we show some possible results concerning stored resources in supply centre, y^t_g, and resources delivered to demand centre, z^t_g, obtained by scenario analysis, solving the above optimisation chance model.
Figure 1 shows stored resources, y^t_g, obtained by scenario analysis and those obtained by a deterministic optimisation model when the "scarce" scenario *g1* is assumed as database. When scenario *g1* is considered independently, it is referred to as *s1*. The resulting graph represents the decisions that would be made for transferring resources in a deterministic optimisation process.
The zone between the two graphics of the aggregated scenarios, *g1* and *g2*, represents the possible decisions that can be made for stored resources. Therefore we can say that any part of *s1* not between *g1* and *g2* represents the error that the

manager would have made if he had adopted decision *s1*.

Figure 2 shows the resources delivered, z^t_g, from supply centre to the demand centre. The behaviour of these flows shows that in the scenario *g2* demand is fulfilled while in scenario *g1* deficits are present after branching time τ. But, comparing this with results in deterministic optimisation under scenario *s1*, we can see that as regards the scarcity of resources conditions, scenario optimisation gives a smoother distribution, i.e., with a lower variance of resource distribution in scenario g1 even though the average is almost the same as scenario *s1*. Thus, when planning for scarce resources, scenario analysis provides less dramatic and more easily implementable results then using deterministic optimisation to determine management policy.

3.2 A barycentric chance reoptimisation model

In the previous section we showed how scenario analysis could be more useful than the deterministic approach in deciding management policy. This can be crucial if scarce resources events occur and a rationing policy must be adopted. But, an effective management policy must be able to establish a target value for delivering resources to the demand centre. The community suffers less from resource rationing if it has been forewarned of a possible shortage. This target value should take into account the entire range of possible scenarios of resource availability, neither too pessimistic in case of abundance, nor too optimistic in case of scarcity of resources. In other words, a target value should be sufficiently barycentric in respect to the different possible scenarios that could take place. Establishing the resource demand level at this target value would permit notifying the resource users (the community) in a timely fashion. As a consequence, preventive measures could be adopted in order to avoid, at least in part, damages derived from an unexpected drastic cut in resources (water, oil, raw materials, currency, transportation and telecommunications, etc.).

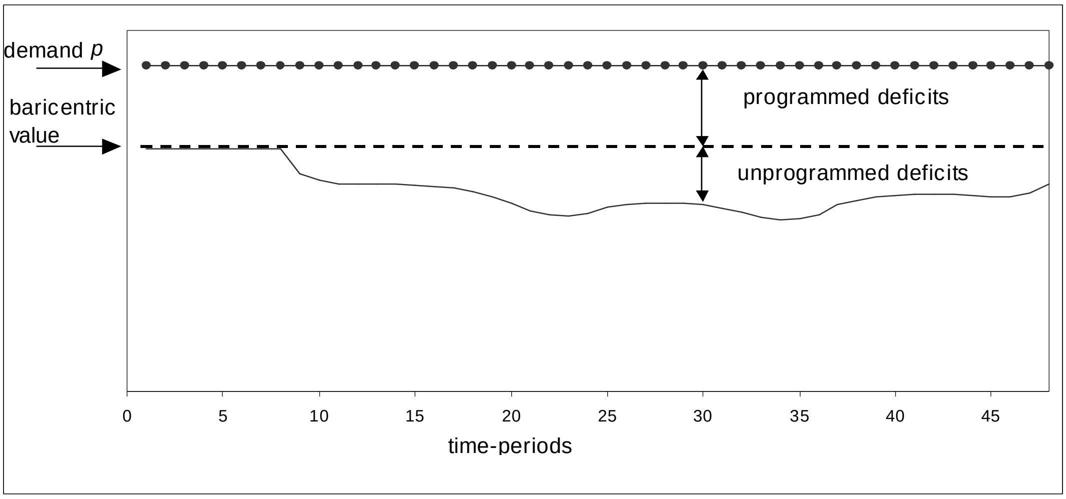

Figure 3. Resources delivered to demand centre in deterministic reoptimisation.

If $\hat{x}^t_g$ are the decision variables representing the resources that can be delivered to a demand centre in time-period *t* under scenario *g*, we want to determine a target demand as the value x^b that is barycentric with respect to all $\hat{x}^t_g$. To obtain this value we introduce in the objective function of problem ($\boldsymbol{P_C}$) a function measuring the weighted distance from x^b to $\hat{x}^t_g$ for all *g* and *t*. If we adopt the Euclidean norm to measure this distance, the chance barycentric model ($\boldsymbol{P_B}$) can be expressed as:

$$(\boldsymbol{P_B}) \qquad \min \Sigma_g w_g f_g(\mathbf{x}_g) + \Sigma_g \Sigma_t \lambda_g (\hat{x}^t_g - x^b)^2$$
$$s.t.$$
$$\mathbf{x}_g \in \mathbf{X}_g \qquad \forall g \in \mathbf{G}$$
$$\mathbf{x}^* \in \mathbf{S}$$

where λ_g.is the weight associated to the norm.

Once the value x^b is determined, a reoptimisation process can be adopted in order to identify the sensitivity of the examined system with respect to deficit programming.

We construct a deterministic dynamic model in which the predefined demand is settled equal to the barycentric value x^b and adopting as data input, those corresponding to the most crucial scenario (e.g. what the manager considers the most risky for the system). The difference between the new configuration of delivered resources in each time-period t and the value x^b, identifies the set of programmed deficits for the system.

In the sample system illustrated in the previous section we determine a value z^b in such a way that it is barycentric with respect to all z^t_g. We then reoptimise the system solving a deterministic model assigning to the demand centre the obtained value z^b as target value and adopt, as data input, those corresponding to scarce scenario. Figure 3 shows the resources delivered to the demand centre in the reoptimisation phase together with the programmed deficits (difference between the new configuration of delivered resources in each time-period t and the value x^b) and unprogrammed deficits (difference between the original resource demand and the value x^b). Moreover, comparing the behaviour of delivered resources with that showed in figure 2, we observe that management policy is even better than the policy corresponding to scenario *g2*. The programming of deficits makes it possible to set up adequate preventive measures, which permit a notable reduction in the event of resources scarcity.

4 CONCLUSIONS

In this paper we showed how scenario analysis can be more useful than the deterministic approach in deciding system management policy when a level of uncertainty affects data input such as supply and demand patterns. Decision policy under uncertainty condition can be crucial if scarce resources events occur and a rationing programme must be adopted. The scenario analysis approach considers a set of statistically independent scenarios, and exploits the inner structure of their temporal evolution in order to obtain a "robust" decision policy, in the sense that the risk of wrong decisions is minimised. This can be done by a reoptimisation deterministic process using a barycentric value derived from a previous scenario optimisation. Finally, this make it possible to identify programmed deficits to control the negative consequences deriving from wrong decisions allowing the system manager to adopt preventive measures avoiding, at least in part, damages derived from an unexpected drastic cut in resources.

5 REFERENCES

Dembo, R., 1991. Scenario optimisation. *Annals of Operations Research* 30, 63-80.

Glockner, G. D., 1996. Effects of air traffic congestion delays under several flow management policies. *Transportation Research Record* 1517, 29-36.

Glockner, G. D., Nemhauser G. L., 2000. A dynamic network flow problem with uncertain arc capacity: formulation and problems structure. *Operations Research* 48 (2), 233-242.

Hoyland, K., Wallace, S. W., 2001. Generating scenario trees for multistage decision problems. *Management Science* 47 (2), 295-307.

Infanger, G., 1994. Planning under uncertainty: Solving large-scale stochastic linear programming. Boyd & Fraser Publishing Company, Danvers, MA.

Kall, P., Wallace, S. W., 1994. Stochastic Programming, John Wiley and Sons, New York.

Mulvey, J. M., Vladimirou, H., 1989. Stochastic network optimisation models for investment planning. *Annals of Operation Research* 20, 187-217.

Onnis, L., Sechi, G. M, Zuddas, P., 1999. Optimisation processes under uncertainty, A.I.C.E., Milano, 238-244.

Pallottino, S., Sechi, G.M., Zuddas, P., 2004. A DSS for Water Resources Management under Uncertainty by Scenario Analysis, to appear in Environmental Modelling & Software, Special Issue.

Rockafellar, R. T., Wets, R. J. B., 1991. Scenarios and policy aggregation in optimisation under uncertainty. *Mathematics of Operations Research* 16, 119-147.

Ruszczynski, A., 1997. Decomposition methods in stochastic programming. *Mathematical Programming* 79, 333-353.