Non-Lorentzian Gauge Fields in Maxwell Electrodynamics

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It is shown that in the standard Maxwell electrodynamics the class of gauge fields is much larger than it is usually stated. The possible significance of the new unexploited gauge fields is discussed.

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1. Introduction

It is well-known that the famous Poincaré covariance of Maxwell electrodynamics may be violated by imposing Lorentz non-covariant gauge conditions on the corresponding electromagnetic potentials. This happens, for example, for the Coulomb and axial gauges used in elementary particle physics[1]. In modern physics gauge fields play a fundamental role because it is generally believed that these fields describe all significant in Nature interactions. It is also generally believed that the nature of the gauge fields is almost completely explored. In the present paper we shall however show that already in the standard Maxwell electrodynamics, apart from the standard electromagnetic potentials, there exits an infinite chain of Lorentz non-covariant gauge fields the physical meaning of which is completely unknown. We shall also show that these unexploited new gauge fields may have some application in the theory of magnetism.

We restrict ourselves to the standard Maxwell electrodynamics [2] without entering any discussion of its domain of validity. We take it as the best theory for all electromagnetic phenomena.

The usual way of introducing electromagnetic gauge fields $\overrightarrow{A}(\overrightarrow{x},t)$ and $\Phi(\overrightarrow{x},t)$ consists in writing the electromagnetic fields $\overrightarrow{E}(\overrightarrow{x},t)$ and $\overrightarrow{B}(\overrightarrow{x},t)$ in the forms

$$\overrightarrow{E}(\overrightarrow{x},t) = -\overrightarrow{\nabla}\Phi(\overrightarrow{x},t) - \frac{\partial\overrightarrow{A}(\overrightarrow{x},t)}{\partial t}, \tag{1}$$

$$\overrightarrow{B}(\overrightarrow{x},t) = rot\ \overrightarrow{A}(\overrightarrow{x},t)\,. \tag{2}$$

From the vacuum inhomogeneous Maxwell equations with the charge density $\rho(\overrightarrow{x},t)$ and the current density $\overrightarrow{j}(\overrightarrow{x},t)$ we get then the field equations

$$\Box\Phi(\overrightarrow{x},t) = \frac{\rho(\overrightarrow{x},t)}{\epsilon_0}, \tag{3}$$

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$$\Box \overrightarrow{A}(\overrightarrow{x},t) = \mu_0 \overrightarrow{j}(\overrightarrow{x},t) \tag{4}$$

provided the Lorenz gauge condition

$$\frac{1}{c^2}\frac{\partial \Phi(\overrightarrow{x},t)}{\partial t} + div\ \overrightarrow{A}(\overrightarrow{x},t) = 0 \tag{5}$$

is satisfied. The gauge fields which satisfy this condition are still not unique but may be gauged according to the rules

$$\overrightarrow{A}(\overrightarrow{x},t) \rightarrow \overrightarrow{A}'(\overrightarrow{x},t) = \overrightarrow{A}(\overrightarrow{x},t) + \overrightarrow{\nabla} G(\overrightarrow{x},t) \tag{6}$$

and

$$\Phi(\overrightarrow{x},t) \rightarrow \Phi'(\overrightarrow{x},t) = \Phi(\overrightarrow{x},t) - \frac{\partial}{\partial t} G(\overrightarrow{x},t) \tag{7}$$

where the functions $G(\overrightarrow{x},t)$ are arbitrary solutions of the wave equation

$$\Box G(\overrightarrow{x},t) = 0. \tag{8}$$

In all non-Abelian gauge theories the above scheme is copied with the necessary changes implied by the structure of the underlying local symmetry group.

2. Non-Lorentzian gauge fields

As a starting point let us take the observation that the representations (1) and (2) are highly non symmetric in the gauge fields. With such non symmetric representations it is impossible to formulate in terms of the gauge fields the famous duality symmetry[3] of electrodynamics. Let us consider therefore more symmetric representations of the form

$$\overrightarrow{E}(\overrightarrow{x},t) = -\overrightarrow{\nabla}\Phi(\overrightarrow{x},t) - \frac{\partial \overrightarrow{A}(\overrightarrow{x},t)}{\partial t} + \lambda\ rot\ \overrightarrow{X}(\overrightarrow{x},t), \tag{9}$$

and

$$\overrightarrow{B}(\overrightarrow{x},t) = -\overrightarrow{\nabla}\Psi(\overrightarrow{x},t) - \frac{\partial \overrightarrow{X}(\overrightarrow{x},t)}{\partial t} + rot\ \overrightarrow{A}(\overrightarrow{x},t) \tag{10}$$

where we have introduced two new gauge fields, a scalar field $\Psi(\overrightarrow{x},t)$ and a vector one $\overrightarrow{X}(\overrightarrow{x},t)$. For dimensional reason we introduced also a dimensional parameter λ the meaning of which will be clear soon. The fields represented by (9) and (10), contrary to those in (1) and (2), do not "solve" the first two Maxwell equations

$$rot\ \overrightarrow{E}(\overrightarrow{x},t) = -\frac{\partial \overrightarrow{B}(\overrightarrow{x},t)}{\partial t} \tag{11}$$

and

$$div\ \overrightarrow{B}(\overrightarrow{x},t) = 0. \tag{12}$$

Indeed, substituting the representations (9) and (10) into these equations we get the equations

$$\frac{\partial^2 \Psi(\overrightarrow{x},t)}{\partial t^2} + \lambda \bigtriangleup \Psi(\overrightarrow{x},t) = 0, \tag{13}$$

$$\frac{\partial^2 \overrightarrow{X}(\overrightarrow{x},t)}{\partial t^2} + \lambda \triangle \overrightarrow{X}(\overrightarrow{x},t) = 0 \tag{14}$$

where on the basis of the non uniqueness of all the potentials we adopted the gauge condition for the gauge fields in the form

$$\frac{\partial \Psi(\overrightarrow{x},t)}{\partial t} - \lambda \, div \, \overrightarrow{X}(\overrightarrow{x},t) = 0. \tag{15}$$

Equations (13) and (14) deserve some discussion. The easiest way to remove all problems connected with them is to assume, as it is usually done, that all the new gauge fields vanish. But this is not a complete resolution of the problem because it does not allow to see that in some situations the new gauge fields may play a very important physical role.

First of all we should notice that, except the case $\lambda = -c^2$, none of the equations (13) - (15) is Lorentz covariant. Therefore the new gauge fields can manifest their significance only in phenomena which by their very nature are non relativistic.

Second, let as substitute the new representations into the remaining Maxwell equations

$$rot \, \overrightarrow{H}(\overrightarrow{x},t) = \frac{\partial \overrightarrow{D}(\overrightarrow{x},t)}{\partial t} + \overrightarrow{j}(\overrightarrow{x},t) \tag{16}$$

and

$$div \, \overrightarrow{D}(\overrightarrow{x},t) = \rho(\overrightarrow{x},t). \tag{17}$$

Using the standard most general constitutive (or material) relations

$$\overrightarrow{D}(\overrightarrow{x},t) = \epsilon_0 \, \overrightarrow{E}(\overrightarrow{x},t) + \overrightarrow{P}(\overrightarrow{x},t) \tag{18}$$

and

$$\overrightarrow{H}(\overrightarrow{x},t) = \frac{1}{\mu_0} \, \overrightarrow{B}(\overrightarrow{x},t) - \overrightarrow{M}(\overrightarrow{x},t) \tag{19}$$

with some polarization and magnetization fields, instead of (3) and (4) we get the following field equations

$$\Box \Phi(\overrightarrow{x},t) = \frac{\rho(\overrightarrow{x},t) - div \, \overrightarrow{P}(\overrightarrow{x},t)}{\epsilon_0}, \tag{20}$$

and

$$\Box \, \overrightarrow{A}(\overrightarrow{x},t) = \mu_0 \left[\overrightarrow{j}(\overrightarrow{x},t) + rot \, \overrightarrow{M}'(\overrightarrow{x},t) + \frac{\partial \, \overrightarrow{P}(\overrightarrow{x},t)}{\partial t} \right] \tag{21}$$

where

$$\overrightarrow{M}'(\overrightarrow{x},t) = \overrightarrow{M}(\overrightarrow{x},t) + \frac{1}{\mu_0} \left(1 + \frac{\lambda}{c^2} \right) \overrightarrow{X}(\overrightarrow{x},t). \tag{22}$$

For the choice $\lambda = -c^2$ the new gauge fields do not produce any physical effect but for different choices they influence the magnetization field of the medium. The shape of the magnetization field is however a directly measurable quantity and we must conclude that the new gauge fields $\overrightarrow{X}(\overrightarrow{x},t)$ have a direct physical meaning. The gauge condition (15) says that the scalar gauge field $\Psi(\overrightarrow{x},t)$ is related to the new vector field $\overrightarrow{X}(\overrightarrow{x},t)$. The parts of the magnetic field $\overrightarrow{B}(\overrightarrow{x},t)$ determined by the gauge fields $\Psi(\overrightarrow{x},t)$ and $\overrightarrow{X}(\overrightarrow{x},t)$ have different properties than the other parts have. Since any relation between parts of a physical field $\overrightarrow{B}(\overrightarrow{x},t)$ must be treated as a physical law any way to relate the new gauge fields to the old one must be considered as a new physical law. Since also the gauge condition (15) is a relation between different part of the magnetic field it expresses therefore some physical law for the magnetic properties of the medium.

The new gauge fields are completely determined by their field equations (13) and (14). These equations have a hyperbolic character only for negative values of the parameter λ . In such case the square of the propagation velocity of any disturbance described by the new gauge fields is given by $-\lambda$. The velocity of the disturbance described by the new gauge fields need not to be however less than the velocity of light. If the propagation velocity is less than the velocity of light, we clearly have to do with a normal situation in which all disturbances propagate with some physically allowed velocities.

It is however easy to see that it is also possible to have disturbances described by the new gauge fields which propagate with velocities larger than the velocity of light in vacuum ! To understand this let us remind that in any medium the velocity of electromagnetic signals is always proportional to the inverse of the dielectric constant of the medium. In order to have in some medium velocities larger than c we must admit that the dielectric constant of such a medium is less than the dielectric constant of the vacuum. We have recently shown [4] that the experimental data on the charge, mass and radius of the electron indeed predict that the dielectric constant ϵ of the medium inside the electron is of the order $10^{-7}\epsilon_0$. This implies that the velocity of signal propagation in such a medium must be of the order $10^{+7}c^2$. The standard tachyons may therefore exist only inside elementary particles ! We should also take into account that quick transmission of signals may mean quick transmission of interaction and this usually means that the interaction is strong. Is this indeed so that strong interactions are the electromagnetic ones but occurring in non standard media ?

The above scheme may be generalized by replacing in (10) the new gauge fields by superpositions of many fields $\Psi_k(\overrightarrow{x},t)$ and $\overrightarrow{X}_k(\overrightarrow{x},t)$with the new term in (9) of the form

$$\sum \lambda_k \overrightarrow{X}_k(\overrightarrow{x},t)\,. \tag{23}$$

This leads to theories with many signals each of which propagates with its own velocity.

3. Some general remarks

It is a common belief that all physically valid models of Maxwell electrodynamics should have their Lorentz covariant realizations. It is however easy to argue that this belief is not sufficiently founded. Indeed, the maximal symmetry groups of Maxwell equations[5] are very large. These symmetries are different for the pair of fields $\overrightarrow{E}(\overrightarrow{x},t)$ and $\overrightarrow{B}(\overrightarrow{x},t)$ and for the pair of fields $\overrightarrow{D}(\overrightarrow{x},t)$ and $\overrightarrow{H}(\overrightarrow{x},t)$. The maximal symmetries are reduced to lower symmetries by the material equations which connect these pairs of fields. The shapes of the material equations depend on the properties of the material bulks and on the parameters of the fields like, for example, frequency. The electromagnetic properties of some matter bulks, contrary to the general belief, may depend on the choice of the reference frames because in different reference frames the ranges of field frequencies are different. The frequency dependence of material equations was not known at the beginning of our century when the present shape of electrodynamics was finally accepted. Now the variety of known materials is much larger and there is no fundamental reason to restrict fundamental research to the commonly accepted narrow interpretation of Maxwell electrodynamics. Therefore it is physically reasonable to consider nonrelativistic and relativistic models of Maxwell electrodynamics as well. By nonrelativistic we mean here theories in which fundamental signals travel with velocities both less and greater than the velocity of light. Such theories obviously cannot be invariant under Lorentz transformations.

4. Conclusions

We have shown that Maxwell electrodynamics is very rich in gauge fields. By no means the discussion of gauge fields and gauge invariance should be restricted to its customary range.

References

[1] T-P. Cheng and L-F. Li; *Gauge Theory of Elementary Particle Physics*, Clarendon Press-Oxford, 1984

[2] See, for example, the standard textbook by J. D. Jackson; *Classical Electrodynamics*, John Wiley and Sons Inc., N.Y., 1962

[3] The best discussion of this duality can be found in the book: V. I. Striazhev; *Polarization Symmetry*, (in Russian) Minsk, 1996

[4] E. Kapuścik; *Foundations of Physics* **28**, 717-727 (1998)

[5] V. I. Fushchich and A. G. Nikitin; *Symmetry of Maxwell Equations* (in Russian), Kiev, 1983