Communication Complexity and Coordination by Authority*

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Abstract

We prove that the simplest communication allowing two players to coordinate on a course of action is authority (letting one player choose an action). We also consider the case where each player possesses valuable information about the benefits of a large number of actions. For this case, we identify conditions under which authority can only be asymptotically improved upon by protocols of exponential complexity in the number of actions (i.e. those describing an unbounded number of actions).

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"Fundamentally, communication is required to translate purpose into terms of the concrete action required to effect it - what to do and when and where to do it [...] Under very simple and usually temporary conditions and with small numbers of persons the communication problem often appears simple, but under many conditions, even with small numbers, a special channel of communication is required. For if all talk at once there is confusion, and there is indecision particularly as to timing of actions. This creates the necessity for a leader."

Chester Barnard (1938)

1 Introduction

Before the 1840s, modern industrial enterprise administered by a set of managers did not exist in the United States. Even though production had grown increasingly specialized, coordination was achieved mostly by the "invisible hand" of the market mechanism. As the volume and speed of economic activity grew and technology permitted many production activities to be carried on simultaneously within a single technological unit, however, administrative coordination became increasingly important. As Chandler (1977) writes, "in all these new enterprises - the railroads, the telegraph, the mass marketers, and the mass producers - a managerial hierarchy had to be created to supervise several operating units and to coordinate and monitor their activities."

Consider, for example, the revolution in American transportation which occurred in the middle of the last century. Before the advent of railroads, "the enterprises that constructed and maintained the canals and turnpikes rarely operated the canal boat companies, stage lines, or mail routes that used them" (Chandler, 1977). On the railroad, however, safety and efficiency required "careful coordination of the movements of trains and the flow of traffic". This coordination could only be achieved by administrative means, by authority of the newly created class of professional managers. This brought about "the first modern, carefully designed internal organizational structure used by an American business enterprise".

It is clear that technological changes played an important role in inducing the organizational changes. However, as Williamson (1985) points out, technological parameters by themselves do not determine organizational form. For example, in theory trains could be owned independently, and the owners could coordinate their schedules by a market mechanism.

Determination of organizational form has been largely unexplained by economic theory. In an early economic analysis of organizations, Simon (1951) introduced economists to the concept of authority as an important feature of administrative organizations:

> "We will say that B[oss] exercises *authority* over W[orker] if W permits B to select [his behavior] x. That is, W accepts authority when his behavior is determined by B's decision."

This concept has been later used by Grossman and Hart (1986) and the subsequent literature on incomplete contracts which models firms as clusters of authority in general, and control rights over assets in particular. However, none of these contributions has formally explained why organizations so often make decisions by authority, rather than using other decision mechanisms. Indeed, incomplete contracts models usually admit only slightly more complicated mechanisms which achieve significant improvements upon authority.

This paper puts forward a formal explanation of authority as the predominant decision mechanism in organizations. This explanation is based on the idea put forward by Barnard (1938), Arrow (1974), Chandler (1977), and, more recently, Milgrom and Roberts (1992), that administrative authority is an efficient way of coordinating activities of many persons in complex environments with limited communication opportunities (see e.g. the opening citation). In order to develop this idea formally, we consider a model where two individuals need to coordinate on a course of action. The necessity of coordination may, for example, come from the two individuals' playing an $n \times n$ coordination game with a very high payoff to coordination. (This payoff can be interpreted as the benefit from avoiding collision on a railroad or explosion in a factory.) Following Crawford and Haller (1990), we assume that coordination is difficult because the parties do not share a labeling of actions which could provide them with a focal point.

Standard economic theory usually dismisses the coordination problem on the grounds that if players can talk *long enough* before the game starts, they will eventually agree on an action. To avoid this criticism, Crawford and Haller restrict attention to the case where pre-play communication is impossible, and coordination can only be achieved by repeated play. We are instead interested in analyzing pre-play communication. Of course, in absence of barriers to communication, coordination is not a problem. For example, the players can eventually publicly label all the actions, and base their behavior on this common labeling. However, if the number of possible actions is large, this communication would take extremely long. The question we ask, therefore, is, *how long is 'long enough'*? More precisely, how can coordination be achieved with a minimum amount of communication?

To address this question, we apply a computer science notion of communication complexity (Yao (1979), Karchmer (1989), Marschak (1995)) to measure the complexity of coordinating communication. Our first result shows that the simplest way to coordinate is by giving one of the players the authority to specify what action to take. For example, consider two people who need to lift a log. To do this, they need to apply their efforts simultaneously to both ends of the log, otherwise efforts would be wasted. The simplest way to achieve such coordination is to give one person the authority to say "up".[1,2]

When authority serves a purely coordinating purpose, the allocation of authority is not important, as long as the bearer of authority is commonly known. Authority could be delegated to any participant of the coordination game, to an outsider, or even to a publicly observed mechanical device. For example, two cars at an intersection may be coordinated by either driver, by a policeman, or by a traffic light. To extend our model to the more interesting situations where allocation of

[1] Unless some commonly observed characteritics of the two individuals (position, seniority, etc.) establish a focal allocation of authority, the parties may need to communicate in advance in order to coordinate on the source of authority. In the words of Barnard (1938), "every communication must be authenticated", which means that "the person communicating must be known to actually occupy the 'position of authority' concerned." Since there are only two possible authority allocations in the example, coordinating on such an allocation is bound to be much simpler than coordinating on when to lift the log.

[2] For more examples of coordination by authority, see Milgrom and Roberts (1992, ch.4).

authority is important, we postulate that the parties possess private information about the benefits of different actions. In this case, an outsider or a mechanical device may be unlikely to pick a good action. Authority should optimally be allocated to the party which has the best information to make the decision.[3]

When different individuals possess different information, authority mechanisms have an obvious disadvantage: they only utilize the information available to one of the players. For example, a traffic policeman does not bother to find out the destinations and time preferences of the cars he coordinates, which may result in certain inefficiencies. As a general matter, if the parties could pool all their information costlessly, they would arrive at a better decision than that achieved by authority. However, as has been emphasized by Hayek (1945), full revelation of all the information relevant to a particular decision may require prohibitively extensive communication. According to our first result, authority mechanisms achieve coordination in the simplest way possible, by describing just one action. Our second result identifies conditions under which improvement upon authority requires communication protocols of exponential complexity in the number of possible actions, i.e. those which asymptotically describe infinitely many actions.

One reason communication is costly is because of associated delays. When parties communicate using a channel with a finite bandwidth (transmission rate), a bound on communication complexity is essentially a bound on communication time. Under this interpretation, our results make a strong case for using authority in complicated situations requiring quick decision making, where time limitations put a severe constraint on the amount of communication. The leading examples include emergency, combat, and operating room. In these situations, a clear line of authority is a prerequisite for success.

However, we think that a model of communication costs can also be used to explain the role of formal authority in situations where decisions are not urgent and informal communication is

[3] Barnard (1938) uses the term *authority of position* for the form of authority which is "to a considerable extent independent of the personal ability of the incumbent of the position", and the term *authority of leadership* for the authority of individuals "whose knowledge and understanding regardless of position command respect". For example, a policeman substituting for a broken traffic light exercises authority of position. But when the same policeman performs a more creative coordinating job, utilizing his knowledge of the location or traffic conditions, his authority becomes that of leadership.

extensive. Indeed, whenever parties' interests diverge, formal (contractual) communication verifiable by a court has the additional benefit of committing the parties to act in certain ways on the basis of messages they receive. When formal communication is much more costly than informal, an organization may economize on these costs by writing simple contracts which allocate formal authority over various decisions, while using extensive informal communication to complement the formal mechanisms. We believe that an extension of our model along these lines can prove useful for analyzing organizations as incomplete contracts.

The rest of the paper is organized as follows. Section 2 discusses the relation of our approach to existing literature. Section 3 formalizes the notion of coordinating communication, and proves that the simplest form of such communication is authority. Section 4 shows that under certain conditions, improvement upon authority requires exponential communication. Section 5 discusses how our analysis can be extended to incorporate the distinction between formal and informal communication. Section 6 concludes. A non-technical reader is recommended to skip the subsections marked with an asterisk (*) in the first reading.

2 Related Literature

Our approach is closely related to the team-theoretic literature which attempts to explain economic organization with the costs of information processing, including those of communication. However, most papers in this literature either do not model communication costs at all, or model them as fixed costs of transmitting pieces of information (Radner (1993), Bolton and Dewatripont (1994), Marschak and Reichelstein (1995)). Team-theoretic explanations of hierarchies can be traced back to Arrow's (1974) observation that when the number of agents is large, centralization yields substantial savings on communication costs.

As noted by Geanokoplos and Milgrom (1991), in the traditional team-theoretic models with a fixed cost of communication "there is no role for 'instructions' from any manager to any other. That is, at an optimum, a superior may communicate information to his subordinate, but he never limits the set of actions that the subordinate may undertake [...]". This stands in stark contrast to

Arrow's (1974) observation that in presence of communication costs " it may be cheaper for a central individual or office to make the collective decision and transmit it rather than retransmit all the information on which the decision is based". In this view, complexity of the communication problem stems not from the number of agents, but from the complexity of environment and the richness of the agents' knowledge (see also the opening citation). In order to formalize this view, we need to allow for the agents' communicating their information piecemeal, and to measure the amount of such communication in a more refined way than it has been usually done in team theory. For this purpose, we adopt the computer science definition of communication complexity which is roughly equivalent to counting the number of bits transmitted between the two players (see Section 3.5). This definition captures the huge difference between fully revealing one's knowledge and sending an 'instruction' on the basis of this knowledge.

3 The Coordination Problem

3.1 Modeling Coordination

Consider a situation where two individuals - player 1 and player 2 - need to coordinate on an action from a set A. To model the coordination problem, we need to postulate that the players do not have a focal action or group of actions. Following Crawford and Haller (1990), we do this by assuming that the players do not share an ordering of A. Indeed, if the players had a common ordering, it could provide them with focal points: they would be able to coordinate on, say, "action number one" without any communication.

At the same time, our players ought to have a common language to describe the actions. If they did not, they would not be able to communicate at all and the coordination problem could not be solved. Therefore, we would like to model a situation where the players do have a common language, but this language does not provide them with a common ordering. To fix ideas, suppose the players need to coordinate on one of three colors: *Red*, *Blue*, and *Green*: $A = \{R, B, G\}$. The two players have a *common language* over the set of actions - both players know what the words *Red*, *Blue*, and *Green* mean. However, the players do not have a *common ordering (dictionary)* of these three words.[4]

Absence of a common dictionary not only rules out tacit coordination, but also imposes a serious constraint on how players can communicate. Consider, for example, the following communication games:

Game 1. Player 1 *describes* an action, which is then undertaken.

Game 2. Player 2 *describes* an action. Player 2 *describes* another action, which is then undertaken.

In order for the players to understand each of the two communication games above, they need not have a focal action or group of actions. The following two communication games, in contrast, cannot be understood by the players unless they already have a focal action:

Game 3. Action R is chosen without any communication.

Game 4. Player 1 says "yes" or "no". If player 1 says "yes", action R is undertaken. Otherwise, player 2 chooses between actions G and B.

In Game 3, action R is undertaken without being described first. This is clearly impossible unless action R was focal to begin with. In Game 4, player 2 is supposed to understand that player 1's "yes" refers to action R, rather than to G or B. Player 2 can only understand this if action R has a focal significance. But in that case, the two players could have coordinated on "the action that player 1 can impose" without actually performing any communication! On the other hand, if action R has no focal significance, player 1 must make his message clearer: for example, instead of "yes" he must say "R", and instead of "no" he must say "not R". But since player 2 will have no prior idea what action player 1 will be talking about, he will expect one of six possible messages: "R","not R", "B","not B", "G", and "not G". But this basically means that player 1 actually has to *describe* the action he is talking about, which requires more extensive communication than that performed in Game 4.

[4] If both players are literate and know the alphabet, they could possibly coordinate on the alphabetic ordering BGR, and tacitly agree to choose the "first" action, B. We essentially assume that the players do not know the alphabet (they are illiterate), or that they do not understand how alphabetical ordering can be used for coordination.

Another way to understand the fundamental difference between Communication Games 1 and 2 on the one hand, and Games 3 and 4 on the other, is to think of the players trying to agree on a communication game in advance, when they do not yet know the names of actions.[5] They will have no trouble establishing common knowledge of Communication Games 1 and 2 without using the names of actions. This is because these two communication games treat all the actions symmetrically. On the other hand, it is impossible to explain Games 3 and 4 without using the names of actions. This is because Games 3 and 4 do not treat all the actions symmetrically; in particular, action R has a focal significance in both games.

Intuitively, if a communication protocol which treats all actions symmetrically is ever to achieve coordination on an action, the action's name will have to be publicly announced - i.e., the action will have to be *described* by a player. This intuition suggests that the simplest coordinating communication is one where one player tells the other which action to choose (as in Game 1). We will call such communication protocols "authority protocols". In the rest of this section, we formally define what it means for a protocol to treat all actions symmetrically. Using a computer science notion of communication complexity, we prove that the simplest such protocol achieving coordination is an authority protocol.

3.2 *Dictionaries

Suppose that the two players want to coordinate on an action from a finite set A, and that they cannot base their behavior on the names of actions. At the same time, each player can privately order the actions and base his behavior on this private ordering.[6] We will call a player's private numbering of actions his *dictionary.* Formally, a dictionary could be represented as a one-to-one map from the set of numbers $\{1, ..., |A|\}$ which a player uses to rank actions onto A, the set of actions.

[5] This setup is similar to that of Maskin and Tirole (1995), who consider designing a contract when future actions are not foreseen. The notion of welfare-neutrality which they use to capture this "unforeseebility" is analogous to the notion of neutrality introduced in subsection 3.4 below. For more on the relationship between our setup and that of Maskin and Tirole, see Conclusion.

[6] He can orders actions randomly or on the basis of some privately observed characteristics. These characteristics may be payoff-relevant (as they will be in Section 4), but they do not have to.

For example, if $A = \{R, B, G\}$, a dictionary d could have G as the first action, B as the second, and R as the third, in which case we will write $d = GBR$, and $d(1) = G$, $d(2) = B$, $d(3) = R$. Define the set of all possible dictionaries on A by $\mathbf{D}_A$. Observe that $|\mathbf{D}_A| = |A|!$.

The two players have two different dictionaries d_1, d_2 - two different orderings of the same set of actions, and each player's dictionary is not known to the other player. In order to achieve coordination, the players need to communicate some information about their dictionaries.

3.3 *Communication Protocols

Suppose that player i's private information takes the values from the set Θ_i, $i = 1, 2$, and that A is the set of possible outcomes. (For modeling the coordination problem, we will be interested in the case where each player's private information is his dictionary, i.e. $\Theta_1 = \Theta_2 = \mathbf{D}_A$.) The players communicate in order to implement an outcome contingent on their private information. As in the standard mechanism design setup, we allow the players to communicate using an arbitrary extensive-form mechanism (game form, message game). It will be convenient to adopt a computer science definition of a communication protocol, which consists of an extensive-form game form and of the two players' strategies in this game form:

Definition. A communication protocol P on $\langle A, \Theta_1, \Theta_2 \rangle$ consists of an extensive game form with outcomes from A, and of the two players' strategies defined in the standard game theoretic sense (Player i's strategy specifies his moves as a function of his private information from Θ_i and his information set.)[7]

Observe that when incentive-compatibility is not an issue (and it will not be in our formal analysis), without loss of generality one could restrict attention to protocols using games of perfect information, where players' moves may depend on all the history.

[7] In order to follow a communication protocol, the players need to have common knowledge of the protocol. Assuming this common knowledge in a coordination problem may be problematic. However, ruling out coordination on a protocol would trigger an infinite regress (how to coordinate how to coordinate how to ...). Coordination on a protocol can result from the players' having been in similar coordination problems before (but with different sets of actions). Also, when the protocol takes the form of authority, coordinating on the source of authority may be easier than coordinating on action (see footnote 2).

As a result of the players' communication, an action from A will be chosen. This action will be a function of both players' private information, and we will call this function $f : \Theta_1 \times \Theta_2 \to A$ *the choice rule*. We will say that the choice rule f is *implemented* by the protocol P.

A useful way to represent communication protocols is by associating each terminal node of the corresponding game form with a subset of $\Theta_1 \times \Theta_2$ containing all the states of the world in which this terminal node is achieved. Given the extensive game form and the players' strategies, $\Theta_1 \times \Theta_2$ can be partitioned into such sets corresponding to the terminal nodes of the game. It can be easily seen that these sets have a special structure: each such set $T \subset \Theta_1 \times \Theta_2$ is a "rectangle" $T_1 \times T_2$ with the choice rule f constant on it. To see this, suppose that player 1 moves first. Since his move cannot depend on player 2's private information, it will partition $\Theta_1 \times \Theta_2$ horizontally into several rectangular subsets (events), and from this move player 2 can infer which of the events has occurred. In his turn, player 2 partitions the received rectangular subset vertically, and so it continues until a terminal node is reached. This simple inductive argument shows that the subset of $\Theta_1 \times \Theta_2$ containing all the states of the world in which a terminal node is achieved has to be rectangular. Moreover, since the node is terminal and there is no more communication, the choice rule f has to be constant on every such subset. Karchmer (1989) calls such subsets "monochromatic rectangles".[8]

Note that a monochromatic rectangle does not have to look like a geometric rectangle, since the rows and columns of $\Theta_1 \times \Theta_2$ need not be adjacent. While any given monochromatic rectangle can be transformed into a geometric rectangle with appropriate permutations of rows and columns of $\Theta_1 \times \Theta_2$, it is not always possible to find permutations which represent all monochromatic rectangles as geometric rectangles.

3.4 *Coordinating Communication

To model the coordination problem, we assume that each player's private information is described by his dictionary on the finite set A of actions, i.e. $\Theta_1 = \Theta_2 = \mathbf{D}_A$. We will assume that the two

[8] While for every communication protocol we have a corresponding partition of $\Theta_1 \times \Theta_2$ into monochromatic rectangles, not every such partition represents a deterministic protocol - see Marschak (1995).

players cannot coordinate on an action on the basis of its name. Therefore, an action will have to be chosen on the basis of its position in the two players' dictionaries. As a result of the players' communication, they reveal enough information about their dictionaries to establish a common understanding of which action to take. Thus, the choice rule f specifies the action $f(d_1, d_2)$ the players choose when their dictionaries are $d_1, d_2 \in \mathbf{D}_A$.

The following examples with $A = \{G, B, R\}$ present four protocols whose extensive-form communication games correspond to the games discussed in Subsection 3.1:

Example 1 *Player 1 specifies the action $d_1(1)$, which is implemented.*

Example 2 *Player 1 specifies $a = d_1(1)$. If $d_2(1) = a$, player two specifies a. Otherwise, player 2 specifies $d_2(2)$.*

Example 3 *The action R is implemented without any communication.*

Example 4 *If $d_1(1) = 1$, player 1 says "yes", and action R is implemented. Otherwise, player 1 says "no", and player 2 chooses from the remaining two actions the one which comes first in his dictionary (i.e. $\arg\min_{a \in \{2,3\}} d_2^{-1}(a)$).*

Extensive-form games and communication partitions corresponding to these protocols are depicted in Figures 1-4 respectively. The protocol in Example 1 simply lets player 1 choose an action. Player 1 chooses the action which comes first in his dictionary. In Example 2, player 1 describes the action which comes first in his dictionary. Player 2 agrees if this action also comes first in his dictionary, otherwise player 2 describes the action which comes second in his dictionary. In Example 3, action R is chosen without any communication. In Example 4, player 1 can either insist on implementing action R or let player 2 choose between B and G. Player 1 insists on R only if this action comes first in his dictionary.

As discussed above, the fundamental difference between Examples 1 and 2 on one hand, and Examples 3 and 4 on the other, is that the former protocols treat all the actions symmetrically, while the latter do not. More precisely, in Examples 1,2 an action is never chosen on the basis of its name, while in Examples 3,4 an action is sometimes chosen just because its name is R.

To formalize this idea, we will consider the set of $\mathbf{S}_A$ of all permutations of the finite set A. This set can be treated as a group, with multiplication of permutations naturally defined as their composition. The unit element e of this group is the permutation keeping all elements of A constant. This group is called the symmetric group of degree $|A|$ and is denoted by $\mathbf{S}_A$; it contains $|A|!$ elements, and it is non-commutative for $|A| \geq 3$ (Kargapolov and Merzljakov, 1979).[9]

Consider an arbitrary permutation $\pi \in \mathbf{S}_A$. This permutation can be interpreted as a change of a language for describing the same physical actions. A physical action whose name used to be $a \in A$ in the old language obtains the name $\pi(a) \in A$ in the new language. If the name of the physical action occupying position i in player 1's dictionary used to be $d_1(i)$, now the same physical action is called $\pi(d_1(i)) = \pi d_1(i)$, where πd_1 is defined as the composition of π and d_1. Therefore, the two players' dictionaries are expressed in the new language as πd_1, πd_2.

The choice rule f prescribes that for these dictionaries, action $f(\pi d_1, \pi d_2)$ in the new language should be chosen. We want to require that this is the same physical action as the action $f(d_1, d_2)$ in the old language, whose name in the new language is $\pi f(d_1, d_2)$:

Definition. A choice rule $f : \mathbf{D}_A \times \mathbf{D}_A \to A$ is *neutral* if $f(\pi d_1, \pi d_2) = \pi f(d_1, d_2)$ for every $d_1, d_2 \in \mathbf{D}_A, \pi \in \mathbf{S}_A$.

This definition says that the physical outcome does not depend on the names of actions, but only on their positions in the two players' dictionaries. This is nothing else but the property of neutrality among alternatives, which may be familiar from the social choice context. This property would be satisfied, for example, when f chooses an action yielding the highest payoff and the names are payoff-irrelevant. In a world of boundless communication, therefore, the imposition of neutrality does not constrain payoffs. It will thus be surprising to find that when communication is bounded,

[9]Formally, $\mathbf{D}_A$, the set of all dictionaries on A, is isomorphic to $\mathbf{S}_A$. This becomes clear once we let actions' names be numbers: $A = \{1, ..., n\}$. However, for expositional purposes we prefer not to think of actions' names as being numbers.

the neutrality requirement becomes a serious constraint not only on protocols used (Section 3), but also on payoffs achieved by these protocols (Section 4)!

Consider again Examples 1-4. By inspecting Figures 1-4, one can convince himself that the choice rules in Examples 1,2 are neutral, while the those in Examples 3,4 are not. Intuitively, in Examples 1,2 an action is picked on the basis of its place in the two players' dictionaries, and not because of its name. The choice rules in Examples 3,4, on the other hand, are not neutral since, for example, switching the names of actions R and B affects the physical action chosen.

Definition. A protocol P on $\langle A, \mathbf{D}_A, \mathbf{D}_A \rangle$ is *coordinating* if its choice rule is neutral.

3.5 *Communication Complexity

To measure the amount of communication we will use the notion of tree complexity from Karchmer (1989):

Definition. The tree complexity of a protocol P, $\Gamma(P)$, is the number of terminal nodes in the corresponding extensive game form, or, equivalently, the cardinality of the corresponding communication partition.

This definition will prove very convenient for our purposes. However, to give some perspective and motivation for this definition, we will point out its relationship to the widely used notion of "worst-case complexity" (Karchmer, 1989). This notion is defined for so-called binary protocols where players' moves consist of sending bits to each other (but clearly, every protocol can be approximated by a binary protocol implementing the same choice rule):

Definition. The worst-case complexity of a binary protocol is the maximum over $\Theta_1 \times \Theta_2$ of the number of bits transmitted between the players.

In other words, the worst-case complexity of a binary protocol is the (maximum) depth of the corresponding game tree. It is easy to see that the tree complexity of a binary protocol, which is the number of terminal nodes of the binary tree, is bounded from above by two to the power of its depth.

On the other hand, there exist very "unbalanced" binary protocols whose tree complexity can be as low as their depth plus one. It turns out, however, that for a given choice rule such unbalanced protocols need not be used, and that tree complexity and worst-case complexity are closely related when we look at minimum complexity protocols implementing a given choice rule:

Definition. The tree [worst-case] complexity of a choice rule f, which we denote by $C_\Gamma(f)$ [$C_W(f)$], is the minimum tree [worst-case] complexity of a [binary] protocol P implementing f.

The following fact has been established by Karchmer (1989):

Fact. $\log C_\Gamma(f) = O(C_W(f))$ and $C_W(f) = O(\log C_\Gamma(f))$ for any choice rule f in any environment.

In words, the worst-case complexity and the log of tree complexity of any choice rule are within constants of each other. For simplicity, all our results will be formulated for tree complexity. But it is useful to keep in mind that a bound on tree complexity is equivalent to a bound on the number of bits the parties exchange. Suppose, for example, that the parties communicate using a channel with a finite bandwidth.. Then a bound on tree complexity is equivalent to a bound on the amount of time the parties can spend communicating before reaching a decision.

3.6 *Authority as the Simplest Coordinating Protocol

A lower bound on communication complexity required to achieve coordination is given by the following lemma:

Lemma 1 *Every coordinating protocol P on $\langle A, \mathbf{D}_A, \mathbf{D}_A \rangle$ satisfies $\Gamma(P) \geq |A|$.*

Proof. Pick any dictionary $d \in \mathbf{D}_A$. Let f be the choice rule of P, and let $f(d,d) = i$. Since f is neutral, we must have $f((ij)d, (ij)d) = (ij)f(d,d) = (ij)i = j$ for any $j \in A$. Therefore, $((ij)d, (ij)d)$ all belong to different monochromatic rectangles for different $j \in A$, and the communication partition of P has at least $|A|$ elements. ■

The intuition behind Lemma 1 is very simple. Since a coordinating protocol is neutral, all actions should be a priori equally likely to be chosen. Therefore, a coordinating protocol should have at least $|A|$ possible outcomes.

This lower bound on complexity of coordinating communication can be achieved by protocols which we will call authority protocols. We define an authority protocol on $\langle A, \mathbf{D}_A, \mathbf{D}_A \rangle$ as a protocol where, for some $i = 1, 2$, player i specifies an action on the basis of its position in his dictionary. If player i specifies the action which comes kth in his dictionary, this will be action $d_i(k)$.

Definition. A protocol P on $\langle A, \mathbf{D}_A, \mathbf{D}_A \rangle$ is an authority protocol if for some $i = 1, 2$ and for some $k \in \{1, ..., |A|\}$ its choice rule is $f(d_1, d_2) = d_i(k)$, and its communication partition is $\{\{(d_1, d_2) \in \mathbf{D}_A \times \mathbf{D}_A |\ d_i(k) = a\} |\ a \in A\}$.

The protocol in Example 1 is an authority protocol, while the protocols in Examples 2-4 are not. Authority protocols are coordinating: $f(\pi d_1, \pi d_2) = \pi d_i(k) = \pi f(d_1, d_2)$, and it is easy to see that authority protocols have complexity $|A|$. The non-authority protocol in Example 2 is coordinating, but its complexity is 9, which is greater than $|A| = 3$. The protocol in Example 3 has complexity 1, and the protocol in Example 4 has complexity $|A| = 3$, but both these protocols are not coordinating. Our first theorem establishes that this is unavoidable: every coordinating protocol of complexity $|A|$ is an authority protocol.

Theorem 1 *If P is a coordinating protocol on $\langle A, \mathbf{D}_A, \mathbf{D}_A \rangle$ and $\Gamma(P) = |A|$, then P is an authority protocol.*

Proof. See the Appendix.

This result says that the simplest way to coordinate is by authority, but it has nothing to say about comparing different allocations of authority. Coordination can be achieved by giving authority to player 1, player 2, an outsider, or a mechanical device. In practice, however, allocation of authority is very important, because different partied have different information about the benefits of different actions. This situation is modeled in the next section.

4 Coordination and Information Transmission

4.1 An Informal Argument

In the previous section, communication served a purely coordinating role. The two players possessed the same information about possible actions, even though they used different dictionaries to order them. In this section we consider a situation where the two players possess different information about the benefits of different actions, and ideally an action should be chosen on the basis of both players' information. However, when the set of actions A is large, the optimal choice of action may require extensive communication.

Let $|A| = n$. Suppose that player 1 privately knows a subset of k_1^n actions which are better than others, and that player 2 privately knows a subset of k_2^n actions which are better than others. Suppose that all the actions are a priori equally likely to be good for either player, and that the two players' subsets are independently realized. Suppose furthermore that $k_1^n/n \to 0$ and $k_2^n/n \to 0$ as $n \to \infty$. This guarantees that for a large n, an outsider is very unlikely to pick an action which is good from either player's viewpoint. It is clearly better to give authority to one of the players - at least he will optimally choose an action on the basis of his own information.

Ideally, however, an action should be chosen using both players' information. In order to do this, the players need to find an "intersection" - an action which is good for both players. But will this intersection exist with a positive probability? The probability of intersection can be computed as

$$1 - \binom{n-k_1}{k_2} / \binom{n}{k_2} = 1 - \frac{(n-k_1)!(n-k_2)!}{n!(n-k_1-k_2)!}.$$

Using Stirling's formula $N! \sim N^N e^{-N}\sqrt{2\pi N}$, this probability can be asymptotically expressed as

$$\begin{aligned}
\lim_{n\to\infty}[1 - \frac{(n-k_1)!(n-k_2)!}{n!(n-k_1-k_2)!}] &= \lim_{n\to\infty}[1 - \frac{(n-k_1)^{n-k_1}(n-k_2)^{n-k_2}}{n^n(n-k_1-k_2)^{n-k_1-k_2}}\sqrt{\frac{(n-k_1)(n-k_2)}{n(n-k_1-k_2)}}] = \\
&= 1 - \lim_{n\to\infty}\frac{(1-k_1/n)^{n(1-k_1/n)}(1-k_2/n)^{n(1-k_2/n)}}{(1-(k_1+k_2)/n)^{n(1-(k_1+k_2)/n)}} = \\
&= 1 - \lim_{n\to\infty}\frac{\exp\{-k_1(1-k_1/n)\exp\{-k_2(1-k_2/n)\}}{\exp\{-(k_1+k_2)(1-(k_1+k_2)/n)\}} =
\end{aligned}$$

$$= \quad 1 - \lim_{n\to\infty} \exp\{k_1^2/n^2 + k_2^2/n^2 - (k_1+k_2)^2/n\} =$$
$$= \quad 1 - \lim_{n\to\infty} \exp\{-2k_1k_2/n\},$$

provided that the last limit exists. Therefore, as long as the proportion of relevant actions for each player does not converge to zero too quickly (e.g. not faster than $n^{-1/2}$), the probability of intersection may be bounded away from zero and may even converge to one. Note that a necessary condition for this is that both k_1^n and k_2^n are unbounded.

Finding an intersection when it exists requires extensive communication. Indeed, when a player describes an action which is good for him, the probability that the described action is also good for the other player goes to zero as $n \to \infty$. The same would be true if a player described a fixed finite number of actions. Therefore, in order to find an intersection with a positive probability, the players need to describe an unbounded (asymptotically infinite) number of actions! Contrast this extensive communication with a simple authority protocol, where just one action needs to be described. As this informal argument shows, authority can only be improved upon by protocols which require infinitely longer communication.

4.2 *The Formal Result

We begin with stating two assumptions on the players' information structure:

Assumption 1. Each player's private information is fully described by his dictionary. More precisely, $\Theta_1 = \Theta_2 = \mathbf{D}_A$, and each player's information about the benefit of an action is fully determined by the place of this action in his dictionary.

For example, it may be common knowledge between a general and a foot soldier that defending a "strategic" position slows down the enemy's advancement, and that defending a "safe" position increases his chances of survival. But it is only in combat that the general finds out which positions are "strategic" and the soldier finds out which positions are "safe". Formally speaking, Assumption 1 says that uncertainty about benefits of different actions is purely permutational. If the players also acquired non-permutational private information, the players will communicate it and on its basis determine who should have authority. For example, if the general finds out that the all the positions are equally strategic, he will delegate the authority to choose the position to the soldier. Even though such message-contingent authority allocations are realistic, for simplicity we rule them out with Assumption 1.

According to Assumption 1, player 1's information about the benefit of an action is fully determined by the position this action has in his dictionary. Suppose that $|A| = n$, and define player i's component of the benefit of the action which has rank k in player i's dictionary as $v_i^n(k)$ (we keep the superscript n as we are going to vary the total number of actions). We assume that if the action a is chosen, the total benefit will be $U^n(v_1^n(d_1^{-1}(a)), v_2^n(d_2^{-1}(a)))$, where $U^n(\cdot, \cdot)$ is increasing in both arguments.

In order to rule out improvement by an uninformed outsider or by a randomization, we assume that for each player, most of the actions are "irrelevant", i.e. do not generate a significant improvement over the average. Formally, we make the following assumption:

Assumption 2.

1. $|v_i^n|$ are bounded;
2. $v_i^n(k) = 0$ for $k > k_i^n$;
3. $k_i^n/n \to 0$ as $n \to \infty$ (for $i = 1, 2$).

This assumption says that only the first k_i^n actions in player i's dictionary yield a positive benefit for player i. We will call such actions $a \in d_i(\{1, \ldots, k_i^n\})$ the *relevant* actions for player i. The proportion of relevant actions for each player goes to zero, and so does the probability that a randomly chosen action generates a positive payoff for either player.

For the sake of generality, we allow for randomized choice rules, i.e. those with outcomes from the space $\Delta(A)$ of probability distributions over A. The probability that action a is chosen when the two players' dictionaries are d_1, d_2 will be denoted by $f_a(d_1, d_2) \in [0, 1]$, with $\sum_{a \in A} f_a(d_1, d_2) \equiv 1$. The notion of neutrality naturally extends to such randomized rules:

Definition. A randomized choice rule $f : \mathbf{D}_A \times \mathbf{D}_A \to \Delta(A)$ is *neutral* if $f_{\pi(a)}(\pi d_1, \pi d_2) = f_a(d_1, d_2)$ for every $d_1, d_2 \in \mathbf{D}_A, \pi \in \mathbf{S}_A, a \in A$.

We assume that all dictionaries are equally likely for each player, and that the two players' dictionaries are independently realized. A neutral randomization (or an outsider) would result in $f_a(d_1, d_2) \equiv 1/n$. As $n \to \infty$, asymptotically this would result in an action which is irrelevant for either player with probability one. An authority protocol where player i describes $d_i(\arg\max_{[n]} v_i^n(k))$ (i.e. puts probability one on this action) yields an action which is optimal for player i, but asymptotically irrelevant for the other player with probability one. While such protocols have the advantage of being simple: their tree complexity is n, they clearly do not achieve the maximum expected benefit. Which protocols yield a higher expected benefit than authority?

It is clear that in order to improve upon authority, it is necessary to find an "intersection" - an action which is relevant for both players - with a positive probability. Finding such an intersection may require extensive communication:

Example 5 *Consider a "full revelation" protocol where player 1 describes all the actions which are relevant for him. Whenever player 2 finds that some of the actions relevant for player 1 are also relevant for him, he announces one of those actions. Under this protocol, player 1 can send* $\binom{n}{k_1^n}$ *possible messages. In the interesting case where the probability of intersection is asymptotically positive,* k_1^n *must be unbounded, therefore the tree complexity of this protocol cannot be bounded by a polynomial in* n*.*

It is natural to ask whether we can improve upon authority and find an intersection with a positive probability while using polynomially bounded protocols. It turns out that this may be possible if coordination is not a problem:

Example 6 *Let* $k_1^n = k_2^n = n/\sqrt{\log n}$*, and let* $A' \subset A$ *with* $|A'| = \log n$*. Then the probability that an intersection exists* A' *converges to one. Consider the protocol where player 1 first describes each of the actions from* A' *as "relevant" or "irrelevant". Whenever player 2 finds that some of* *the actions described as relevant by player 1 are also relevant for him, he announces one of those actions. Under this protocol, player 1 can send* $2^{|A'|} = n$ *possible messages, and player 2 can send* $|A'|$ *possible messages in response to each message by player 1. The tree complexity of this protocol is therefore* $n \log n$*, and the protocol asymptotically finds an intersection with probability one.*

Intuitively, the protocol in Example 6 requires a lot less communication than the "full revelation" protocol in Example 5, because player 1 only describes the characteristics of actions from A' which is asymptotically "infinitely smaller" than A. Therefore, by arbitrarily restrict the set of actions they are dealing with, the players they may be able to achieve the first best with only a slight increase in the amount of communication relative to authority protocols.

The problem with the protocol in Example 6 is that it is not coordinating, since its outcome depends on which actions fall into A'. Without knowing this, player 2 would be unable to figure out which physical actions player 1 is describing by the $|A'|$ bits transmitted. Given our assumptions coordination on the subset A' is impossible without prior communication. For example, before following the protocol in Example 4, player 1 could describe all the actions from A'. In order to do this, however, player 1 should be able to send $\binom{|A|}{|A'|} = \binom{n}{\log n}$ possible messages, which grows exponentially in n.

Our next result shows that this exponential complexity is unavoidable: coordinating protocols of polynomial complexity cannot improve upon authority. The reason is that asymptotically it is impossible to find an intersection with a positive probability using coordinating protocols of polynomial complexity:

Theorem 2 *Take a sequence of action sets* A^n *such that* $|A^n| = n$*, and a sequence of coordinating protocols* P^n *on* $\langle \Delta(A^n), \mathbf{D}_{A^n}, \mathbf{D}_{A^n} \rangle$*. Let* k *be a positive integer, and* $\gamma > 0$*. If* $\Gamma(P^n) \leq \gamma n^k$*, then the probability of finding an intersection by the protocol* P^n *goes to zero as* $n \to \infty$*.*[10]

[10]It is interesting to note that it is very easy to find intersection if non-deterministic communication is allowed (see Marschak (1995) for a definition of non-deterministic communication and its history in economics). If someone guesses an action from the intersection, it is easy to verify whether the guess is correct by announcing the action and asking each player to say "yes" if it is relevant for them, and "no" if it is not. This takes essentially the same amount of

Proof. See the Appendix.

The intuition behind the proof of Theorem 2 runs as follows. The parties start with a situation where all the actions are symmetric, and coordinating communication serves to describe (pin down) some of the actions. There are $\binom{n}{m}$ possible ways to describe m actions, and *a priori* all these ways are possible. Therefore, the tree complexity of describing m actions is $\binom{n}{m}$. Since the tree complexity is bounded by a polynomial of degree k, we should have $m \leq k$ for large enough n. (The case where $m \geq n-k$ can be ruled out.) In words, at most k actions can be described by the two players for a large enough n. Observe that when a player describes one action, the asymptotic probability that it is relevant for the other player is zero. Thus, if at most a finite number of actions is described, the probability of finding an intersection converges to zero.

Theorem 2 straightforwardly implies that asymptotically it is impossible to improve upon authority by protocols of polynomial complexity. Indeed, the maximum asymptotic payoff achieved by authority protocols is $\max\{V_1, V_2\}$, where

$$\begin{array}{l} V_1 = \limsup_{n\to\infty, k\in[n]} U^n(v_1^n(k), 0), \\ V_2 = \limsup_{n\to\infty, k\in[n]} U^n(0, v_2^n(k)). \end{array}$$

Theorem 2 implies the following corollary:

Corollary 1 *Take a sequence of action sets A^n such that $|A^n| = n$, and a sequence of coordinating protocols P^n on $\langle \Delta(A^n), \mathbf{D}_{A^n}, \mathbf{D}_{A^n} \rangle$, which implement choice rules f^n. Let let k be a positive integer, and $\gamma > 0$. If $\Gamma(P^n) \leq \gamma n^k$, then*

$$\limsup_{n\to\infty} E_{d_1,d_2} U^n(v_1^n(d_1^{-1}(f^n(d_1,d_2)), v_1^n(d_2^{-1}(f^n(d_1,d_2))) \leq \max\{V_1, V_2\}.$$

In words, coordinating protocols of polynomial complexity in the number of actions asymptotically cannot improve upon authority protocols.

Using the fact presented in Subsection 3.5, this result can be reinterpreted in terms of the number of information bits the parties need to exchange in order to improve upon authority protocols.

communication as authority. In our view, this example suggests that the non-deterministic notion of communication complexity is not appropriate for use in organization theory.

Remember that the tree complexity of an authority protocol is n, therefore the number of bits transmitted by such a protocol (the worst-case complexity) is $\log n$. If the tree complexity of a protocol is bounded by a polynomial of degree k in n, then the worst-case complexity is bounded by $\log \gamma n^k = \log \gamma + k \log n \sim k \log n$ as $n \to \infty$. In words, the number of bits transmitted by the protocol is asymptotically at most k times that for an authority protocol. If the parties communicate using a channel with a finite bandwidth, such a protocol asymptotically takes at most k times longer than an authority protocol. Conversely, if the tree complexity of a protocol cannot be bounded by a polynomial in n, asymptotically such a protocol takes infinitely many times longer than an authority protocol. The result of this section means that asymptotically the parties cannot improve upon authority without using such extremely long protocols.

5 Formal and Informal Communication

Our analysis oversimplifies communication in organizations in two ways. First, it does not distinguish between formal and informal communication. In practice, extensive informal communication in organizations coexists with very simple formal arrangements. Second, our analysis ignores strategic barriers to communication, resulting from the parties' incentives to misrepresent information. Both criticisms can be dealt with by extending our approach to situations where the parties' interests diverge, informal communication is cheap, and formal communication is costly. When the parties' interests diverge, the parties may find it beneficial to commit themselves to act in certain ways on the basis of messages they receive. For this commitment to be feasible, communication has to be verifiable by an honest outsider - in most practical cases, by a court. (Such verifiable communication is usually called a "message game".)

Despite the advantage of formal communication, in practice communication in organizations is mostly informal. Informal communication in organizations has been modeled as "cheap talk" (Aghion-Tirole, 1996) and renegotiation (Grossman-Hart, 1986). In absence of a cost differential between formal and informal communication, however, informal communication would always be dominated by formal and would never have to be used in equilibrium (this is the idea behind the "renegotiation-proofness" principle).[11]

We believe that it is the costs of formal communication relative to informal that explain why most communication in organizations is informal. An extension of our model which introduces divergence of interests and bounds the complexity of formal communication could explain why formal communication usually takes the simple form of allocating authority, and coexists with extensive informal communication, as in the models of Aghion-Tirole and Grossman-Hart.

6 Conclusion

If players are allowed to communicate without a bound before playing their game, they eventually establish "common knowledge of everything", and the coordination problem is resolved. We believe that assuming boundless communication is not only unrealistic, but that it contradicts the notion of organizations as systems designed for coordination of economic activity. In this paper we have modeled a coordination problem with barriers to communication, and we have shown that decision making in complicated situations requires the use of administrative authority. At the same time, we have given an example demonstrating that if coordination is not a problem, the parties can do at least as well as authority, and often much better, by using protocols which are only slightly more complicated. Therefore, understanding the coordination problem is both logically necessary and sufficient for explaining authority.

The model we have developed can be interpreted as a model where the set of possible actions cannot described in advance. The importance of such undescribability for the study of organizations has been clear since at least March and Simon (1958), who write:

> Of course, writers on organization theory are aware that coordination is a highly significant problem ... To fill this gap between formal theory and wisdom, we need a framework that recognizes that the set of activities to be performed is not given in advance, except in a most general way.

[11] The possibility of informal communication out of equilibrium may still provide constraints on achievable outcomes (e.g. renegotiation-proofness, collusion-proofness).

More recently, the incomplete contracts approach has been informally justified by the impossibility to describe all possible activities in advance. One interpretation of this has been through bounded rationality of the parties who cannot foresee all the possible contingencies and actions in advance. However, the difficulty in modeling unforeseen contingencies is that they seem to be in blatant contradiction with rational decision-making. Maskin and Tirole (1995) tried to reconcile "unforeseen contingencies" with individual rationality ("foreseen payoffs"). However, they found that in absence of barriers to communication, impossibility to describe all contingencies in advance does not put any restrictions on contracting by rational individuals who foresee their payoffs.

Our paper suggests why the set of possible actions could be "hard to describe" in advance without appealing to bounded rationality. While each individual in our model is unboundedly rational and can describe all the possible actions to himself, barriers to communication do not allow the parties to describe all the possible actions to each other (or to a court) in advance, thus establishing a focal public ordering. Therefore, even though the (infinite) set of all possible actions in our model is commonly known in advance, this is not of much help to the parties trying to design the optimal decision mechanism. In contrast to the analysis of Maskin and Tirole, barriers to communication not only make it hard to "describe" actions in advance, but also make it hard to "describe" actions ex post. The cost of communication makes authority, which only describes one action, the simplest coordination mechanism, and in certain cases rules out any improvement upon authority.

Our analysis suggests that decisions should be made by authority when the number of possible actions is large, communication opportunities are bounded, and the parties have not discussed all the possibilities in advance. On the other hand, if an organization repeatedly faces a fixed decision set, its members eventually develop a common structure ("dictionary") over the set of actions. This allows the organization to economize on communication by "routinizing" decision-making (March and Simon, 1958). In a simple situation where environment is constant over time, communication may be completely eliminated by choosing the same action over and over again. Where environment changes, the players could still substantially simplify communication by restricting attention to a set of previously discussed actions (as in Example 6). Authority will only be used when all the previously discussed actions are not admissible any more, and a non-routine decision has to be made.

It is instructive to compare our analysis to the traditional view of markets as coordinators of economic activity. One commonly proclaimed virtue of markets is that they coordinate activities and achieve efficient outcomes using relatively little communication (see, e.g., Hayek, 1945). For example, in order to verify that an allocation is first-best efficient, it suffices to announce the vector of prices of all commodities, so that everyone could check if this vector is proportional to his marginal utilities. However, even the announcement of prices becomes problematic when the number of all potential commodities becomes large. When prices are quoted and marginal utilities revealed only for the commodities with a positive trading volume, and only a small subset of commodities is traded in equilibrium, markets exhibit serious coordination failures (for a survey, see Matsuyama, 1995).

In our model, an action can be interpreted as a possible trade between the two players. A market can be modeled in a stylized way, for example, as player 1's fully revealing his valuations for all trades by quoting prices to player 2.[12] When the number of different trades is small or when all the possible trades have a natural ordering, market coordination may indeed be achieved with a small amount of communication. For example, when different trades are known to be different amounts of the same commodity, it is sufficient for the players to report their marginal valuations for the commodity. On the other hand, when the number of possible trades is very large and they are not ordered, market coordination requires an enormous amount of communication. In such situations, coordination by authority may be preferable.

[12]This abstracts from player 1's incentives to misrepresent his valuations.

Appendix: Proofs

Proof of Theorem 1.

Let f be the neutral choice rule implemented by P, and $\wp$ be the communication partition corresponding to P. Pick any dictionary $d \in \mathbf{D}_A$, and consider the following claims:

Claim 1. If $\Gamma(P) = |A|$, then we must necessarily have $\wp = \{T_1, ..., T_{|A|}\}$ with $f(T_i) = i$ for every $i \in A$.

This clearly follows from the proof of Lemma 1.

Claim 2. Let $f(d, d) = i$, and let $C_i = \{\pi \in \mathbf{S}_A | \ \pi(i) = i\}$ be the subset of the symmetric group keeping action i constant. Then $C_i d \times C_i d \subset T_i$.

Indeed, take any $(c_1, c_2) \in C_i \times C_i$. Since f is neutral, we must have

$$
\begin{aligned}
f(c_1 d, c_1 d) &= c_1 f(d, d) = c_1 i = i, \\
f(c_2 d, c_2 d) &= c_2 f(d, d) = c_2 i = i.
\end{aligned}
$$

Using Claim 1, this implies that $(c_1 d, c_1 d) \in T_i$ and $(c_2 d, c_2 d) \in T_i$. But since T_i is a rectangle, we must also have $(c_1 d, c_2 d) \in T_i$, which together with Claim 1 implies Claim 2.

Claim 3. Either $T_i \subset C_i d \times \mathbf{D}_A$ or $T_i \subset \mathbf{D}_A \times C_i d$. In words, the rectangle T_i containing $C_i d \times C_i d$ cannot extend both vertically and horizontally beyond $C_i d \times C_i d$.

Indeed, suppose in negation that neither inclusion holds. Then we have $\{d'\} \times C_i d \subset T_i$ and $C_i d \times \{d''\} \subset T_i$ for some $d', d'' \in \mathbf{D}_A \backslash C_i d$ (see Figure 5). Since T_i is a rectangle, we must also have $(d', d'') \in T_i$.

Let $d' i = j \neq i$, $d''_2 i = k \neq i$. Then by neutrality of f we have

$$f((jk)d', (jk)d'') = (jk)f(d', d'') = (jk)i = i,$$

and therefore by Claim 1 $((jk)d', (jk)d'') \in T_i$. But since T_i is a rectangle, we must also have $(d', (jk)d'') \in T_i$. On the other hand, $d' \in (ji)C_i d$ and $(jk)d'' \in (jk)(ik)C_i d = (ji)C_i d$. Then by neutrality of f we have

$$f(d', (jk)d'') \in f((ji)C_i d, (ji)C_i d) = (ji)f(C_i d, C_i d) = (ji)\{i\} = \{j\},$$

which contradicts $(d', (jk)d'') \in T_i$. Thus, the rectangle T_i containing $C_i d \times C_i d$ cannot extend both vertically and horizontally beyond $C_i d \times C_i d$.

For definiteness, suppose the rectangle only extends horizontally: $T_i = C_i d \times B$ with $C_i \subset B \subset \mathbf{S}_A$ (otherwise, we can switch players 1 and 2).

Claim 4. We must have $B = \mathbf{D}_A$.

Indeed, suppose in negation that $B \neq \mathbf{D}_A$. Then for some $c \in C_i$, $d' \in \mathbf{D}_A \backslash C_i d$ we have $(cd, d') \in T_j$ for some $j \neq i$, and therefore by Claim 1 $f(cd, d') = j$. Since f is neutral, this implies $f((ij)cd, (ij)d') = (ij)j = i$, and therefore by Claim 1 $((ij)cd, (ij)d') \in T_i$. But clearly $(ij)cd \notin C_i d$, therefore we have obtained a contradiction to the fact that $T_i = C_i d \times B$.

Claim 5. $\wp$ corresponds to the authority protocol where player 1 announces $d_1(i)$.

Indeed, from Claim 4 we have $T_i = C_i d \times \mathbf{D}_A$, and $f(C_i d \times \mathbf{S}_A) = \{i\}$. Since f is neutral, we must also have

$$f((ij)C_i d \times \mathbf{D}_A) = f((ij)C_i d \times (ij)\mathbf{D}_A) = (ij)f(C_i \times \mathbf{D}_A) = (ij)\{i\} = \{j\}$$

for any $j \in A$. Observe also that the sets $\{(ij)C_i d\}_{j \in A}$ constitute a partition of $\mathbf{D}_A$. Therefore, from Claim 1 we have $\wp = \{(ij)C_i d \times \mathbf{S}_A |\ j \in A\}$. This partition corresponds to the authority protocol where player 1 announces $d_1(i)$. ■

Proof of Theorem 2.

In order to simplify notation, we will number the set of actions, so that we can write $A^n = \{1, ..., n\}$. This allows us to define $\mathbf{S}_n = \mathbf{S}_{A^n} = \mathbf{D}_{A^n}$.

We introduce some useful space-saving notation:

- $[m] = \{1, ..., m\}$.
- $\mathbf{S}_n^2 = \mathbf{S}_n \times \mathbf{S}_n$.
- $d \in \mathbf{S}_n^2$ is equivalent to $(d_1, d_2) \in \mathbf{S}_n^2$.
- When $\pi \in \mathbf{S}_n$ and $d \in \mathbf{S}_n^2$, $\pi d = (\pi d_1, \pi d_2) \in \mathbf{S}_n^2$.
- When $B \subset \mathbf{S}_n^2$, B_1 and B_2 will denote the two projections of B.
- When $\wp$ is a partition of $\mathbf{S}_n \times \mathbf{S}_n$ and $\pi \in \mathbf{S}_n$, define the partition $\pi\wp = \{\pi T | T \in \wp\}$.
- $\Pr\{B|C\} = \frac{|B \bigcap C|}{|C|}$.

While the proof heavily uses some concepts of group theory, knowledge of basic definitions is sufficient to understand it. These definitions can be found in any textbook on group theory, see e.g. Kargapolov and Merzljakov (1979).

The concept of group theory that is most important for our proof is that of an orbit:

Definition. If G is a subgroup of $\mathbf{S}_n$, then the set $Gi = \{g(i)|\ g \in G\}$ is called *the orbit of G containing $i \in [n]$*.

It is easy to see that the orbits of a subgroup G form a partition of $[n]$, which we denote by $O(G) = \{Gi | i \in [n]\}$. The following result will be useful:

Claim 0. When G is a subgroup of $\mathbf{S}_n$ and $Y \subset [n]$, for every $i \in [n]$ we have

$$\Pr\{g(i) \in Y | g \in G\} = \frac{|Y \bigcap Gi|}{|Gi|}.$$

Proof: For every $j \in Gi$ there exists $\pi_{ij} \in G$ such that $\pi_{ij}(i) = j$, therefore we have

$$\Pr\{g(j) \in Y | g \in G\} = \Pr\{g\pi_{ij}(i) \in Y | g\pi_{ij} \in G\pi_{ij} = G\} = \Pr\{g(i) \in Y | g \in G\},$$

which enables us to write

$$\Pr\{g(i) \in Y | g \in G\} = \Pr\{g(j) \in Y | j \in Gi, g \in G\}. \tag{1}$$

On the other hand, for any fixed $g \in G$ we have

$$\Pr\{g(j) \in Y | j \in Gi\} = \Pr\{j \in g^{-1}(Y) | j \in Gi\} = \frac{|g^{-1}(Y) \bigcap Gi|}{|Gi|} = \frac{|g^{-1}(Y \bigcap Gi)|}{|Gi|} = \frac{|Y \bigcap Gi|}{|Gi|},$$

which enables us to write

$$\Pr\{g(j) \in Y | j \in Gi, g \in G\} = \frac{|Y \bigcap Gi|}{|Gi|}.$$

Combining with (1), we obtain the Claim.□

Our first task is to restrict attention to protocols where all the players' messages are neutral with respect to the names of actions. We define such protocols as follows:

Definition. An event $N \subset \mathbf{S}_n^2$ is a *node* of a communication protocol P on $\langle \Delta([n]), \mathbf{S}_n, \mathbf{S}_n \rangle$ if there is a node of the extensive-form game of P which is achieved if and only if the state of the world belongs to N.

As the argument in subsection 3.3 shows, every node N of a protocol is a rectangle, i.e. $N = N_1 \times N_2$.

Definition. A node $N \subset \mathbf{S}_n^2$ of a communication protocol P on $\langle \Delta([n]), \mathbf{S}_n, \mathbf{S}_n \rangle$ is neutral if for every $\pi \in \mathbf{S}_n$, πN is also a node of P. A communication protocol P on $\langle \Delta([n]), \mathbf{S}_n, \mathbf{S}_n \rangle$ is *neutral* if every its node is neutral.

Intuition suggests that if our objective is to implement a neutral choice rule, we need not transmit messages which depend on actions' names, and can therefore restrict attention to neutral protocols without any sacrifice in communication complexity. This is formally stated in the following lemma:

Lemma A.1. If a neutral choice rule f is implemented by a protocol P on $\langle \Delta([n]), \mathbf{S}_n, \mathbf{S}_n \rangle$, then it can also be implemented by a neutral protocol $\overline{P}$, with $\Gamma(\overline{P}) \leq \Gamma(P)$.

The lemma allows us to restrict attention to neutral protocols in the rest of the proof.

For every set $B \subset \mathbf{S}_n^2$, define $G(B)$ to be the set of permutations which keep B fixed, i.e.

$$G(B) = \{\pi \in \mathbf{S}_n | \pi B = B\}.$$

It is easy to see that $G(B)$ is a subgroup:

- $e \in G(B)$ trivially.
- $g \in G(B) \Rightarrow g^{-1}B = g^{-1}(gB) = B \Rightarrow g^{-1} \in G(B)$.
- $g_1, g_2 \in G(B) \Rightarrow (g_1 g_2)B = g_1(g_2 B) = g_1 B = B \Rightarrow g_1 g_2 \in G(B)$.

Observe also that any rectangle can be written as $B_1 \times B_2 = G(B_1 \times B_2)B_1 \times G(B_1 \times B_2)B_2$.

Let $L(G)$ denote the largest orbit from $O(G)$ (if there are many such orbits, choose any one of them). Define $l(\wp) = \min_{T \in \wp} |L(G(T))|$. Intuitively, for a terminal node T, $L(G(T))$ is the largest set of actions which an outside observer of the communication cannot distinguish between on the basis of the two players' messages. We will say that these actions are left "undescribed" by the players. Then $l(\wp)$ is the minimal number of actions which are left undescribed by the protocol $\wp$. Observe that there are $\binom{n}{m}$ ways to describe m actions out of n, and a priori all these ways are equally likely. Therefore, the number of possible outcomes of such communication (terminal nodes) is at least $\binom{n}{m} \sim \frac{n^m}{m!}$ as $n \to \infty$. Therefore, if the protocol's tree complexity is bounded by a polynomial of degree k, we must have $m \leq k$, i.e. at most k actions can be described. This is formally established in the next lemma:

Lemma A.2. If $\Gamma(P^n) \leq \gamma n^k$, for n large enough we must have $l(\wp^n) \geq n - k$.

Proof. Suppose in negation that there is a subsequence of protocols P^n and terminal nodes $T^n \in \wp^n$ (the communication partition of P^n) such that $|L(G(T^n))| < n - k$.

Let $G^n = G(T^n)$. Number the set $O(G^n)$ in an arbitrary fashion: $O(G^n) = \{M_i^n\}_{i=1}^{|O(G^n)|}$. Define the set

$$B^n = \begin{cases} L(G^n) \text{ if } |L(G^n)| > k, \\ \bigcup_{i=1}^{I_n} M_i^n \text{ otherwise, where } J_n = \max\{J : \sum_{i=1}^{J} |M_i^n| \leq \frac{n}{2}\}. \end{cases}$$

Observe that in the second case we must have $|n/2 - |B^n|| \leq \max_i |M_i^n| = |L(G^n)| \leq k$. Therefore, in both cases we must have $|B^n| \in [k+1, n-k-1]$ for n large enough. Also, in both cases B^n is a union of orbits of G^n, thus we must have $G^n(B^n) = B^n$.

Now, define $\Omega^n = \{C \subset [n] : |C| = |B^n|\}$. Observe that $|\Omega^n| = \binom{n}{|B^n|}$. For every $C \in \Omega^n$, we can construct a permutation $\pi_C \in \mathbf{S}_n$ such that $\pi_C(B^n) = C$.

Whenever C and C' are two different elements of Ω^n, we can write $\pi_C^{-1}\pi_{C'}(B^n) = \pi_C^{-1}(C') \neq \pi_C^{-1}(C) = B^n$. Therefore, we must have $\pi_C^{-1}\pi_{C'} \notin G^n$. By definition of G^n, this implies that $\pi_C^{-1}\pi_{C'}T^n \neq T^n$, which can be rewritten as $\pi_{C'}T^n \neq \pi_C T^n$. Since the protocol is neutral, we must have $\pi T^n \in \wp^n$ for all $\pi \in \mathbf{S}_n$. Therefore, $\pi_C T^n$ constitute different elements of $\wp^n$ for different $C \in \Omega^n$. This implies that

$$\Gamma(P^n) = |\wp^n| \geq \{\pi_C T^n | C \in \Omega^n\} = |\Omega^n| = \binom{n}{|S^n|} \geq \binom{n}{k+1} \sim \frac{n^{k+1}}{(k+1)!}$$

which contradicts the assumption that $\Gamma(P^n) \leq \gamma n^k$. ■

For every $d \in \mathbf{S}_n^2$, define $I(d) = d_1([k_1]) \bigcap d_2([k_2]) \subset [n]$, i.e. the set of intersections at d, and let $T(d)$ be the element of $\wp$ containing d. Now we are going to bound the probability that the described actions do not contain an intersection from below. Intuitively, consider a communication stage where u actions have not yet been described. When player 1 describes an action at this stage, the probability that it is also relevant for player 2 is at most k_2/u. Using the fact that at most $n - l(\wp)$ actions are described and that $u \geq l(\wp)$ at each stage, we can obtain the following lower bound on the probability that an intersection is not found among the described actions when both players can together describe at most $n - l(\wp)$ actions:

Lemma A.3. When $|l(\wp)| > n/2$, we must have

$$\Pr\{I(d) \subset L(G(T(d)))\} \geq 1 - \widehat{k}\frac{n - l(\wp)}{l(\wp)},$$

where $\widehat{k} = \max\{k_1, k_2\}$.

Proof. Consider the following claims:

Claim 1. For any node N of P, $G(N) = N_1 N_1^{-1} \bigcap N_2 N_2^{-1}$.

Proof:

- If $g \in G(N)$, then we can write

$$\begin{aligned} g &= ge \in g(N_1 N_1^{-1}) = (gN_1)N_1^{-1} = N_1 N_1^{-1}, \\ g &= ge \in \pi(N_2 N_2^{-1}) = (gN_2)N_2^{-1} = N_2 N_2^{-1}, \end{aligned}$$

  which implies that $g \in N_1 N_1^{-1} \cap N_2 N_2^{-1}$. Therefore, $G(N) \subset N_1 N_1^{-1} \bigcap N_2 N_2^{-1}$.

- If $g \in N_1 N_1^{-1} \bigcap N_2 N_2^{-1}$, then we can write $g = s_1 t_1^{-1} = s_2 t_2^{-1}$, where $s_1, t_1 \in N_1$, $s_2, t_2 \in N_2$. Then we can write

$$N \ni (s_1, s_2) = (gt_1, gt_2) \in gN.$$

  Thus, gN and N intersect. Since P is a neutral protocol, gN must be also a node of P. Two nodes of a protocol can only intersect if one lies inside another. But since the sizes of N and gN are the same, we must have $gN = N$, i.e. $g \in G(N)$. Therefore, $N_1 N_1^{-1} \bigcap N_2 N_2^{-1} \subset G(N)$.

Claim 2. When N is a node of P and $\pi \in \mathbf{S}_n$, we have $L(\pi G(N)) = \pi L(G(N))$.

Proof: By Claim 1,

$$G(\pi N) = \pi N_1 N_1^{-1} \pi^{-1} \bigcap \pi N_2 N_2^{-1} \pi^{-1} = \pi (N_1 N_1^{-1} \bigcap N_2 N_2^{-1}) \pi^{-1} = \pi G(N) \pi^{-1}.$$

Furthermore, it is easy to see that $O(\pi G \pi^{-1}) = \pi O(G)$ for every subgroup G of $\mathbf{S}_n$. Substituting $G = G(N)$, we find that $O(G(\pi N)) = O(\pi G(N) \pi^{-1}) = \pi O(G(N))$. This implies that $L(G(\pi N)) = \pi L(G(N))$.

Claim 3. When N and N' are two nodes of P such that $N' \subset N$, we must have $L(G(N')) \subset L(G(N))$.

Proof: Claim 1 implies that $G(T) \subset G(N') \subset G(N)$, where $T \in \wp$ is chosen so that $T \subset N'$. This implies that $O(G(T))$ is a refinement of $O(G(N'))$, which is in turn is a refinement of $O(G(N))$. This implies that $n/2 < l(\wp) \leq |L(G(T))| \leq |L(G(N'))| \leq |L((G(N)))|$. If we did not have $L(G(N')) \subset L(G(N))$, then the two orbits would not intersect, which would imply that $n \geq |L(G(N'))| + |L(G(N))| \geq 2|L(G(T))| \geq 2l(\wp) > n$ - a contradiction.

Claim 4. For every node N of P,

$$\Pr\{I(d) \subset L(G(T(d)))|I(d) \subset L(G(N)), d \in N\} \geq 1 - \widehat{k}\frac{|L(G(N))| - l(\wp)}{l(\wp)}.$$

Proof: By backward induction on the nodes of P. For a terminal node $T \in \wp$, we have $T(d) = T$ for every $d \in T$, and

$$\Pr\{I(d) \subset L(G(T))|I(d) \subset L(G(T)), d \in T\} = 1 \geq 1 - \widehat{k}\frac{|L(G(T))| - l(\wp)}{l(\wp)},$$

since by definition $l(\wp) \leq |L(G(T))|$. Thus, the statement is true for every terminal node.

Let N be a node of P, and let $\Pi(N)$ denote the partition of N into nodes which are immediate successors of N. Suppose the statement has been proven for every $N' \in \Pi(N)$. Since P is neutral, we must have $g\Pi(N) = \Pi(N)$ for all $g \in G(N)$, which enables us to write

$$\begin{aligned}
\Pr\{I(d) &\subset L(G(T(d)))|I(d) \subset L(G(N)), d \in N\} = \\
\Pr\{I(d) &\subset L(G(T(d)))|I(d) \subset L(G(N)), d \in N' \in \Pi(N)\} = \\
\Pr\{I(d) &\subset L(G(T(d)))|I(d) \subset L(G(N)), d \in gN', N' \in \Pi(N), g \in G(N)\} \geq \\
\min_{N' \in \Pi(N)} \Pr\{I(d) &\subset L(G(T(d)))|I(d) \subset L(G(N)), d \in gN', g \in G(N)\} = \\
\min_{N' \in \Pi(N)} \Pr\{I(d) &\subset L(G(T(d)))|I(d) \subset L(G(gN')), d \in gN', g \in G(N)\} \cdot \\
\cdot \Pr\{I(d) &\subset L(G(gN'))|I(d) \subset L(G(N)), d \in gN', g \in G(N)\} \qquad (2)
\end{aligned}$$

For any $g \in G(N)$ and $N' \in \Pi(N)$ we have $gN' \in \Pi(N)$, and using Claim 2, $|L(G(gN'))| = |gL(G(N'))| = |L(G(N'))|$. Using the inductive hypothesis, we can bound the first factor:

$$\begin{aligned}
\Pr\{I(d) &\subset L(G(T(d)))|I(d) \subset L(G(gN')), d \in gN', g \in G(N)\} \geq \\
&\geq \min_{g \in G(N)} \left(1 - \widehat{k}\frac{|L(G(gN'))| - l(\wp)}{l(\wp)}\right) = 1 - \widehat{k}\frac{|L(G(N'))| - l(\wp)}{l(\wp)}. \qquad (3)
\end{aligned}$$

As for the second factor, suppose for definiteness that player 1 moves at N under P. Define $R(d_2) = d_2[k_2] \bigcap L(G(N))$, and observe that whenever $R(d_2) \subset L(G(gN'))$ and $I(d) \subset L(G(N))$, we must have

$$I(d) = I(d_1, d_2) \bigcap L(G(N)) = d_1[k_1] \bigcap (d_2[k_2] \bigcap L(G(N))) \subset R(d_2) \subset L(G(gN')).$$

Therefore,

$$\begin{aligned} \Pr\{I(d) &\subset L(G(gN'))|I(d) \subset L(G(N)), d \in gN', g \in G(N)\} \geq \\ &\geq \min_{d_2 \in N} \Pr\{R(d_2) \subset L(G(gN'))|g \in G(N)\}. \end{aligned} \tag{4}$$

By Claims 2 and 3, $L(G(gN')) = gL(G(N')) \subset L(G(N))$ for all $g \in G(N)$. Now we can use Claim 0 to write

$$\begin{aligned} \Pr\{R(d_2) &\subset L(G(gN'))|g \in G(N)\} = \Pr\{R(d_2) \bigcap gL(G(N)) \backslash gL(G(N')) = \emptyset | g \in G(N)\} \geq \\ &\geq 1 - \sum_{i \in L(G(N)) \backslash L(G(N'))} (1 - \Pr\{g(i) \in R(d_2)|g \in G(N)\}) = \\ &= 1 - (|L(G(N))| - |L(G(N'))|) \frac{|R(d_2) \bigcap L(G(N))|}{|L(G(N))|}. \end{aligned}$$

Since $|R(d_2) \bigcap L(G(N))| = |d_2[k_2] \bigcap L(G(N)| \leq |d_2[k_2]| = k_2 \leq \widehat{k}$, and $|L(G(N))| \geq l(\wp)$, we can now rewrite (4) as follows:

$$\Pr\{I(d) \subset L(G(gN'))|I(d) \subset L(G(N)), d \in gN', g \in G(N)\} \geq 1 - \widehat{k} \frac{|L(G(N))| - |L(G(N'))|}{l(\wp)}.$$

Substituting this inequality and (3) into (2), we obtain the inductive statement for N.

Substituting the node $N = \mathbf{S}_n^2$ in the statement of Claim 4, we obtain the statement of the Lemma. ■

To complete the proof of the Theorem, we combine the results of the Lemmas A.2 and A.3. Lemma A.2 says that at least $n-k$ actions must be left "undescribed" for n large enough. Therefore, the proportion of actions relevant for either player among the *undescribed* actions goes to zero, and choosing an undescribed action achieves intersection with a vanishing probability as $n \to \infty$. To see this formally, for any terminal node $T \in \wp^n$ and any "undescribed" action $i \in L(G(T))$, we can write using Claim 0:

$$\begin{aligned} \Pr\{i &\in d_1[k_1] | d_1 \in T_1\} = \Pr\{g(i) \in gd_1[k_1] | gd_1 = d_1' \in gT_1 = T_1, g \in G(T)\} \leq \\ &\leq \max_{d_1' \in T_1} \Pr\{g(i) \in d_1'[k_1] | g \in G(T)\} = \max_{d_1' \in T_1} \frac{|d_1'[k_1] \bigcap L(G(T))|}{|L(G(T))|} \leq \frac{k_1}{|L(G(T))|}. \end{aligned}$$

Similarly, $\Pr\{i \in d_2[k_2] | d_2 \in T_2\} \leq \frac{k_2}{|L(G(T))|}$. Therefore, when $|\wp^n| \leq \gamma n^k$, we can use Lemma A.2 to write

$$\begin{aligned} \Pr\{i &\in I(d) | d \in T\} = \Pr\{i \in d_1[k_1^n] | d_1 \in T_1\} \cdot \Pr\{i \in d_2[k_2^n] | d_2 \in T_2\} \leq \\ &\leq \frac{k_1^n k_2^n}{|L(G(T))|^2} \leq \frac{k_1^n k_2^n}{l(\wp^n)^2} \leq \frac{k_1^n k_2^n}{(n-k)^2} \to 0 \text{ as } n \to \infty. \end{aligned}$$

We could still hope to find an intersection by implementing an action which has been *described* (i.e. $i \notin L(G(T(d)))$). However, Lemma A.3 implies that when at most k actions are described, the probability that they contain an intersection is also very small when n is large. Formally, using the above bound and Lemma A.3, the probability of finding an intersection by a choice rule f which is implemented by $\wp^n$ can be bounded from above as follows:

$$\begin{aligned} &\tfrac{1}{|\mathbf{S}_n^2|} \textstyle\sum_{i \in I(d), d \in \mathbf{S}_n^2} f_i(d) = \tfrac{1}{|\mathbf{S}_n^2|} \textstyle\sum_{i \in I(d) \subset L(G(T(d))), d \in \mathbf{S}_n^2} f_i(d) + \tfrac{1}{|\mathbf{S}_n^2|} \textstyle\sum_{i \in I(d) \not\subseteq L(G(T(d))), d \in \mathbf{S}_n^2} f_i(d) \leq \\ &\leq \tfrac{1}{|\mathbf{S}_n^2|} \sum_{T \in \wp^n} \sum_{i \in L(G(T))} f_i(T) \left|\{d \in T | i \in I(d)\}\right| + \tfrac{1}{|\mathbf{S}_n^2|} \textstyle\sum_{d \in \mathbf{S}_n^2 | I(d) \not\subseteq L(G(T(d)))} \displaystyle\sum_{i \in [n]} f_i(d) = \\ &= \tfrac{1}{|\mathbf{S}_n^2|} \sum_{T \in \wp^n} \sum_{i \in L(G(T))} f_i(T) \Pr\{i \in I(d) | d \in T\} |T| + 1 - \Pr\{I(d) \subset L(G(T(d)))\} \leq \\ &\leq \tfrac{1}{|\mathbf{S}_n^2|} \sum_{T \in \wp^n} \sum_{i \in [n]} f_i(T) |T| \cdot \tfrac{k_1^n k_2^n}{l(\wp^n)^2} + \widehat{k} \tfrac{n - l(\wp^n)}{l(\wp^n)} \leq \tfrac{k_1^n k_2^n}{(n-k)^2} + \widehat{k} \tfrac{k}{n-k} \to 0 \text{ as } n \to \infty. \blacksquare \end{aligned}$$

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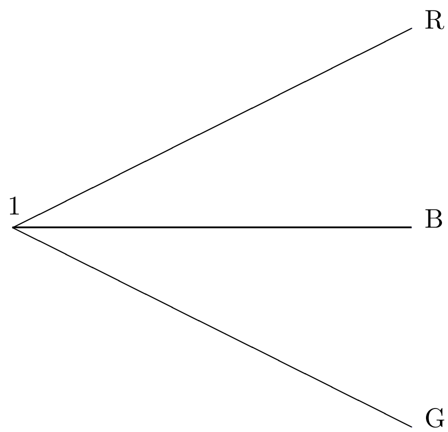

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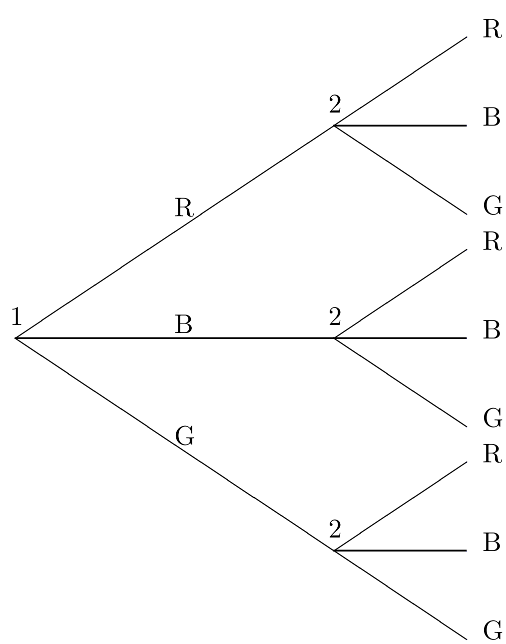

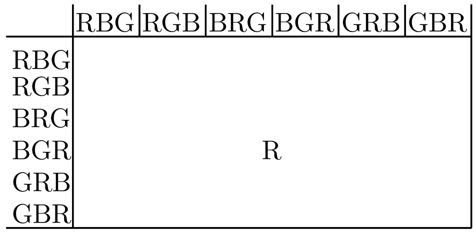
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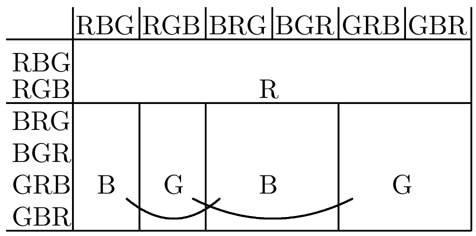
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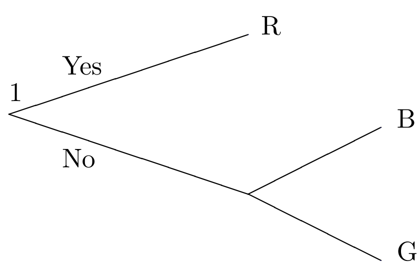
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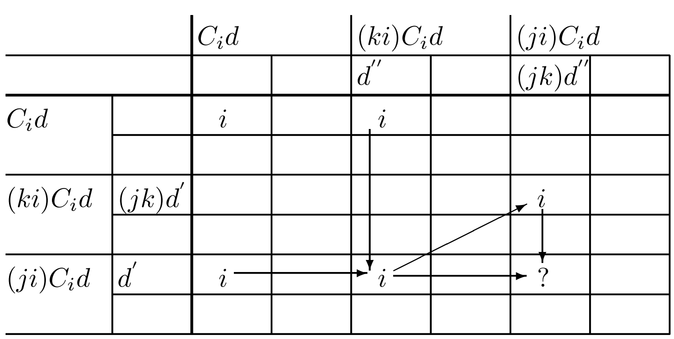
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